NVL ... NG SCHOOL MATHEMATICS

TEACHERS' EDITION

VESTIGATION DISCUSSION UTILIZATION

	Topic	Kindergarten	Book One	Book Two
		Units P and R	Units a, b, c, and d	Units e, f, g, and h
A	Sets, Logical Reasoning, and Patterns	Concept of sets: P13–14 Comparisons of sizes: P1–8, P11–12, P15 Similarities and differences: P17–32 Patterns: P9–10, P16 Maze: P32	Comparing sets: a1–9 Equivalent sets: a5–8 Cardinal number of a set: a13–14 Empty set: a19 Union of sets and addition: b1–13, b25–32, b41 Sets and subtraction: b15–23, b33–37, b45 Sets of ten: c1–4 Skip counting: d54, d62 Logic: a59–60, d42 Patterns: c14	Cardinal number of a set: e1–7 Maze: e40, g22 Sets and addition: e43 Sets and subtraction: e47 Counting sequences: e15, 62; f36; g6, 11, 13; h38 Informal logic: e52, 57; f55; g12, 34, 37; h20, 50 Patterns: f10 Attribute pieces: g23 Puzzle problems: g34, g62, h8 Multiplication and sets: g50–52, g55, g57
B	Numeration and Place Value	More than and less than concept: P33–37, P43–47 Matching, one-to-one correspondence: P37–42 Numerals and numbers 0 to 10: R1–7, R11–12, R17–32 Recognition of number of a set: R8–10, R13–16, R37–38	Introducing ten: c1–4 Grouping by tens: c5–13 Counting to 99: c15–26, d44 One more than: a9, a27, 29, 31, 33, 35 Greater than and less than: a41–49, d55–61 Numeral writing: a15–23, a27–36	Numeral writing: e3 2- and 3-digit numerals: e7–11, f13–18, g1–1 Counting to 99: e13–17, h62 Greater than and less than: e17–23, g12 Roman numerals: e32–39 Expanded notation: f49, g16–17 Regrouping: h33, h41–44, h52–56 Fraction numerals: h21–29
C	Addition and Subtraction of Whole Numbers	Addition combinations 1 to 10 (intuitive): R32–36, R39–44 Subtraction related to sums of 10 or less (intuitive): R45–46 Order of numbers 1 to 10: R48	Addition and subtraction facts, sums through 9: b1–51, c35–45 Zero principle for addition: b9 Inverse relation (+ and −): c57–60, d49–52 Missing addends: c53–56, d45–53 Sums of ten: d1–11 Addition and subtraction facts, sums greater than 10: d13–23, d35–43 Commutative (order) principle: d41, d51	Addition and subtraction facts, sums of 10 or less: e41–51, e59 Missing addends: e53–61 Inverse relation (+ and −): e57–58, h13–14 Basic principles: f1–9 Addition and subtraction facts, sums to 18: f11–35, g35–47, h1–19 Addition and subtraction without regrouping: f47–61, g15–21 Addition and subtraction with regrouping: h31–61
D	Multiplication and Division of Whole Numbers			Multiplication concepts: g49–61
E	Fractional Numbers		Introduction to fractions, $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$: d25–33	Halves, thirds, and fourths: h21–29
F	Problem Solving and Applications		Solving money picture stories: c47–51 Discussions: a2, 14, 26, 42; b2, 16, 26, 40; c16, 36, 46, 54; d2, 14, 26, 46, 56 Picture story for zero: a20	Money problems: e25–31, g14 Addition and subtraction problems: e60; f22, 32, 34, 54, 60; g20, 46, 60; h16, 18, 45, 58, 60 Multiplication problems: g59–60 Addition and subtraction problems with regrouping: h45, h58, h60 Discussions: e2, 26, 42; f48
G	Estimation			Introductory concepts leading to regrouping: h31, h39
H	Number Theory		Skip counting sequences: d54 Skip counting even numbers: d62	Odd and even numbers: e12 Skip counting: e62, f36, h38 Nomograph addition: f24 Square numbers: f62
I	Measurement		Concept of length: b53–57 Centimetres: b57–61 Length of paths: b62, d34 Readiness for area concepts: c62 Readiness for volume concepts: d24	Time: e33–39 Centimetres: f37–40 Perimeter: f41–42 Area: f43–44 Litres: f46 Volume: g48
J	Geometry, Number Line, and Graphs	Recognition of simple shapes: P17, P27, P28, P30, P48	Number line: a47; b11, 21, 29, 35, 43, 47; d5, 19–22, 48, 54 Recognition of basic shapes: a12, a50–54 Open and closed curves: a55 Segments and paths: a57 Same size figures: a59–61 Symmetry: d25	Number line: e6, 46, 49; f12, 16, 21, 29; g54, 58; h35 Triangles, squares, rectangles, circles: g23–26 Segments: g27–28 Congruence: g28–30 Similarity: g31–32 Graphing: h48
K	Special Topics		Telling time: c27–33 Money: c45–51 Let's have fun: a12, 24, 40, 50, 62; b14, 24, 38, 52, 62; c14, 26, 34, 44, 52, 62; d12, 24, 34, 44, 54, 62	Time: e33–39 Roman numerals: e32–39 Money: e25–31, g14 Magic squares: h8 Let's have fun: e12, 24, 32, 40, 52, 62; f10, 24, 36, 46, 62; g14, 22, 34, 48, 62; h8, 20, 30, 38, 48, 62

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Teachers' Edition to accompany

Investigating School Mathematics

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CHARLES R. FLEENOR

Collaborator, Teachers' Edition
THERESA BURKE

Collaborators, Reference Material and Metrication
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Foreword

The *Investigating School Mathematics* series co-ordinates the precise concepts of modern mathematics with an approach that stimulates the child to actively participate in his own learning experiences. The series provides for the necessary mastery of basic number skills, and presents the material in a way that emphasizes the exciting, creative nature of mathematics. As the child becomes involved in exciting explorations and investigations, the structure and beauty of mathematics unfolds. The children are encouraged to investigate and discover ideas for themselves, to look for interesting patterns and relationships, and to develop their own generalizations. New and fascinating topics are explored not solely for their mathematical value, but also because they stimulate interest and motivate children to put forth their best efforts.

In our view, the development of a sound mathematical structure need not be hindered by an exciting, activity-oriented approach. Rather, the activity approach can and should reinforce the child's experiences as he investigates mathematical topics in an orderly, structured fashion. The same, sound mathematical structure that was called "modern" in the 1960's is present in *Investigating School Mathematics*. The important difference in this new series lies in its approach. The child learns through continual active participation in activities and investigations that lead to the discovery of each new idea.

As each new concept unfolds, the child is given an opportunity to investigate the ideas by using a wide variety of manipulative materials and activities. Then, through guided discussion, he is led to a deeper understanding of the ideas and their relation to the overall structure of mathematics. Following the investigation and discussion, he is provided with sufficient problem-solving practice to develop speed and accuracy.

The *Investigating School Mathematics* series is unprecedented in its careful provision for individual differences. Throughout each text, the child is challenged to do what he *can* do, not what someone else *thinks* he can do. Each child has the opportunity to experience individual success in an environment that stresses co-operation and communication rather than competition. This careful provision for individual differences makes the *Investigating School Mathematics* series unusually adaptable to such diverse teaching situations as ungraded schools, individual or small-group instruction, or whole-class instruction.

The essence of the *Investigating School Mathematics* series is reflected in the beliefs to which we are committed: that there are fundamental mathematical concepts which can be isolated and set forth with sharpness and clarity; that these concepts, when truly understood, provide powerful tools for extending knowledge; that children of every level should be encouraged to actively participate, to think, to question, and to seek understanding; that, although a certain body of knowledge must be passed on to each generation from preceding generations, the individual creativity of each new generation must not be stifled by pedagogy which forces upon its pupils patterns of thought which have served us well in the past but which may be inadequate for the future.

Mathematics can be successfully taught in this spirit. At every stage in the learning of mathematics the discovery of new relationships can be a delight. It is in this spirit that *Investigating School Mathematics* has been written.

The authors wish to express their appreciation to Ball State University and to the Educational Research Council of Greater Cleveland, where many of the ideas were generated and tested for the *Elementary School Mathematics* series, which served as a forerunner of *Investigating School Mathematics;* to Edith Biggs and the Nuffield Project in England, for their leadership in bringing the activity-oriented laboratory approach into prominence; to Mrs. Nancy Hildebrand, whose contributions to the teachers' manuals for *Elementary School Mathematics* are still reflected in this manual; to Theresa Burke, who assisted in the preparation of this manual by bringing, from a wealth of classroom experience, many of the activities and teaching suggestions found in each lesson; and finally, to the many teachers and children who have proved that studying mathematics can be an exciting and stimulating experience in the elementary school.

Contents

Mathematics of the Book 5 Program

Before using Book 5 of this series in the classroom, you are urged to take the time to study *A Text for Teachers*, which comprises the last section of this Teachers' Edition. A careful study of this material will help you gain a better understanding and appreciation of the philosophy and the teaching strategy that underlie the mathematics of the *Investigating School Mathematics* program.

All of the important mathematical concepts that were covered in Book 4 of the series are reviewed, expanded, and reinforced in the Book 5 program.

In structuring Book 5, we have assumed that the children know addition and subtraction facts through 18 and multiplication and related division facts through 81. The children should also be able to handle effectively addition and subtraction with regrouping and should have some skill in finding products involving two-digit factors and in working long division with two-digit divisors. Although the preceding ideas are reviewed thoroughly, the review is brief. The following major ideas are introduced or expanded in Book 5.

Sets and logic: quantifiers (all, none, some); attribute cards; geometric and number patterns

Whole-number concepts: extension of the multiplication algorithm to three- and four-digit numbers; extension of long division to two- and three-digit divisors using the traditional division algorithm; estimation of products and quotients; place value through fifteen places; other numeration systems; averages; shortcuts for computing

Fractional-number concepts: equivalent fractions; lowest-terms fractions; inequalities; ratio and scale drawing; basic principles for addition and multiplication of fractional numbers; decimals; money and metric units; multiplication and division of fractional numbers

Number theory: prime factors; union and intersection of sets; greatest common factor; least common multiple

Geometry and measurement: points, rays, lines; congruent segments, angles, and triangles; measure of segments and angles; parallel and perpendicular lines; area and perimeter of polygons; symmetry; space figures; front, top, and side views of space figures; surface area and volume; co-ordinate geometry; graphing functions; rotations, enlargements, tessellations

Positive and negative whole numbers; on the number line; in graphing functions

Modular arithmetic: a "twelve clock"; addition, subtraction, and multiplication for a "twelve clock"; a "four clock"

A wide variety of story problems, optional exercises, challenging *Think* problems, flow charts, and logic problems are also integrated throughout the text to help the children improve their skills in problem analysis and solution.

Teachers' Bibliography

The books in the following list will provide a broader understanding of the philosophy which underlies the *Investigating School Mathematics* series.

Forbes, Jack, and Robert Eicholz, *Mathematics for Elementary Teachers* (Don Mills, Ont.: Addison-Wesley, 1971).

Biggs, Edith, and James MacLean, *Freedom to Learn: An Active Learning Approach to Mathematics* (Menlo Park, Calif.: Addison-Wesley, 1969).

Nuffield Mathematics Project, *I Do, and I Understand* (New York: John Wiley & Sons, Inc., 1967).

Dienes, Z. P., *Building Up Mathematics* (London: Hutchinson Educational Ltd., 1960; available from Humanities Press, New York).

Copeland, Richard, *How Children Learn Mathematics: Teaching Implications of Piaget's Research* (New York: The Macmillan Co., 1970).

The Schools Council, *Mathematics in Primary Schools* (Curriculum Bulletin No. 1; available from Selective Educational Equipment, Newton, Mass., 1964).

Boyer, Carl B., *A History of Mathematics* (New York: John Wiley & Sons, Inc., 1968).

Elliott, H. A., James MacLean, and Janet Jorden, *Geometry in the Classroom: New Concepts and Methods* (Toronto, Ontario: Holt, Rinehart and Winston of Canada, Ltd., 1968).

National Council of Teachers of Mathematics, *Insights into Modern Mathematics* (23rd Yearbook, 1957); *The Growth of Mathematical Ideas, Grades K-12* (24th Yearbook, 1959); *Enrichment Mathematics for the Grades* (27th Yearbook, 1963); *Topics in Mathematics for Elementary School Teachers* (29th Yearbook, 1964); *More Topics in Mathematics for Elementary School Teachers* (30th Yearbook, 1969), Washington, D.C.: National Council of Teachers of Mathematics).

Newman, J. R., *The World of Mathematics* (New York: Simon and Schuster, 1956).

School Mathematics Study Group, *Studies in Mathematics, Volume IX, A Brief Course in Mathematics for Elementary School Teachers*, Revised Edition (Stanford University: 1963).

Members of the Association of Teachers of Mathematics, *Notes on Mathematics in Primary Schools* (New York: Cambridge University Press, 1967).

Williams, E. M., and Hilary Shuard, *Elementary Mathematics Today, Grades 1–8,* (Menlo Park, Calif.: Addison-Wesley, 1972).

Teaching the Book 5 Program

Design Features of Book 5

Each lesson is titled with either a question or a provocative phrase inviting the child to explore a given idea. The core lessons of Book 5 are designed in one of two ways: those lessons which include an investigation activity begin on the left-hand page with the investigation, followed by a set of discussion exercises and then by a set of exercises on the right-hand page for the children to work independently. The titles of these sections are *Investigating the Ideas, Discussing the Ideas*, and *Using the Ideas*, respectively; other lessons are designed around a set of discussion exercises, titled *Discussing the Ideas*, and a set of independent exercises, titled *Using the Ideas*.

Many lessons throughout the book contain starred exercises for enrichment and a *Think* problem for the more able or interested children. Each chapter contains a chapter review (*Reviewing the Chapter*) and every chapter, except the first two, contains at least one cumulative review (*Keeping in Touch*).

Though each lesson in Book 5 is presented on facing pages, this format may be treated with considerable flexibility. You may find that for some lessons you will want to spend your entire allotment of time for mathematics on the Investigation and Discussion, saving the exercises for another day. In other lessons, however, you may find that you can cover as many as two facing-page lessons in one day. Keep in mind, though, that, in general, the core lessons are designed to be used for a single mathematics lesson in one day.

Color is used functionally throughout the text whenever it is felt that color-coding of numerals and symbols will facilitate learning and understanding of key concepts.

Teaching Strategies for Book 5

While specific teaching strategies will be made clear through your teachers' edition notes, there is a broad general plan for the teaching strategy throughout the book. The organization of the teachers' manual, as well as the material in the child's book, continually suggests this strategy. It is intended that each day's lesson in which the child is presented with a new concept be divided into these parts: Preparation, Investigation, Discussion, and Using the Exercises. The preparation usually should be kept fairly short, and care should be taken to see that this work does not preempt either the Investigation or the Discussion. Generally, the Preparation should do nothing more than provide the children with that readiness which they need before they begin the Investigation. The Investigation presents the rudiment of the concept treated in the lesson and should be the "main event" in terms of pupil activity and involvement in the unfolding of the concept.

In general, the Investigation should be done by the children either independently or in small groups. Think of the Investigation as a student-centred activity. It is fully anticipated that the students will grope, question, search, and explore. Most Investigations are designed to provide for individual differences; that is, the child is frequently asked to perform a certain task as many ways as *he* can, or to find how many ways he can do a certain thing. By presenting the child with this type of challenge, at least some degree of success is assured. That is, your slowest student will find that he can do something more than one way, while your more able children will find many ways to do a given task. Thus, as you guide the children through an investigation, it is important for you to recognize that they will achieve in widely differing ways, and that you should give recognition for all levels of achievement. Perhaps the most important thing to remember in working with the children during the Investigation is to encourage them to do the thinking and exploring. It is their section. Do not help them too much.

Following the Investigation, the children are given an opportunity in the Discussion section to talk about what they did and to summarize the mathematical ideas of the lesson in preparation for working independently in the Using the Ideas section. Generally, the beginning discussion exercises are designed to stimulate the children to talk about what they did in the Investigation. You should encourage them to discuss the various methods that they used to investigate and explore the concepts. Also, you should follow your teachers' guide carefully to be sure that whatever mathematical ideas are to be developed in the section are actually summarized and understood by the children.

The section titled *Using the Ideas* (called *Using the Exercises* in the teachers' edition) does exactly what the title implies. The children, having come through the first three parts of the day's lesson – the Preparation, the Investigation, and the Discussion – should be ready to work on their own to use the ideas of the lesson. Again, you should provide for individual differences in assigning work in this section; in other words, base your assignments on the needs and abilities of the children. The exercise sets in this section are generally graded, beginning with the easier exercises and ending with the more challenging (starred) problems. Often, there will be a challenging puzzle-type problem for a given lesson. From time to time, a fairly easy challenge problem is provided to encourage less able children to attempt it. Also, quite often, all of the children will benefit from a discussion of the challenge problem.

At the top of each left-hand page of the teachers' edition, under the heading *Objective(s)*, the goal of the lesson is stated in terms of what the child should be able to do as a result of the lesson. This objective

summarizes the key idea of the lesson in terms of the child's performance. Throughout the teaching suggestions for the Investigation and Discussion, the most important ideas of each lesson are reemphasized and clarified. It is important that you carefully consider the objective of each lesson so that you know how to direct the children through the development of ideas to the desired goal.

Following the statement of the objective for each lesson, the teaching suggestions follow the format of the general teaching strategy mentioned above. The Preparation lists any materials the child will need for the Investigation, and, occasionally, materials recommended for use by the teacher during the Discussion. The preparation section is designed to review ideas, motivate children, or give them necessary background information regarding the Investigation. Depending on your classroom organization, you may choose to use only those Preparations which are essential to the Investigation. At times it is recommended that you have children begin immediately with the Investigation.

The next three sections of the teaching suggestions correspond to the sections of the child's text:

Investigating the Ideas – Investigation
Discussing the Ideas – Discussion
Using the Ideas – Using the Exercises

Each section contains teaching suggestions related specifically to the corresponding section in the child's text. When lessons in the child's text deviate from the standard format, the teaching suggestions fall under the Discussion and/or the Using the Exercises heading.

A section titled *Mathematics* is included in certain lessons to provide background or to clarify for the teacher the principal mathematical concepts treated in the lesson. This section is strictly for the teacher; terms used here need not be used with the children other than as indicated in the teaching suggestions or in the child's text.

The *Follow-up* suggests various activities, games, or worksheets to help reinforce the concepts developed in the lesson. These are included only as suggestions to be used as your schedule allows. However, some of these activities and many of the materials recommended in the *Resources for Active Learning* may be used by the children independently at various activity or free periods throughout the day.

Since effective use of the investigative approach requires knowledge of the materials available for activity-oriented classrooms, suggestions are provided in the section titled *Resources for Active Learning*. The lists of resources in the chapter introductions and in most of the lessons offer an ample start. Provided some of these resources are available, you should be able to use or adapt the ideas contained in them to various situations.

The "Manipulative Devices" can be used to support the lessons. The "Commercial Games" and "General Activities" in the Chapter Introduction should be useful as continuous activities throughout the chapter, for review or practice of basic skills and concepts. The Resources listed for specific lessons are directly related to those lessons' objectives. Choose one or two, and try them out in a variety of situations.

At the time of this writing, the authors cited those resources which are of high quality and which directly complement the active-learning approach. Familiarize yourself with those materials which have been marketed subsequently. Check with your principal or supervisor for more recent materials to support this type of learning approach. Such materials can often be obtained from your local supplier. Additional sources are listed on page xi.

Provision for Individual Differences

Minimum, average, and maximum assignments are provided for each lesson other than review lessons. These assignments are given to assist you in providing for the individual needs of the children. It is *not* intended that you give the minimum assignment to the slower children, the average assignment to the average children, and the maximum assignment to the more able children. Rather, these designations are given to assist you in making individual assignments according to needs, abilities, and time available for each individual child. For example, if time is short and you need to move rapidly through a particular lesson, you may choose to use the minimum assignment for all children. The minimum assignment will, in general, provide the children with sufficient practice and mastery of skills to move ahead to the next lessons. On the other hand, you may sometimes choose to use the maximum assignment with slower children over a period of two or three days. Also, it is highly likely that you will not want to assign the maximum assignment to the more able children, since quite often they need less practice than some average and below average children. For example, when your more able children demonstrate the ability to perform a particular skill with great efficiency, they should not be made to drill excessively in that skill. In some cases, an asterisk is placed beside an assignment to indicate that the lesson could be omitted without loss of continuity in the flow of ideas.

Long-Range Planning Chart

The long-range planning chart is designed to provide the teacher with some guidelines for planning basic, average, and maximum course coverage of *Investigating School Mathematics*.

The basic course outline covers all the essential parts of the program, but provides little in-depth or extension material. The average course covers all the material of the basic course plus considerable extension material. The maximum course provides for nearly total

LONG-RANGE PLANNING CHART						
Chapter	**Basic Course**		**Average Course**		**Maximum Course**	
1	4-13, 18	1½ weeks	4-13, 16-19 14-15	1½ weeks	4-19	1½ weeks
2	20-29, 39 30-31, 36-37	2½ weeks	20-31, 36-37, 39 32-33	2 weeks	20-33, 36-39 34-35	2 weeks
3	40-57, 62-67	3 weeks	40-57, 62-67 58-61	3 weeks	40-67	3 weeks
4	68-81, 88-95	3 weeks	68-81, 88-95 82-87	3 weeks	68-95	3 weeks
5	96-105, 110-113 108-109	2 weeks	96-105, 110-113 106-109	2 weeks	96-107, 110-113 108-109	2 weeks
6	114-125, 132-137, 140-141, 144-150	5 weeks	114-127, 132-141, 144-150 128-131	4 weeks	114-141, 144-150 142-143	4 weeks
7	155-157, 160-163, 166-169	2½ weeks	152-157, 160-169 158-159	2 weeks	152-169	2 weeks
8	170-173, 178-185 174-175	3 weeks	170-175, 178-185 176-177	1½ weeks	170-185	1½ weeks
9	186-195, 204-207 203	2½ weeks	186-197, 204-207 198-203	2½ weeks	186-207	2½ weeks
10	208-215, 222-225 216-217	2 weeks	208-217, 222-225 218-219	2 weeks	208-219, 222-225 220-221	2 weeks
11	226-233, 236-239, 248-251 241-243	4 weeks	226-239, 244-245, 248-251 240-243	3 weeks	226-245, 248-251 246-247	3 weeks
12	252-258, 264-267 260-261	3 weeks	252-261, 264-267 262-263	2 weeks	252-267	2 weeks
13	268-271, 276-277, 286-287	1 week	268-273, 276-282, 286-287 274-275	2 weeks	268-279, 284-287 280-283	2 weeks
14	288-291, 298, 300-301	1 week	288-295, 298-301 296-297	1½ weeks	288-301	1½ weeks
15			302-315, 320-321 316-317	2 weeks	302-317, 320-321 318-319	2 weeks
16			322-325, 328-331	2 weeks	322-325, 328-332 326-327	2 weeks

coverage of all topics presented in the text. Optional material for each of the three courses is shown by page numbers shaded gray.

The suggested time schedule, covering 36 weeks, should be viewed only as an *aid* in helping you plan a time schedule that allows for the individual differences within your own class. You should not view it as a rigid schedule. For example, the basic course may be used with children who are achieving below the average grade level. This same course might be used in other ways, such as with a brighter child who may move rapidly through these same pages to make up for time lost through an absence.

General Suggestions

We offer two general suggestions regarding use of the chapter and page lesson notes:

(1) Read and consider each point as it applies to the immediate objectives for the lesson and the overall objectives of the unit.

(2) Do not allow the teachers' manual notes to deter you from using your own effective teaching methods or to stifle creative efforts.

Your manual does not attempt to dictate all of the activities in the

day-to-day handling of your class and the individuals in it. You should use your manual as a guide to be co-ordinated with those methods which you have found to be most effective in teaching mathematics in the past. One of the key techniques in presenting a structured system of mathematics consists of developing a topic and pursuing it until you have reached a desired level of understanding with regard to that topic. This is one of the guiding philosophies behind the development of *Investigating School Mathematics*. You will notice this philosophy particularly in the development of some of the more intense sections of work. That is, when the topic is being introduced, it is explored in great detail; however, we have tried to provide interesting activities to relieve the intensity of these longer sections. Treat the relief materials with a light touch and in the spirit of having fun and playing games with mathematics.

Classroom Organization

The Book 5 program can be used in situations where the entire class works together on the same lesson, where small groups work together on the same lesson, or where individual students are allowed to proceed at their own rate of speed. You will want to employ the type of classroom organization that best suits the physical facilities of your particular situation. Teachers often find it stimulating to vary group sizes for different lessons and units of work. Smaller-size groups can often work more effectively together and allow greater opportunity to participate in the Investigation and the Discussion. Whatever class organization you choose, keep in mind that the key to the *Investigating School Mathematics* program is active student participation.

Evaluation of Progress

A child's attitude toward mathematics is often influenced by the methods used for evaluating his progress. All too often, evaluation procedures focus attention on what the child *did not* understand or master, rather than on what the child *did* accomplish. In evaluating a child's progress, try to maintain a positive view, one which capitalizes on successes and develops confidence.

Achievement and diagnostic tests for Book 5 may be obtained from the publisher. Chapter reviews as well as cumulative reviews are provided in the text, to aid in evaluation of the child's progress. However, you may find that a day-to-day evaluation of the children, often involving personal interviews, will help you determine how well a child grasps the concepts and how well he is able to apply them.

Children's Bibliography

Books listed in the Books to Explore section of the pupils' text (pages A30–A32) were selected to appeal to the interests of children, as well as to provide historical background and reference material.

Additional books which may be of interest to you and the children are listed below.

Adler, Irving, *Magic House of Numbers* (New York: John Day, 1957).

Bendick, Jeanne, *Names, Sets, and Numbers* (New York: Franklin Watts, 1971). Sets, numbers, and classifying (science)

Berger, Melvin, *For Good Measure* (New York: McGraw-Hill Book Co., 1969).

Boeke, Kees, *Cosmic View: The Universe in 40 Jumps* (New York: John Day Co., 1957).

Diggins, Julia E., *String, Straightedge, and Shadow* (New York: Viking Press, 1965).

Feravolo, Rocco, *Wonders of Mathematics* (New York: Dodd-Mead, 1963).

Frédérique and Papy, *Graph Games* (New York: Thomas Crowell, 1971). A Young Math Book. Sets and relations

Froman, Robert, *Faster and Faster: A Book About Speed* (New York: Viking Press, 1965).

Hertzberg, Hendrik, *One Million* (New York: Simon & Schuster, 1970). Number

Kaplan, Philip, *Posers* (New York: Harper and Row, 1963).

Kettlekamp, Larry, *Kites* (New York: Morrow, 1959). Geometry and measurement

Kohn, Bernice, *Computers at Your Service* (Englewood Cliffs, N.J.: Prentice-Hall, 1962).

Krahn, Maria and Fernando, *The Life of Numbers* (New York: Simon and Schuster, 1970).

Razzell, Arthur, and K. G. O. Watts, Exploring Mathematics Series (Garden City, New York: Doubleday & Co.). *Circles and Curves* (1969); *Probability—The Science of Chance* (1967); *A Question of Accuracy* (1969); *Symmetry* (1968); *This is 4: The Idea of a Number* (1967); *Three and the Shape of Three* (1969).

Shulman, Alix, *Bosley on the Number Line* (New York: McKay Co., 1970). Adventure story

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Thurber, James, *The Great Quillow* (New York: Harcourt Brace Jovanovich, 1944). A fantasy utilizing number and measurement concepts. *Many Moons* (1943).

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Bibliography of Resources for Active Learning

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Wirtz, Robert, Morton Botel, and B. G. Nunley, *Discovery in Elementary School Mathematics* (Chicago: Encyclopaedia Britannica Educational Corp., 1963).

Suppliers of Resources for Active Learning

Addison-Wesley, Don Mills, Ontario
CCM School Materials, Inc., Chicago
Childcraft Education Corp., New York
Concept Co., Belmont, Massachusetts
Creative Playthings, Princeton, New Jersey
Creative Publications, Palo Alto, California
Edmund Scientific Co., Barrington, New Jersey
Educational Teaching Aids, Chicago
Encyclopaedia Britannica Publications Ltd., Toronto, Ontario
Gamco Industries Inc., Big Spring, Texas
Geyer Instructional Aids, Fort Wayne, Indiana
J. L. Hammett Co., Braintree, Massachusetts
D. C. Heath Canada, Ltd., Toronto, Ontario
Herder and Herder, Inc., New York (Available from Methuen Publications, Agincourt, Ontario
Holt, Rinehart and Winston of Canada, Ltd., Toronto, Ontario
Ideal School Supply Co., Oak Lawn, Illinois
Imout, Cleveland, Ohio
Kindrey Mfg. Co., Palo Alto, California
Lakeshore Equipment Co., San Leandro, California
James W. Lang, Mound, Minnesota
LaPine Scientific Co., Chicago
The Macmillan Co., New York (Available from Collier-Macmillan, Don Mills, Ontario)
Math Media Inc., Danbury, Connecticut
Midwest Publications Co., Inc., Birmingham, Michigan
Milton Bradley, Springfield, Massachusetts
Nasco, Fort Atkinson, Wisconsin
Responsive Environments Corp., Englewood Cliffs, New Jersey
Scott, Foresman and Co., Glenview, Illinois (Available from Gage Educational Publishing Ltd., Agincourt, Ontario)
Selective Educational Equipment, Newton, Massachusetts
Sigma Division, Scott Scientific Inc., Fort Collins, Colorado
TUF, Rowayton, Connecticut
Walker Educational Book Corp., New York (Available from Fitzhenry and Whiteside, Ltd., Don Mills, Ont.)
Webster Division, McGraw-Hill Ryerson Ltd., Scarborough, Ontario
Wff 'N Proof, New Haven, Connecticut
World Wide Games, Delaware, Ohio

PAGES 2–3

Objective

Given a typical lesson in the Investigating School Mathematics *text, the child will be able to distinguish its three principal parts: Investigating the Ideas, Discussing the Ideas, and Using the Ideas.*

Preparation

This first step of the overall teaching plan for each lesson in the text is the briefest. However, it will often be the key to inspiring children with an enthusiastic approach to the lesson. In some lessons, it is essential to the investigation; in other lessons, it simply serves as a review. In most cases, the preparation should be limited to a maximum of five minutes. Brisk and lively, it should ensure that each lesson is begun with a positive attitude.

For this introductory lesson, you might simply encourage the children to flip through the pages of the book, stopping to look at whatever interests them. After one or two minutes, refer them to the title and author page and the table of contents, asking them which chapters they think they will enjoy most. Finally, refer them to the first introductory lesson.

Investigation

As stated in the introductory material, the investigation presents the child with an opportunity to explore, often with minimum teacher direction, the ideas presented. Sometimes this exploration consists of an investigation with concrete objects; at other times, the child simply uses paper and pencil to explore ideas with number. In any case, this section should be totally child-centred. It is the child's opportunity to explore new ideas, or known ideas in a new way; it launches the lesson. The attitude to be fostered is that the child cannot make a mistake. Since he will often learn as much from his "incorrect" approaches as from finding the correct methods, he should be encouraged to explore things on his own.

Let's explore your mathematics book.

Investigating the Ideas

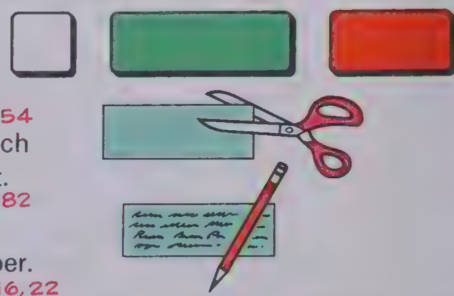
This is a sample lesson to help you understand how to use your book. In this part of a lesson you will find things to **investigate** and discover.

? Can you find Investigations in your book like the ones below?

A Find an "Investigating the Ideas" section that uses colored strips.
Sample answers: pages 52, 54

B Find an Investigation in which you must cut something out.
Sample answers: pages 78, 82

C Find an Investigation which requires only pencil and paper.
Sample answers: pages 12, 16, 22



Discussing the Ideas

In this part of a lesson you will **discuss the ideas** of the lesson with your classmates and teacher. You will share your ideas with others. You will be getting ready to use the ideas.

1. Does each "Investigating the Ideas" section contain a key question for you to answer? Yes

2. A What color do you find on the border of the page beside each Investigation section? A shade of blue
   B What color is the strip next to the "Discussing the Ideas" section? A shade of green

3. Is there always a "Discussing the Ideas" section following an "Investigating the Ideas" section? Yes

In this introductory lesson, you will probably want to read with the children the paragraph at the top of the page. Explain that you want them to read and answer questions A, B, and C by themselves. Move around the room giving encouragement to children who find the required sections and suggesting that they look for more. Stress the importance of recording their answers even when the text does not explicitly request it.

Discussion

The discussion section provides an opportunity for children to discuss the ideas they investigated and for you to guide the discussion so that the emphasis is on the main point of the lesson. It is in this section that you will sometimes want to demonstrate various computational skills or use visual materials to highlight a particular concept. The questions in the text help the children focus on the most important concepts, but your skill in guiding discussion and presenting ideas will be of inestimable value in this section.

Notice that discussion questions

Using the Ideas

In this part of a lesson you will be **using the ideas** that you investigated and discussed on the opposite page. You will work problems to improve your understanding of those ideas. Try the problems below.

1. What color is the strip beside the "Using the Ideas" problems? A shade of red
2. How many "Investigating the Ideas" sections are in Chapter 3? 10
3. Look up the word **angles** in the index. What page numbers are given? Pages 75-79
4. How many special flags referring you to additional exercises can you find in Chapter 5? 4
5. On page 67 you are invited to explore one of the "Mathematical Activities." Find the activity. Activity Card 3, page 334
6. Is there a **glossary** in your book? Find it and look up the meanings of some words you choose. Yes; choices will vary.

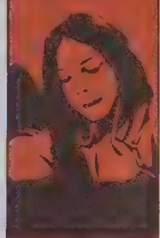
Problems in these boxes are special challenge problems for you. Be sure to try some of them. See if you can do this one.

think

Make four copies of this region and cut them out.

Can you fit the four pieces together to form a square?

3



1 and 2A give children an opportunity to talk more about the investigation sections of their texts. Point out that every investigation section is followed by a discussion section. However, in some lessons no investigation precedes the discussion section.

Using the Exercises

The section in the child's text entitled *Using the Ideas* is referred to in the teacher's manual as "Using the Exercises." As indicated by its title, this section is intended to provide the children with an opportunity to use the ideas they have investigated and discussed. Ordinarily, children should be expected to work by themselves for this section. Occasionally, however, you might want to use a few of the exercises as basis for introduction, directions, and discussion. Exercises on these pages may be assigned selectively; it is rarely essential that each child do all of the exercises. However, on many pages the exercises progress developmentally so children should do parts of each exercise, excepting those which are starred. Starred exercises extend the concept of the lesson or present more difficult applications of it. They are not intended for all children.

Explain to the children the purpose of this section and assign the exercises. Due to the amount of reading in some of these sections, you might find it necessary to read through the material with some children. As explained in the text, the *Think* problem is a special challenge. Encourage children to try these, even though they are intended primarily for enrichment.

CHAPTER 1 | Sets, Logic, and Patterns

Pages 4–19

General Objectives

To provide an interesting and different beginning to the study of mathematics

To provide a variety of opportunities for mathematical thinking, such as work with informal logic, discovery of patterns, and special attributes and categorizations of sets

One of the chief purposes of this chapter is to get school started in an interesting and stimulating way. The children are not expected to master all of the mathematical ideas that are presented in the chapter; rather, this material should be treated with a light touch. The intention is for the children to have fun with the ideas of the chapter rather than to attempt to master them. Hopefully, the positive attitudes which this chapter is intended to develop will prevail throughout the school year, and will continually reflect a realization of the fact that mathematics can be fun and that there are many mathematical ideas and concepts which can profitably be explored and discussed without specific attempts at memorization or mastery.

Mathematics

A comprehensive treatment of the mathematics in this chapter would require considerably more space than is available in this mathematics section. A number of topics concerning sets, logic, and patterns are explored in an informal way. Hence, rather than attempting in this introduction to present a comprehensive mathematics section for all of the diverse topics of this chapter, a brief mathematics section is presented in each lesson that introduces a new mathematical idea, to assist you in understanding the ideas as they are presented to children. It is not essential that you completely master all of the logical ideas that are presented informally to the children in this chapter. Actually, sufficient mathematical background can be attained from a careful study of the ideas as they are presented on the child's page.

Teaching the Chapter

Materials

Crayons, red and blue

Index cards, small (at least 22 per child)

Yarn, red, 75-cm pieces (1 per child); blue, 75-cm pieces (1 per child)

Vocabulary

input	sequence
logic	set
output	transformation
pattern	

Although the manipulative materials needed for this chapter are relatively few, the investigations should provide stimulating and rewarding experiences for the children. It is important that your enthusiasm be apparent to the children and that you motivate them to enjoy thinking. The interest and enthusiasm sparked by this first chapter should set the tone for the entire year. It also presents an opportunity to prepare the children for the independent thinking which they should develop. By having these first few lessons well organized, you can start the children on the path toward an exciting year of investigation and discovery. However, the phrase "well organized" is not intended to suggest that you will be continuously directing the activities of the children. Rather, it implies that you understand the goal of the lesson, acquire and prepare the materials necessary, ensure that the children realize what the primary investigation question is, and then give them the freedom to explore it. If this attitude of investigative freedom is new to you or to the children, be patient with yourself and the children as you become accustomed to a more open-ended approach to teaching mathematics.

Lesson Schedule

Plan to spend approximately two weeks on this chapter. If children have found any of the topics particularly interesting, you might suggest that they continue with some of the activities as follow-up or free time projects. Keep in mind that one of the main purposes of this chapter is to stimulate interest in such topics. Thus, children should be encouraged to continue working with them if they choose.

Evaluation of Progress

Any evaluation of children's mastery of the ideas of this chapter should be informal. Observation of the children's responses to the various investigations should indicate which children need particular help in building a positive attitude both toward the study of math and toward themselves as learners. Also, children who demonstrate a highly developed ability to reason and think logically should be challenged to continue with similar problem-solving activities, or other games with the attribute pieces.

Resources for Active Learning

GENERAL ACTIVITIES

[The first five references below are games of logic and patterns.]

Experiments in Mathematics, Stage 2, "3-D Noughts and Crosses," p. 52; "Game of Hex," p. 53, Houghton Mifflin (Available from Thomas Nelson and Sons)

Math Activity Cards, "Special Mill," C45; "Switcheroo Function," D44, Macmillan

Maths Mini-Lab, Cards 135, 136, Selective Educational Equipment

Notes on Mathematics in Primary Schools, "Squares," p. 233; "Hex," pp. 234–235; "Go," p. 235; "Nim," p. 236; "Lucas' games," pp. 237–238; Cambridge University Press (Available from Macmillan of Canada)

SMSG: *Puzzle Problems and Games Project,* "Nim-Type Games," pp. 13–20, Stanford University

Madison Project: *Explorations in Mathematics:* A Text for Teachers, "Logic," pp. 118–160, Addison-Wesley

Nuffield Project; *Problems*—Red Set, Nos. 9, 9A, Wiley

Math Activity Cards, C39–43, Macmillan [Puzzles and patterns]

Nuffield Project: *Computation and Structure* 3, pp. 48–55, Wiley [Puzzles and patterns]

Manipulative Devices

Attribute Games and Problems (Selective Educational Equipment; Webster, McGraw-Hill)

Dienes Logical Blocks (Herder and Herder)

Pattern Blocks (Selective Educational Equipment; Webster, McGraw-Hill)

Setsplay (Selective Educational Equipment)

Sigma Chips (Sigma, Scott Scientific)

Commercial Games

Beeline (Selective Educational Equipment)

Haar Hoolim Perception Games (Selective Educational Equipment)

Hindu Pyramid Puzzle (Tower of Hanoi) (Creative Publications; World Wide Games)

Jumpin' (Selective Educational Equipment)

Kalah (Oh-Wah-Ree) (Creative Publications; Math Media)

Math Games and Puzzles Kit (Concept Co.)

Mem (Math Media; Selective Educational Equipment)

Nine Men Morris (World Wide Games)

On-Sets (Nasco; Wff 'N Proof)

Psyche-Paths (Cuisenaire Co.)

Shuttle Puzzle (World Wide Games)

Think-a-Dot (Childcraft; Cuisenaire Co.)

Tri-Nim (Childcraft; Gamco; Wff 'N Proof)

Wff 'N Proof (Childcraft; Wff 'N Proof)

PAGES 4–5

Objective

Given a set of objects containing some elements with a common property, the child will be able to identify those objects which have the common property.

Preparation

Your treatment of these first few pages of the text will generate the mood which is likely to prevail in the classroom throughout the entire year. Thus, in this first lesson, show enthusiasm in introducing the chapter, motivating the children for some *fun* with *thinking*. Move quickly into the investigation, for it should serve to stimulate children's interest.

Investigation

Children would benefit from discussing this investigation in small groups. Direct them to read through the investigation section only, and to see if they can figure out which cards are in Eric's set. It may take some children quite a bit of discussion to realize that the only acceptable objects are those on which a circular shape appears. As you move around the room, encourage creative thinking. Avoid giving answers to children's questions too quickly. Stimulate thought with questions of your own, such as: "What is alike about all the objects in the first set? in the second set?" "Would a waste paper basket be in Eric's set?"

1 Sets, Logic, and Patterns

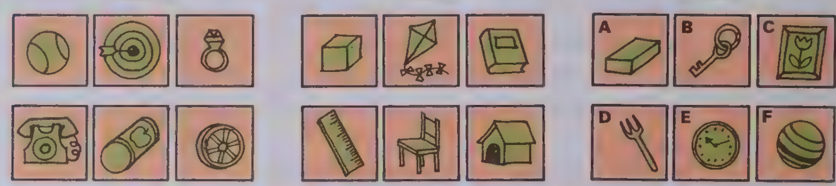
● *Let's explore sets.*

Investigating the Ideas

Imagine that there is a giant stack of cards with a different object or symbol on each one.

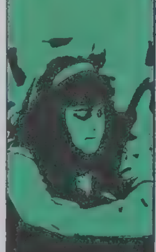
Eric is thinking of a special set of the cards.

Clue number 1	Clue number 2	Question
All of these are in Eric's set.	**None** of these are in Eric's set.	Which of these are in Eric's set?



? Can you use the clues above to tell which cards are in Eric's set? B, E, F (Cards picturing objects on which a circular shape appears)

Discussing the Ideas



1. What are some other objects or symbols that might be on the cards in Eric's set? Sample answers: balls of all kinds, globe, spools, circular plates (any circular object).
2. How can you describe the objects in Eric's set? All of them are circular or have circular parts.
3. Can you explain why none of the objects in the set under Clue number 2 are in Eric's set? None are circular or have circular parts.
4. Choose a set of objects and describe it to your classmates. Answers will vary. See Discussion.

4

Discussion

Although, mathematically, "set" is undefined, there are ways by which we can help children build up an intuitive notion of set. For example, we can think of a set as a collection of objects about which we can determine whether a particular object is or is not in the set. This is precisely what the children did in the investigation. Any object on which the shape of a circle appears is in Eric's set; any object on which the shape of a circle does not appear is not in Eric's set. Thus, given any object, we can determine whether or not it is in Eric's set. Stress these ideas as you discuss exercises 1–3.

When you discuss exercise 4, help the children describe their sets so that there is minimal ambiguity. Help the children realize that the test of a good description is whether or not a classmate could think of any object and know whether this object is or is not in the described set.

Using the Ideas

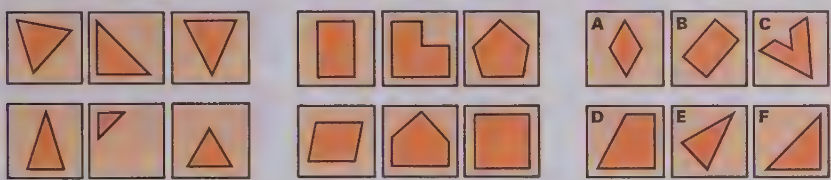
In each exercise you are given clues to help you figure out what special set of cards is involved. Study the clues and answer the question.

1. B, C, D

All of these are in the set.			None of these are in the set.			Which of these are in the set?		
A	e	I	r	P	n	A b	B E	C O
U	a	o	S	t	j	D i	E Z	F x

2. E, F

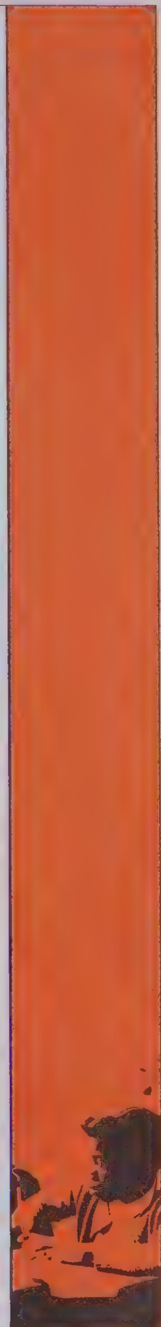
All of these are in the set. — None of these are in the set. — Which of these are in the set?



★ 3. A, C, D

All of these are in the set.			None of these are in the set.			Which of these are in the set?		
Ten	Wow	Jan	Four	Sixty	Born	A Yes	B Nice	C Two
Man	Six	Pop	Golly	No	Sandra	D Cat	E Kitten	F To

5



Mathematics
Although the emphasis in this first lesson is on logical reasoning, several mathematical concepts underlie the material presented in this lesson.

The lesson is basically concerned with classification of *elements* of *sets*. The word *set* is used in this lesson in a natural way. Only the language of sets is needed here, not set notation.

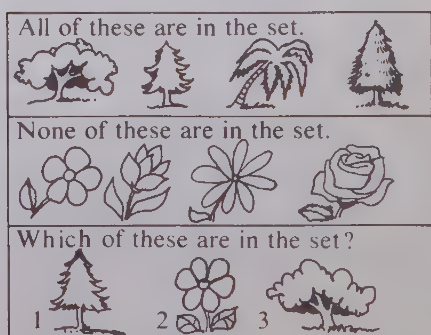
Considerable emphasis is placed on the words *all* and *none* in this lesson and in succeeding lessons. Both words are called *quantifiers* in mathematical logic because they specify, in a broad sense, the quantity or number of objects of a given set. Another familiar quantifier is *some*. As used in mathematics, *some* means *at least one*. Thus, in the third set given in the exercises, this statement could be made: "Some of these are in the set. Which ones?"

This lesson introduces the child to concepts by means of examples and non-examples and deals with classification or categorization of objects possessing similar properties. For a more extended discussion of these ideas, refer to *Investigating Mathematics Learning* Section III, pages I-9 through I-18 in the appendix of this Teachers' Edition.

Using the Exercises
Assign the exercises on page 5 as independent work. You might wish to suggest that the children also list other objects which would be in the set illustrated in each exercise.

When the children have finished the exercises, provide time for checking answers and clarifying any points in question.

Follow-up
Children might enjoy making classification cards similar to those in the exercises on page 5. Their first pictures should show objects in their set; their second pictures, objects not in their set; and their third group of pictures, a mixture of objects in and not in their set. Remind the children to make an answer key for each card they make.



Assignments (page 5)
Minimum: 1–2, oral. Average 1–2, oral. Maximum: 1–3.

PAGES 6–7

Objective

Given a set of numbers all of which have a simple arithmetic property, such as "an even number" or "a multiple of 5," the child will be able to identify other numbers that have the same property.

Preparation

Your adaptation of the following preparation will depend on your classroom organization. You may choose to have the children begin the investigation immediately. However, if you prefer, spend no more than five minutes giving children examples of descriptions of sets and ask them to list the members in the set. Or, name some objects in a chosen set and ask children to describe your set. For example, say something such as: "I'm thinking of the set of all the days of the week. List the members in my set." Or: "I'm thinking of the set of all the girls in this room with blond hair. List the members of my set." (If no girls in your class have blond hair, this set will be the empty set.)

To ask children to describe a set, list objects whose common property is easily recognized, such as the chalk, a certain pen, a certain pencil, for the set of writing objects in the classroom. Or, choose a color and then name objects in the room that are that color.

Investigation

In this investigation, children must identify the common arithmetic property shared by all the numbers in the first illustration. Most will have no difficulty recognizing the multiples of 5. Remind children to list 5 more numbers in the set. For those who finish quickly, you might suggest another exercise on the chalkboard, as in the example below.

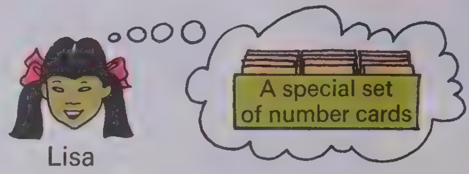
All of these are in the set:
49, 21, 35, 91, 140, 364
None of these are in the set:
10, 24, 12, 100, 81, 72
List more members in the set.

Let's explore sets of numbers.

Investigating the Ideas

Suppose the numerals 1 through 1000 were printed on cards.

Lisa is thinking of a special set of these number cards.



? Can you use the clues below to tell which cards are in Lisa's set? 35,45,50

List 5 more numbers in her set. Sample answer: 40,55,60,65,70 (Any multiple of 5)

Clue number 1	Clue number 2	Question
All of these are in the set.	**None** of these are in the set.	Which of these are in the set?
5, 10, 15, 25, 20, 30	2, 28, 6, 14, 1, 23	4, 35, 18, 27, 45, 39, 51, 50

Discussing the Ideas

1. How can you describe the numbers in Lisa's set?
   Sample answer: Each is a multiple of 5.
2. All these number cards are in the same set. 232 32 2 23 323
   A What are some clues for cards that are not in the set? Cards showing nos. with digits other than 2 and/or 3
   B Give some more number cards that would be in the set.
   Sample answer: Cards showing the numbers 3, 233, 22
3. Think of a special set of number cards.
   Then give clues like those above and see if your classmates can tell what numbers are in your set.
   Answers will vary. See Discussion.

6

Discussion

As children discuss the investigation and exercise 1, stress that it is necessary to identify a particular common property of the numbers to be chosen from the given set. Their new set of the multiples of 5 is a *subset* of the given set. For exercise 3, children should list five or six members of their set under the heading "All of these are in my set." Then they should list five or six numbers not in the set under "None of these are in my set." They may then exchange papers and try to list other members in the set their classmate designated. If you choose, the exercise might also be handled with the class as a group orally or with the use of the chalkboard.

During these first few lessons on sets, recall that one of the broad objectives is to help the children build up an intuitive notion of set. Also, keep in mind that these lessons should be treated as enjoyable mental activities rather than rigorous problems.

Using the Ideas

Each exercise gives you clues to help you figure out what special set of number cards is involved. Study the clues and answer the question.

1. A,D,F

All of these are in the set.	**None** of these are in the set.	Which of these are in the set?
2, 4, 6, 16, 28, 40	1, 3, 5, 15, 27, 39	A 8, B 9, C 21, D 32, E 17, F 10

2. B,C,F

All of these are in the set.	**None** of these are in the set.	Which of these are in the set?
10, 30, 70, 90, 100, 420	11, 53, 45, 115, 96, 1	A 25, B 20, C 50, D 105, E 238, F 150

3. A,B,C,F

All of these are in the set.	**None** of these are in the set.	Which of these are in the set?
3, 6, 9, 18, 27, 30	2, 4, 5, 20, 31, 100	A 12, B 21, C 33, D 25, E 44, F 99

7

Using the Exercises

Assign the exercises on page 7 as independent work. Encourage children to think up more sets of their own when they finish. They might put them on task cards, similar to those shown on page 7, and exchange these cards among themselves. Note that although sets of multiples of a number are excellent for this type of exercise, other properties may also be chosen. For example, in the set {24, 134, 294, 704, 994}, the common characteristic is that each numeral ends with the digit 4.

Assignments (page 7)
Minimum: 1–2, oral. Average: 1–3, oral. Maximum: 1–3.

Mathematics

In this lesson, the child must select from a given set all the elements that have a particular property. The elements so chosen form a new set, called a *subset* of the original set. The following statement precisely defines the idea of a subset of a set.

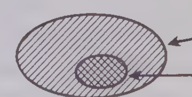
> Set B is a *subset* of set A if B contains no element that is not also in A.

An alternative definition follows:

> Set B is a subset of set A if every element of B is also an element of A.

From this second definition, however, it is not immediately apparent that the empty set is a subset of every set. Using the first definition, we can easily argue that the empty set contains no element that is not in set A. As a matter of fact, it contains *no element*. Hence the empty set satisfies the definition of being a subset of set A regardless of the nature of the given set A.

The idea of a subset can be represented by a diagram. Set B in the diagram is contained entirely within set A, and nothing outside A is in B. Hence, B is a subset of A.



Some further examples will help clarify the idea.

Set	Subset
Your class	The girls in your class
A baseball team	The pitchers on that team
A set of dishes	The cups from that set

Notice that the subsets may contain no elements at all: if you teach in an all boys' school, then the set of all the girls in your class would contain no members at all. Also, the subset may be the same set as the original set: if you teach in an all girls' school, the set of all the girls in your class is the same set as the set of all children in your class.

Workbook, page 1

PAGES 8–9

Objective

Given a set of figure cards, the child will be able to classify them in sets according to the properties of the figures and the relationships among them.

Preparation

Materials

small index cards (Have available at least 12 per child but do not distribute in advance; let children take them as they decide how many they need.); red and blue pencils or crayons

The nature of this lesson, makes it appropriate to begin immediately with the investigation.

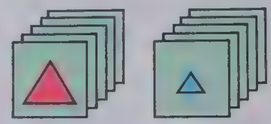
Investigation

It is important for children to figure out for themselves which cards they can make. As you move around the room, refrain from answering children if they ask you to check to see whether or not they have made all possible cards. Instead, encourage them to make sure for themselves whether or not they have made all possibilities. (A finished set will have 12 cards.) The figures may be made freehand; it is not essential that the circles be drawn exactly, with a compass. It is only essential that a difference in shape and size (large or small) be apparent.

Let's explore logic and sets.

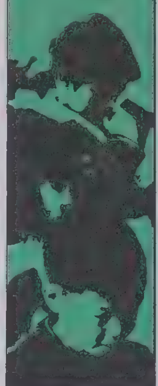
Investigating the Ideas

A **figure card** must have one colored figure on it.



To make each figure card you may use:
- one of the 2 **colors,** red or blue
- one of the 3 **figures,** square □, circle ○, or triangle △
- one of the 2 **sizes,** large or small

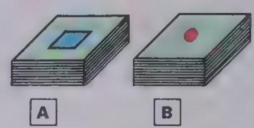
? How many different figure cards can you make? 12



Discussing the Ideas

1. Use your cards to form the set of circles. How many cards are in this set? 4 What other sets can you form and name? See Discussion.
2. How many cards have figures which are
   A red? 6 B red circles? 2 C small red circles? 1
3. Mix up a complete set of cards and have someone secretly take one of them. How can you find which one he took? See Discussion.
4. Suppose one of the cards is turned facedown. Someone who knows tells you the figure on the card is **not** blue. What do you then know for sure about the figure? It is red.

5. If you have one set of cards, can you put all the cards with blue figures in pile A and all the cards with circles in pile B? Explain. No; see Discussion.



8

Discussion

Once the children have made all possible cards, they are equipped with material that is highly beneficial for the study of sets and for developing skill in distinguishing like and unlike properties and in classifying objects accordingly.

As you work through discussion exercise 1, do not try to exhaust the number of possible sets children may form and name, as there is a large number of sets. Only discuss some samples which the children suggest. Note that sets need not always be neatly categorized as "all red figures" or "all blue squares"; also included may be sets such as "all blue triangles and the large red square."

In exercise 3, after the single card has been removed, the remaining cards may be grouped according to various methods. For example, the cards might be separated by color, then the color which has only five cards may be put in pairs according to shape until the missing card is discovered.

In exercise 4, stress the fact that a figure cannot be both blue and not blue at the same time. In exercise 5, stress the fact that since two figures are both blue and circles they cannot be put in separate piles.

Using the Ideas

1. Use the cards you made in the Investigation.
How many are in each set?

A Squares 4
B Triangles 4
C Circles 4
D Small figures 6
E Large figures 6
F Blue figures 6
G Red figures 6
H Blue circles 2
I Small squares 2
J Large triangles 2
K Large red figures 3
L Small blue triangles 1

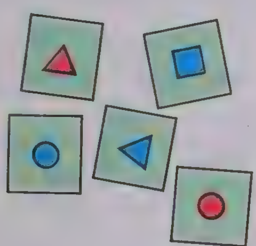
2. One of the cards is shown facedown in each exercise.
Some clues have been written on the back.
Which figure is on the card?

A Small blue square.
It is small
It is a square.
It is **not** red.
What is it?

B Large red circle
It is **not** small.
It is **not** blue.
It is a circle.
What is it?

C Small blue triangle
It is **not** large.
It is **not** red.
It is **not** a circle.
It is **not** a square.
What is it?

3. This is part of a special set of the cards. Which card is missing?



★ **4.** Make up a different set of cards and write some problems about them.
Answers will vary.
See Using the Exercises.

think

After the 3rd inning the baseball scoreboard looked like this.

Mets	2
Expos	4

The final score was
7 to 3
Who won the game? Expos
How do you know?
The Expos had more than 3 runs at the end of the 3rd inning.

9

Using the Exercises

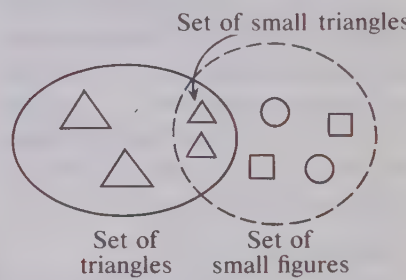
You might assign the exercises on page 9 as independent work or as a small-group activity. Encourage all the children to try starred exercise 4. When children have finished making up their own problems, suggest that they exchange problems and try to solve those their classmates wrote.

Let those who try the *Think* problem explain their reasoning to the class.

Assignments (page 9)
Minimum: 1–2. Average: 1–3.
Maximum: 1–4.

Follow-up

If commercial attribute blocks are not available, children may use their figure cards for a number of thought-provoking activities. For example, two children can make up games together, such as the following: one child thinks of a special set from the cards; the other child inquires, card by card, if it belongs to his partner's set until he is able to describe the set his partner is thinking about. One color yarn may be used to circle the cards which do belong and another to circle those which do not.



"Choose the Subset"

Children can work in pairs or in small groups to play "Choose the Subset." Have each child print numerals 1 through 10 (or, if desired, 1 through 20) on slips of paper or small cardboard squares. One child states a property for a given set of numbers, and the other children choose all the cards that have a number with the given property. Score one point for each correct card selected. Subtract one point for a wrong card or for each omitted card.

Suggest sets having simple properties, such as these:

1. All the numbers that are multiples of 3
2. All the numbers less than 6
3. All the numbers greater than 6 and less than 9
4. All the prime numbers in the set
5. All the numbers in the set that can be divided exactly by 2 or 3

PAGES 10–11

Objective

Given two intersecting sets of numbers, the child will be able to identify a number when given appropriate statements concerning its inclusion in or exclusion from the sets, or when given some properties of the number.

Preparation

Materials

small index cards (3 per child); 75-cm piece of red yarn (1 per child); 75-cm piece of blue yarn (1 per child)

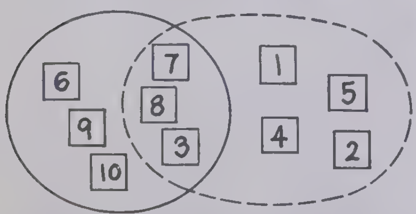
If colored yarn is not available, you may have the children use crayons to draw red and blue loops on large sheets of paper.

The nature of this investigation, makes it appropriate to begin it without specific preparation.

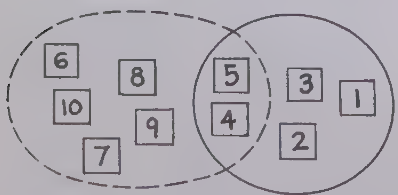
Investigation

Direct the children to fold and cut their cards into fourths and use each for a number from 1 through 10. Some children may try to plan which number they want to use and then arrange the loops and cards accordingly. Others may simply mix the cards and place the loops on top, and then decide which number to use. Notice that the clues should be brief but complete.

Sample possibilities (dashed lines represent red yarn):



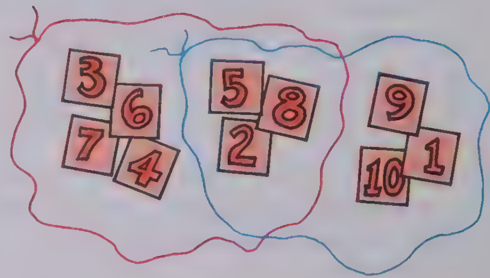
Clues: It is not inside the red loop. It can be divided evenly by 5.



Clues: It is inside the blue loop. It is less than three. It is even.

Investigating the Ideas

Make cards for the numbers 1 through 10. Make two 1-metre loops from red and blue yarn.

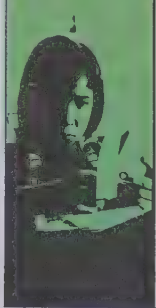


CLUE CARD
It is inside the red loop **and** it is inside the blue loop.
It is odd.
Which number is it? 5

Use the clues to find the number.

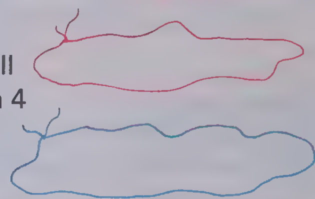
See Investigation.

?	
Can you put the cards into the loops in a different way and make a new clue card?	Challenge a classmate to find **your** number.



Discussing the Ideas

1. Which cards are inside the red loop **and** also inside the blue loop in the Investigation? 2, 5, 8
2. A student is thinking about a card that is **not** inside the red loop. What do you know about his card? It is 1, 9, or 10.
3. A student placed his loops like this. Can he lay the cards down so that all the cards with numbers greater than 4 are inside the red loop and all the cards with numbers less than 7 are inside the blue loop? Explain.



No; see Discussion.

Discussion

The logical use of the words *and* and *not* as used in mathematical logic is in agreement with their ordinary usage. You will want to ascertain that children understand how these words are used as you discuss exercises 1 and 2. (See the mathematics section for a more extended discussion of the mathematics underlying this lesson.)

In exercise 3, help the children realize that because the numbers 5 and 6 have the properties of both statements, they should be included in both loops. So, as long as the loops are separated, the cards cannot be properly placed.

Do not dwell very long on the ideas in this discussion. Children will benefit more from working with actual exercises and challenging each other as the investigation suggests than from overemphasis on simple concepts.

Using the Ideas

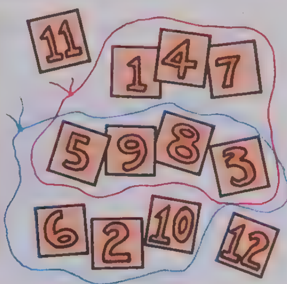
1. Use the clues and give the number for each clue card.

A 12

CLUE CARD
It is not inside either loop.
It is not 11.

B 4

CLUE CARD
It is not inside the blue loop.
It is inside the red loop.
It is even.



C 10

CLUE CARD
It is not inside the red loop.
It is inside the blue loop.
It is greater than 6.

D 8

CLUE CARD
It is inside the red loop
and
it is inside the blue loop.
It is even.

E 1

CLUE CARD
It is not inside the blue loop.
It is not even.
It is less than 7.

2. Use the set of cards for the numbers 1 through 10. How many cards have numbers that are

   A not even? 5
   B not odd? 5
   C less than 5? 4
   D more than 5? 5
   E more than 4 and less than 7? 2
   F even and more than 6? 2

★ 3. Put the cards with odd numbers inside the red loop. Put the cards with multiples of 3 inside the blue loop. Which numbers are inside both loops? 3 and 9

think

These are in a set of "special" words.
EIGHT THREE FORTY SEVEN SIXTY

These are not in the set.
SIX TWENTY ELEVEN FOUR SEVENTY

Which of these is in the set?
ONE NINE TWELVE FIFTY THIRTY

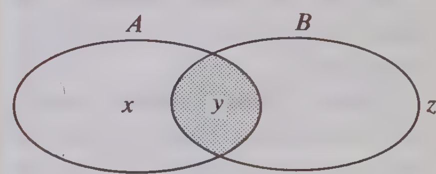
Fifty

Mathematics

The mathematics of this lesson involves informal ideas of set membership and the intersection of sets, as well as simple properties of numbers.

If S and T are any two sets, then their *intersection*, written $S \cap T$, is the set consisting of all elements in both S and T.

In the figure below, the shaded portion of the figure represents $A \cap B$. The figure shows that y is an element of $A \cap B$. This means that y is an element of A *and* y is an element of B. Observe that x is an element of A, but x is not an element of B, so x cannot be an element of $A \cap B$. As shown by the drawing, z is not an element of either set A or set B.



Follow-up

Encourage children to make numerous clue cards to exchange with each other. You might broaden the possibilities by including in the set the numbers 11, 12, 13, 14, and 15, or other numbers.

Workbook, page 2

Using the Exercises

Assign the exercises on page 11 as independent work. When the children finish, check their answers and allow time for discussion of any questions. Note that for starred exercise 3 the loops should be placed so that they overlap.

Encourage everyone to try the *Think* problem. Children will realize how simple it is when those who solve it explain that they simply noted the number of letters in each word.

Assignments (page 11)
Minimum: 1–2. Average: 1–2.
Maximum: 1–3.

PAGES 12–13

Objective

Given a sequence of numbers and a corresponding dot pattern, the child will be able to recognize the sequence and continue the construction of the pattern.

Preparation

It would be appropriate to begin this lesson immediately with the investigation.

Investigation

Children might work independently on this investigation. Direct them to study the pattern and try to figure out how each number is formed in relation to the number preceding it. Some children might see the relationship before drawing another dot figure. Many, however, will rely on the construction of the next dot figure to find the next number in the sequence.

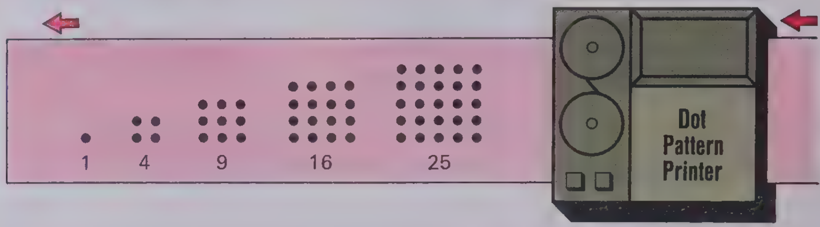
If some children finish quickly, suggest that they build a pattern of their own for a sequence you suggest on the chalkboard, such as the following:

1, 1, 2, 3, 5, 8, 13, . . .

● *Can you find the patterns?*

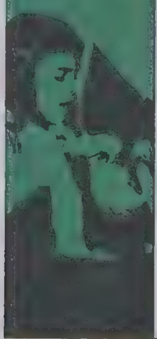
Investigating the Ideas

The **Dot Pattern Printer** printed patterns for the number sequence 1, 4, 9, 16, 25, ...



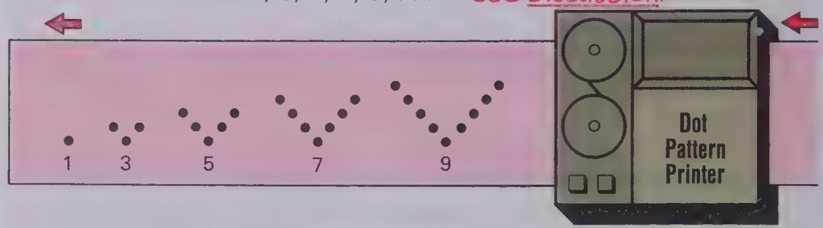
Pretend that the dot patterns can go on and on.

? Can you draw the next two patterns the printer will make and give the next two numbers in the sequence? (Patterns of 6 by 6 and 7 by 7) 36, 49



Discussing the Ideas

1. Why do you think the numbers in the sequence above are called square numbers? What are some other square numbers? 64, 81, 100, ...
   Sample answer: The dots for these numbers can be arranged in a square array.
2. What pattern might the machine print for the sequence of even numbers 2, 4, 6, 8, 10, 12, 14, . . . ? See Discussion.
3. What pattern might the machine print for the sequence of odd numbers 1, 3, 5, 7, 9, . . . ? See Discussion.



4. Can you invent another sequence of numbers and draw the dot patterns that go with the sequence? Answers may vary.

12

Discussion

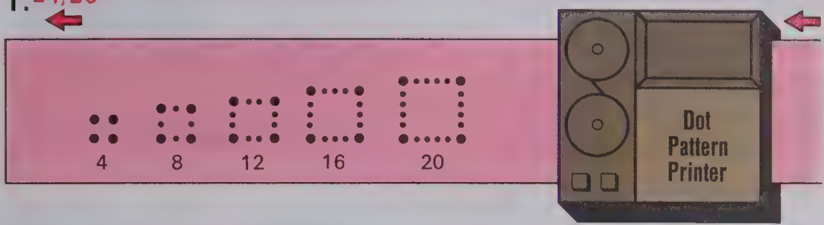
Use the sequence of square numbers to discuss the relation of the number sequence to the geometric pattern represented by the dots. Stress the appropriateness of the term "square numbers" and help children see how this sequence is formed arithmetically: 1 × 1, 2 × 2, 3 × 3, and so on.

For exercises 2 and 3, accept any pattern for a number sequence which children can reasonably explain. Although at times one geometric pattern will seem obvious for a particular sequence, other reasonable patterns may be just as valid.

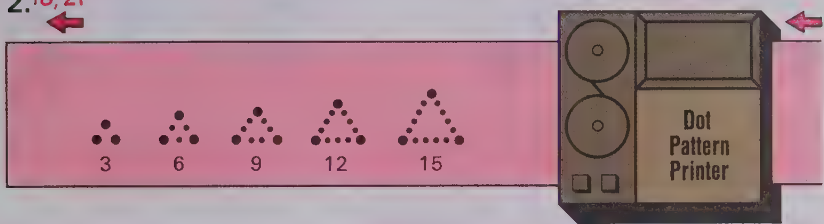
Using the Ideas

Draw the next dot pattern the "printer" will make and give the next two numbers in each sequence.

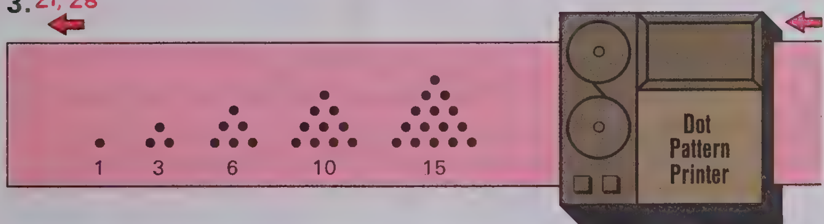
1. 24, 28



2. 18, 21



3. 21, 28



4. 32, 64

13

Follow-up

Children can use the centimetre strips which accompany this series to show simple arithmetic sequences, that is, sequences in which successive numbers have the same difference. Examples are:

1, 4, 7, 10, . . . and
2, 6, 10, 14, . . .

More complicated sequences such as

1, 1, 2, 1, 2, 3, 1, 2, 3, 4, 1, . . .

could also be constructed.

Resources for Active Learning

Developmental Math Cards, H²5, J¹15, Addison-Wesley.
Mathematics in Modules, WN22, Addison-Wesley.

Using the Exercises

Assign the exercises on page 13 as independent work. Suggest that the children look not only at the geometric dot pattern but also at the numbers in sequence, to see if they can both draw the next two patterns and explain how each preceding number was used to form the next number in the sequence. However, the primary objective in this lesson is for children to be able to recognize a pattern and give other numbers in the sequence. Children should not be expected to state arithmetically how the sequence is built.

Assignments (page 13)

Minimum: 1–3. Average: 1–3.
Maximum: 1–4.

PAGES 14–15

Objective

Given illustrations of an object before and after a certain transformation (or application of a function rule), the child will be able to identify the same transformation on another object.

Preparation

To prepare for this lesson, you might remind the children that in the previous lesson they studied number patterns which were represented geometrically with dots. For example, write one or two of the easier sequences and ask children to explain how they are "built." Then explain that patterns may occur in figures as well as numbers.

Investigation

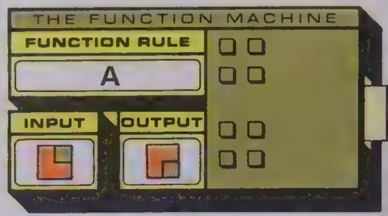
In this lesson, children must examine a particular transformation, or change, which occurs on an illustrated object and then apply this same transformation to another illustrated object. The function rule in the investigation can be described as, "Rotate the input one-fourth turn clockwise."

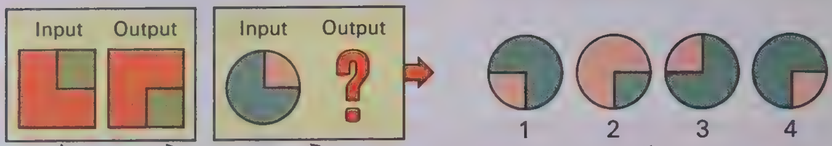
After children have given other examples of the rule in the text, suggest that they make up a rule of their own and challenge a classmate with it.

Let's explore figure patterns.

Investigating the Ideas

When you put a figure in this function machine, the machine changes it in a certain way. The machine prints yellow input-output cards to show what happened.





When this figure is the input | this figure is the output. | When this figure is the input | which of these is the output? 4

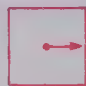
Answer the question above.

See Investigation.

? Can you invent your own function rule like the one above and make some input-output cards to show your rule?

Discussing the Ideas

1. Explain function rule **A** in your own words.
   Sample answer: Rotate the input figure $\frac{1}{4}$ turn clockwise.
2. Which figure did you select as the output in the Investigation? Why? Sample answer: It shows how figure looks after $\frac{1}{4}$ turn clockwise.
   4
3. Can you draw the output that the machine above will produce if this figure is the input?



14

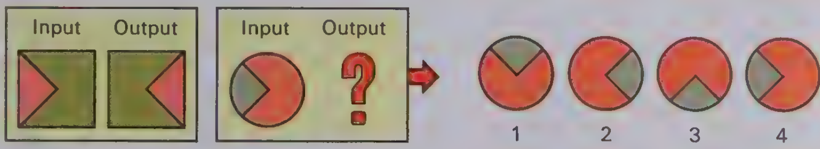
Discussion

One of the principal aims of this lesson is to have children recognize patterns and apply them to similar situations. Do not belabor the discussion exercises. Make sure children realize that the pattern is given by the first illustration of each example and then direct them to the exercises on page 15.

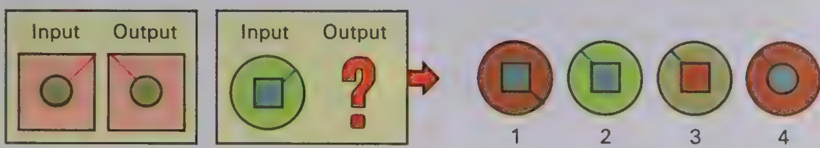
Using the Ideas

The first card in each exercise shows what the machine does. Give the number of the output for the second card.

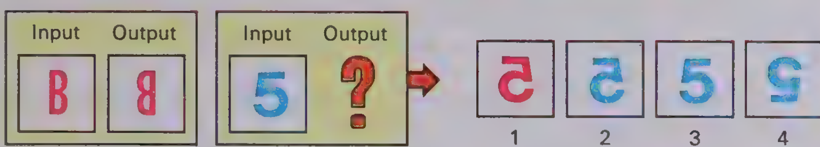
1. 2



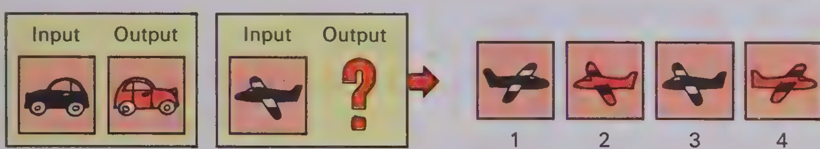
2. 2



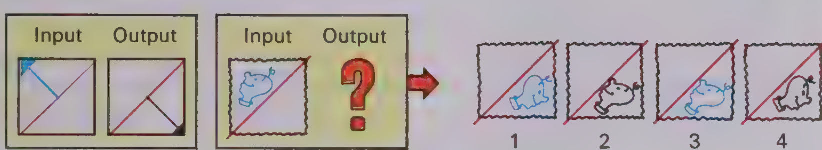
3. 2 if digit is flipped backwards; 4 if it is rotated 180°



4. 4



5. 4



Resources for Active Learning
Experiments in Mathematics, Stage 1, "Transformations," pp. 38–39, Houghton Mifflin (Available from Thomas Nelson and Sons).

Using the Exercises
Assign the exercises on page 15 as independent work. Also, suggest that for each exercise the child give at least two more examples of other objects before and after the transformation. It is unnecessary for children to describe the transformation verbally, although, for the sake of discussion, you may wish to have some children tell why they chose the particular output for a given exercise.

Assignments (page 15)
Minimum: 1–3. Average: 1–4. Maximum: 1–5.

PAGES 16–17

Objective

Given number patterns and sequences, the child will be able to extend the pattern or sequence.

Preparation

To introduce this lesson, you might use an oral drill such as counting by multiples of 5, 4, or 2. Or, write on the chalkboard the numbers 1, 2. Then ask the children to add the second number to the one which comes before it, that is, to add 2 to 1 and write the sum, 3, next in line. Ask them to continue doing this a few times. They may be surprised by how quickly the sums grow. Point out that such a listing of numbers is called a sequence and that they will be studying patterns and sequences in this lesson.

Investigation

Explain to the children that they should first study the four completed equations on each poster. If they have figured out the pattern they should be able to complete the equations without actually performing any operations. However, most children should check their equations by actually performing the operations on the numbers which they have chosen to complete the equation. You might encourage some children to extend the list of equations in the investigation after they have discovered the patterns.

● *Let's explore other mathematical patterns.*

Investigating the Ideas

Study the first 4 equations on each poster.

(1 x 9) − 1 = 8
(21 x 9) − 1 = 188
(321 x 9) − 1 = 2888
(4321 x 9) − 1 = 38 888
(_____ x 9) − 1 = _____

1 + 3 = 2 x 2
1 + 3 + 5 = 3 x 3
1 + 3 + 5 + 7 = 4 x 4
1 + 3 + 5 + 7 + 9 = 5 x 5
1 + 3 + _ + _ + _ + _ = ____

? Can you copy and complete the last equation on each poster?

(54 321 x 9) − 1 = 488 888

1 + 3 + 5 + 7 + 9 + 11 = 6 x 6

Discussing the Ideas

1. Use the pattern on the first poster to guess the number for ***n*** in this equation.

$$(987654321 \times 9) - 1 = n$$

Check your guess by computing. 8 888 888 888

2. A certain number multiplied by itself gives a product equal to this sum: 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17
Explain how to find the number.
The number of addends equals the middle addend. Multiply this number by itself to get product that equals the sum.

3. Can you find the pattern and give three more numbers in this sequence? Double the preceding number and subtract 1.

2, 3, 5, 9, 17, 33, 65, 129, 257 . . .

Discussion

In discussing exercise 1 and the pattern on the first poster, stress to the children that recognizing the pattern will enable them to complete the equation without actually performing any operation. As you discuss the second set of equations, elicit from the children that the middle term, or the average of the two middle terms, indicates the number to be multiplied by itself. Finally, in the last discussion exercise, let the children discover the rule themselves: double the first number, and subtract 1 to get the second number.

Using the Ideas

1. Study the patterns in the first four equations on each poster. Copy and complete the next equation in each part.

A

$0+1+2=3$

$1+2+3=6$

$2+3+4=9$

$3+4+5=12$

$4+\underline{5}+\underline{6}=\underline{15}$

B

$143\times 7=1001$

$143\times 14=2002$

$143\times 21=3003$

$143\times 28=4004$

$143\times\underline{35}=\underline{5005}$

C

$(1\times 9)+2=11$

$(12\times 9)+3=111$

$(123\times 9)+4=1111$

$(1234\times 9)+5=11\,111$

$(\underline{12\,345}\times 9)+\underline{6}=\underline{111\,111}$

D

$(1\times 8)+1=9$

$(12\times 8)+2=98$

$(123\times 8)+3=987$

$(1234\times 8)+4=9876$

$(\underline{12\,345}\times 8)+\underline{5}=\underline{98\,765}$

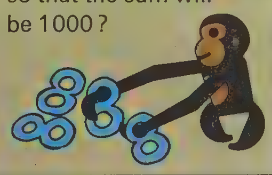
2. Check each equation in exercise 1. Is each "answer" correct? Yes

3. Give the next three numbers in each sequence.
   - A 2, 4, 6, 8, 10, 12, 14, ?, ?, ? 16, 18, 20
   - B 1, 2, 4, 8, 16, 32, ?, ?, ? 64, 128, 256
   - C 10, 11, 20, 21, 30, 31, 40, 41, ?, ?, ? 50, 51, 60

think

Can you arrange eight "eights" in an addition problem so that the sum will be 1000?

888
88
8
8
8
1000



Using the Exercises
Assign page 17 as independent work. Note that in exercise 1 the children need only follow the patterns in order to complete the equations. In exercise 2, they are to work out the equations to see if their patterned work was correct. If some children can find the patterns in exercise 1 quickly, you might ask them to try to write other equations that would follow the same pattern.

Follow-Up/"Buzz"
Children should enjoy the "Buzz" game, based on patterns of multiples. Choose a set of multiples, such as multiples of 4. Ask the children to begin counting by ones. Every time a multiple of 4 is reached, the child whose turn it is should say "Buzz." When the children catch on, suggest another multiple and use another word as its substitute, such as "Clang." You might even challenge the children to use a combination of multiples, such as multiples of both 3 and 4.
Samples:
1. Multiples of 4:
   1, 2, 3, Buzz, 5, 6, 7, Buzz, 9 . . .
2. Multiples of 3:
   1, 2, Clang, 4, 5, Clang, 7, 8, Clang . . .
3. Multiples of 3 and 4:
   1, 2, Clang, Buzz, 5, Clang, 7, Buzz, Clang, 10, 11, Buzz-Clang . . .

Resources for Active Learning

Developmental Math Cards, "Square Numbers," I-19, K-21, Addison-Wesley.
Math Activity Cards, "Magic Squares," D38, Macmillan.
Mathex: Numeration No. 7, "Pattern Seeking," pp. 1–6 (pupil pages 1–5), Encyclopedia Britannica Publications Ltd.

Duplicator Masters, page 1
Workbook, page 3
Skill Masters, page 1

Assignments (page 17)
Minimum: 1–2. Average: 1–3.
Maximum: 1–3.

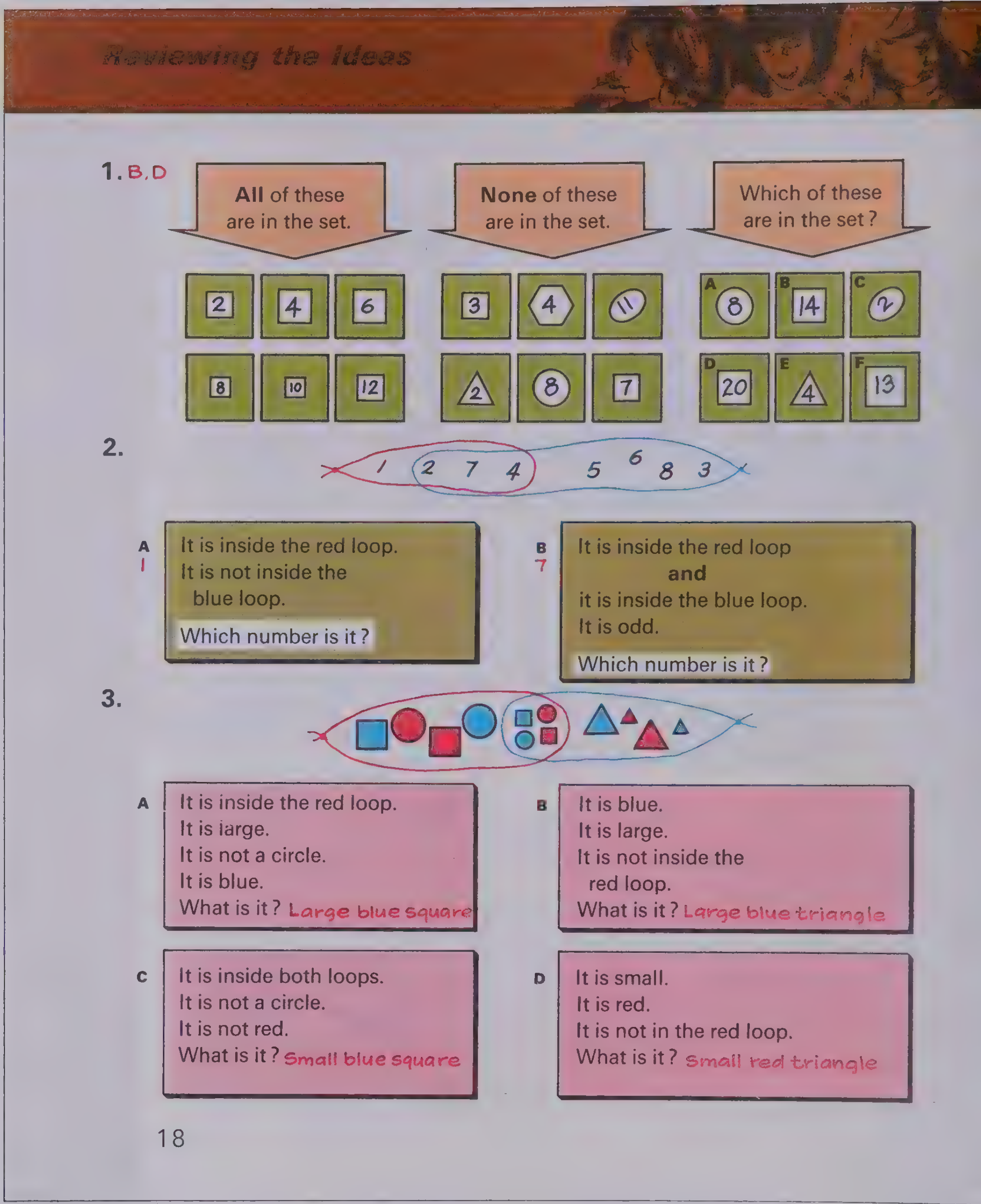
PAGES 18–19

Objective

The child will demonstrate his ability to work with the concepts presented in this chapter.

Preparation

Depending on the needs of your children, you may wish either to begin this lesson immediately or to review any of the topics treated on these pages which the children found troublesome.



Reviewing the Ideas

1. B, D

All of these are in the set.	None of these are in the set.	Which of these are in the set?
2, 4, 6, 8, 10, 12	3, 4, 11, 2, 8, 7	A 8, B 14, C 2, D 20, E 4, F 13

2.

A 1

It is inside the red loop.
It is not inside the blue loop.
Which number is it?

B 7

It is inside the red loop **and** it is inside the blue loop.
It is odd.
Which number is it?

3.

A

It is inside the red loop.
It is large.
It is not a circle.
It is blue.
What is it? Large blue square

B

It is blue.
It is large.
It is not inside the red loop.
What is it? Large blue triangle

C

It is inside both loops.
It is not a circle.
It is not red.
What is it? Small blue square

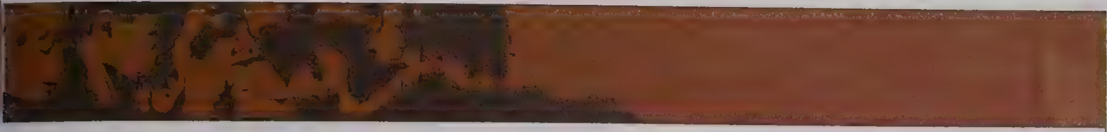
D

It is small.
It is red.
It is not in the red loop.
What is it? Small red triangle

18

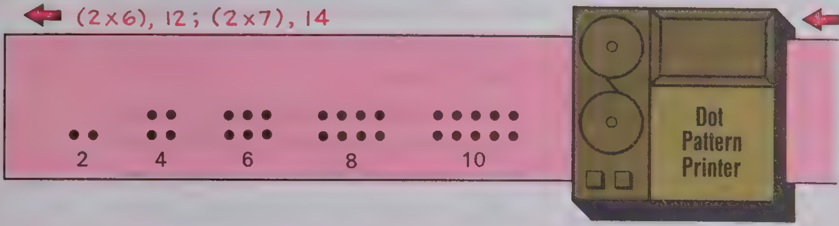
Discussion

Since pages 18 and 19 both contain the kind of exercises with which the children worked in this chapter, you might simply assign them as independent work. The ability of the children to work these exercises independently should help you evaluate their development in the area of logical thinking. However, keep in mind that many children will not yet have reached the level of operation above that of the concrete or semiconcrete level. Therefore, evaluation of their ability to work with the material in the chapter should be treated with a light touch and not "emphasized."

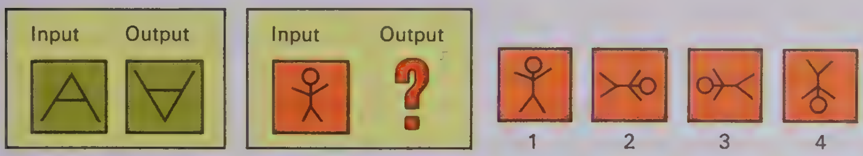


4. Give the next two patterns and numbers.

(2×6), 12; (2×7), 14



5. The first card shows what the function machine does.
Can you give the number for the output for the second card? 4



6. Study the first three equations on each poster.
Then copy and complete the next equation.

A

$0 + 2 + 4 = 6$

$2 + 4 + 6 = 12$

$4 + 6 + 8 = 18$

$\underline{6} + \underline{8} + \underline{10} = \underline{24}$

B

$15 \times 15 = (1 \times 2 \times 100) + 25 = 225$

$25 \times 25 = (2 \times 3 \times 100) + 25 = 625$

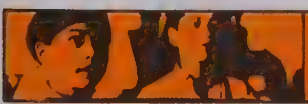
$35 \times 35 = (3 \times 4 \times 100) + 25 = 1225$

$45 \times 45 = (\underline{4 \times 5 \times 100}) + 25 = \underline{2025}$

7. Give the next three numbers in each sequence.

A 1, 6, 11, 16, 21, 26, 31, 36

B 1, 3, 13, 131, 1313, __, __, __
13131, 131313, 1313131



You are invited to explore

ACTIVITY CARD 1
Page 333

Follow-up

Encourage children to develop task cards for the attribute pieces. They might do this in groups of 2 or 3 and then exchange cards. Others might like to invent input and output rules for the function machine, creating cards similar to those in exercise 5. An example of a pattern may be explored by those who remember their multiplication facts for 9:

$$\begin{array}{r} 12345679 \\ \times\ 27 \\ \hline 86419753 \\ 24691358 \\ \hline 333333333 \end{array} \qquad \begin{array}{r} 12345679 \\ \times\ 45 \\ \hline 61728395 \\ 49382716 \\ \hline 555555555 \end{array}$$

Workbook, page 4

Pages 20–39

General Objectives

To provide understanding of place-value concepts

To review and extend work with inequalities

To introduce work with bases other than ten

A clear understanding of place value is one of the most important objectives of the *Investigating School Mathematics* program. Without such an understanding, there is little hope of teaching meaningfully the algorithms (computational rules) of arithmetic and decimal notation. For this reason, the concepts of place value are treated carefully beginning in Book 1 and are followed up throughout the series. This chapter begins with a study of unconventional numerals to review the usual place-value scheme. Then, various methods of showing numbers with groups of objects, number-blocks, and the abacus are investigated. The reading and writing of large numbers, which is studied next, includes an introduction to expanded notation. Following this, inequalities are used to help the children understand place-value concepts and the ordering of the set of whole numbers. Once place value has been presented carefully, the idea of grouping by sets other than groups of ten is introduced. The children are introduced to grouping by fours, or base four. This lesson involves primarily the writing of base-four numerals for a given set of objects. Some work with operations is provided in order to give the children further experience with this interesting idea. The chapter closes with a study of some older systems of numeration, which may spark further exploration, depending on children's interests. A chapter review (pages 38–39) summarizes the content of these lessons.

Mathematics

The digits are the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. A numeral is any symbol that stands for a number. Thus, the symbol 4 is both a digit and a numeral. The symbol 57 is a two-digit numeral for the number fifty-seven.

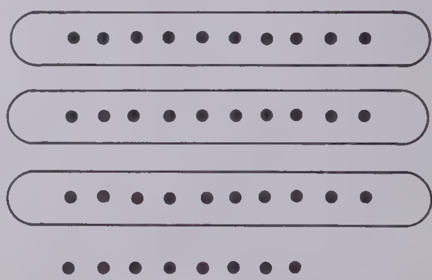
Correct use of the words *number, numeral,* and *digit* may sometimes be awkward. You might find it convenient to ask a child to *write* the "number" 1486, or to refer to this as a "four-digit number." Strictly speaking, these are abuses of the language, because you do not write the *number* and the *number* does not have four digits. You write the *symbol* or the *numeral,* and it is the symbol or the numeral that has four digits. The children, however, will no doubt understand that, when you say, "Write the number," you mean for them to write the symbol for this number; likewise, when you say "a four-digit number," they will understand that you are referring to the symbol used to represent this number. Hence, we consider both expressions acceptable at this level. However, certain abuses of the number-numeral terminology are clearly objectionable and may confuse the children. Do *not* ask the children to *add digits or numerals*. They *add* numbers and *write* digits and numerals.

The paramount consideration regarding the number-numeral terminology is to keep the language simple and meaningful to the children. Whenever you are uncertain about whether you should say number or numeral, say *number*. Avoid making an issue of these words with the children. If a child points to a symbol or numeral and calls it a number, do not criticize his remark. Most children have an intuitive grasp of the difference between number and numeral, and a lengthy discussion of these ideas may serve only to confuse that which was previously clear.

An important property of our numeration system is *place value.* Place value means that the number a digit represents depends on the place it occupies in the symbol. For example, when we write the numeral 4357, the digits 4, 3, 5, and 7 stand for 4000, 300, 50, and 7, respectively.

Another important aspect of our numeration system is that we can represent any number by using only ten symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Each digit used by itself represents a single number; it is only when we write symbols for numbers greater than nine that a given digit may stand for two or more numbers. Thus, in the numeral 636, one 6 stands for six hundred, and the other stands for six.

Our place-value scheme has the number ten as its base, which means that we group by tens. Given a collection of objects, we find out how many disjoint sets (sets which have no common elements) of ten can be formed. Consider the dots shown in the illustration; there are 3 sets of ten, and 8 left over.



The importance of place value is evident when we write the numeral for this number of dots. Instead of writing "3 sets of ten, and 8 more," we write "38" and agree that the digit in the "second place" (3 in this case) represents sets of ten.

In order to represent numbers over 99, we need to group the sets of ten by tens. We say that each set

of 10 tens is one hundred. For example, we might have some objects grouped as follows: 5 sets of one hundred, 3 sets of ten, and 7 more; we write "537."

Teaching the Chapter

Materials

Abacus (optional)
Counters (15 per child)
Number blocks for base ten (optional)
Reference materials (history books, almanac, encyclopedia)
Sugar cubes or blocks for base four (optional)

Vocabulary

abacus	is not equal to
base four	less than
base ten	million
billion	numeral
digit	operation
equals	place value
equation	Roman numeral
greater than	set
hundred million	ten million
hundred thousand	ten thousand
inequality	thousand
	trillion

Although materials such as an abacus and sugar cubes are not essential to the investigations, the availability of these materials would enhance the learning experiences for which they might be used. Most children can successfully follow instructions in making an abacus, given a few simple materials such as copper wire, plastic foam, cardboard and beads; or plywood, nails, and washers. Sugar cubes may be used for work both in base ten and in base four. However, their use in the lessons on bases other than ten (pages 32–37) may be especially helpful in providing the concrete experience children need in order to understand place value with other number bases and the grouping processes involved.

Although the vocabulary list is long, most of the words should be familiar to the majority of the children. Those words that are new should be stressed so that they are mastered by the end of the chapter.

Lesson Schedule

Plan to spend about one-and-a-half to two weeks on this chapter, depending upon the interests, abilities, and background of the children.

Evaluation of Progress

The material presented on base four is included in this chapter to increase understanding of base ten and to serve as a stimulation for the children. Mastery is not expected. The principal value of teaching something unusual, such as base four, is that it arouses the children's interest and stimulates them to do a better job in arithmetic. Keep these objectives in mind as you teach this small section of the chapter. Since you should not expect the children to master all of the topics concerning base four, do not be discouraged if some of them fail to understand the more difficult concepts. Most should understand the rudiments of grouping by four and writing a two-place base-four numeral. Treat this section lightly, and do not allow the children to become discouraged if they find it particularly difficult.

The chapter review on pages 38 and 39 should serve as a guide for evaluating the children's progress. We stress again that, while place value and inequalities are vital to successful study of the remainder of the text, topics such as base four and different types of numerals should not be considered a significant part of the children's progress for this chapter. Encourage more capable children to independently explore these optional items in depth if they are intrigued by them.

Resources For Active Learning

GENERAL ACTIVITIES

Applied Mathematics Cards, "Methods of Counting," Group 2/1, Schofield and Sims (Available from Mafex Associates, Willowdale, Ont.)
Franklin Series: *From Fingers to Computers,* "The Abacus," pp. 18–35, Lyons and Carnahan (Available from McGraw-Hill Ryerson)
Nuffield Project: *Problems*—Green Set, No. 3, Wiley

[The following references deal with bases other than 10.]

Discovery, Section II, Units 8/5; 15/2; 16/2,3, Encyclopaedia Britannica Educational Corp.
Mathex: Numeration No. 7, pupil pages 15–22, Encyclopaedia Britannica Publications Ltd.
Teaching Aids for Elementary Mathematics, "Tables for other Bases," pp. 42–43; "A Quinary Calendar," pp. 46; "A Binary Code . . .," p. 47; "Greeting Cards," pp. 48–51, Holt, Rinehart and Winston

MANIPULATIVE DEVICES

Abacus or abacus board (school supplier)
Cuisenaire Cubes, Squares, and Rods (Cuisenaire Co.)
Dienes Multibase Arithmetic Blocks (Herder and Herder)
Dr. Nim (Childcraft; Gamco)
Multi-base Converter (Math Media; Selective Educational Equipment)

COMMERCIAL GAMES

Base Check Game (Lakeshore)
Radix (Lang)
Ranko—pink and yellow cards (Midwest Publications)

PAGES 20–21

Objective

Given a different set of symbols to replace the base-ten digits, the child will be able to use these symbols to write numerals that have place-value meaning.

Preparation

To stimulate interest in this lesson, write a numeral using the secret code described in the text. For example, write the numeral for the number of children in the class, and explain that you have used a secret code to write a number with which all of them are associated. Then tell the children that as they work through the investigation they will be able to figure out the number you have written.

Investigation

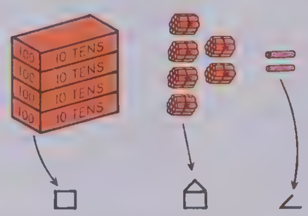
In order for children to make sense of Fran's date of birth, they must recognize that each symbol takes on a particular meaning according to its position in a numeral. Let the children discover this from their own observation of the text material and from their use of Fran's symbols in writing other numbers. You might want the children to work in small groups, although independent work would also be suitable. Encourage them to write many numerals and to exchange papers so that what one child has written, another may decipher. Have suitable reference materials available to assist the children in finding numbers which interest them. Note that Fran's symbols have not been named. They may be read by using the base-ten name of the digit they replace, or children may make up names for them.

2 Numbers and Numerals

● *Let's explore symbols for numbers.*

Investigating the Ideas

Fran decided to invent new numerals for numbers. She used them like this:



Here is Fran's date of birth.

June ∠⌂, /

Fran's Secret Numeral Code

○ = 0
/ = 1
∠ = 2
△ = 3
□ = 4
⌂ = 5
= 6
= 7
= 8
= 9

? Can you use Fran's numerals to show your date of birth and some other interesting numbers? See Investigation.

Discussing the Ideas

1. How many sticks are shown in the Investigation above? 462
2. When was Fran born? June 25, 1964
3. Once Fran decided to use ⌂ for 5, why do you think she used the numerals for 6, 7, 8, and 9? Answers may vary. See Discussion.
4. In the numeral △ ○, the △ indicates which of these numbers: 3, 30, 300, or 3000? 300

20

Discussion

The illustration of the sticks is important to the children's realization that Fran's numerals depend on a base-ten place-value system. As you work through the discussion exercises with the children, stress this idea of the position of a digit in a numeral with respect to the number for which it stands. As you continue through the discussions, also stress the arbitrariness of numeral symbols. Point out that as long as Fran's code is understood, her symbols represent any number just as well as a numeral which employs the usual base-ten digits. It would be helpful to count with Fran's numerals at least through twenty. Also, have some children write the numerals for their dates of birth on the chalkboard and have others find the date of birth by using "ordinary" base-ten numerals. As you discuss exercise 3, allow wide variation in children's explanations of why Fran chose these symbols, but also point out the relationship between the number of strokes used and the number represented.

Using the Ideas

1. Change these to "ordinary" numerals.

A 628 D 400 G 2609 J 6555

B 27 E 91 H 6000 K 3789

C 578 F 161 I 1000 L 7234

2. Use Fran's numerals to rewrite each of these.

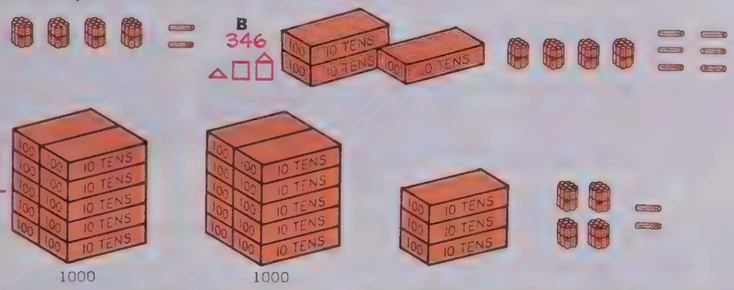
A 36 D 136 G 5265 J 4000

B 423 E 200 H 7068 K 1007

C 58 F 405 I 9000 L 6528

3. Use "ordinary" numerals and then Fran's numerals to tell how many are in each set.

A 42 B 346 C 2342



think

Here is the beginning of a code. Can you complete it to 9 and then write some numerals with it?

0 = •

1 =

2 =

3 =

4 =

5 =

6 =

7 =

8 =

9 =

Sample numeral : (1973)

21

Using the Exercises

Assign the exercises on page 21 as independent work. When the children finish, check their work and allow time for discussion. Continue to stress the place-value meaning of each symbol. You may wish to use an abacus to show the numbers for the sets of objects shown in exercise 3. For those who finish quickly or who show particular interest in symbols, you might suggest that they do the *Think* problem and use these numerals in problems of their own.

Assignments (page 21)

Minimum: 1–2. Average: 1–3.
Maximum: 1–3.

Follow-up

Encourage children to make up their own secret numeral code. They might use it in some problems and have other children try to figure out what the numerals represent. Or, they might write a story using dates and numbers of persons, animals, scores, games, and the like.

PAGES 22–23

Objective

Given numbers in the thousands or millions, the child will be able to interpret them in relation to an abacus and to the number blocks of thousands, hundreds, tens, and ones.

Preparation

To prepare for this lesson, you might demonstrate the inconvenience of using objects to illustrate large numbers. For example, ask someone to show with sticks or similar objects, the number four. Then ask someone to show the number fourteen, then twenty-four, then forty-four, and so on. Soon you will be giving them a number for which not enough sticks are available or convenient. Suggest that they try to think of other ways of exhibiting the numbers for which sticks cannot be used conveniently. Then direct them to the investigation.

Investigation

You might find it appropriate to have the children actually make, rather than draw, an abacus and then show their numbers on it. Long nails, strips of plywood, and washers make very sturdy abacuses. However, a variety of other materials may also be used, such as copper wire, beads, plastic foam, cardboard, coat hangers, etc. In any case, recall that for work with base-ten numerals, nine is a convenient number of counters on each wire. For example, while counting in base ten, as soon as one more than nine is reached, a bead on the next wire to the left should replace the nine on the previous strand.

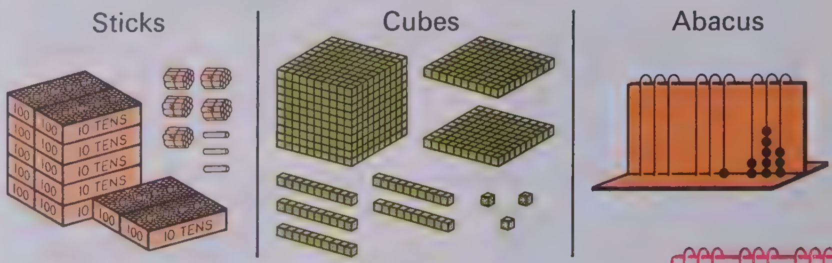
If you choose to have the children simply draw the abacus, you might suggest that they keep the drawing of the abacus simple:

Do not indicate how to illustrate any particular number; let the children figure this out from their study of the text. Suggest other numbers for them to illustrate.

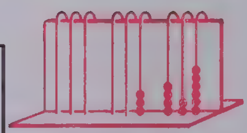
● *How do we use place value?*

Investigating the Ideas

Each of these figures shows 1253.

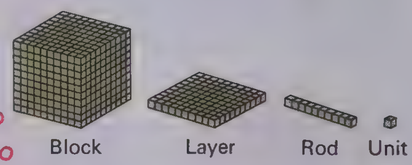


? Can you draw an abacus that shows 3426 ?



Discussing the Ideas

1. We name the parts in the cube figure like this.
   A How many units in a rod ? 10
   B How many rods in a layer ? 10
   C How many units in a layer ? 100
   D How many units in a block ? 1000



2. It is difficult to show thousands and millions with sets of objects, but we can show larger numbers easily on the abacus. Write the number shown on this abacus.
   562 342

3. Explain how you would show these numbers on the abacus.
   A 26 714
   B 132 476
   C 46 205
   D 813 495
   E 2 615 284
   F 39 700 408

   See Discussion.

22

Discussion

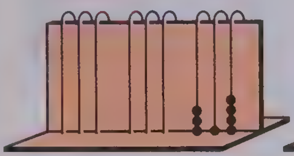
As you discuss exercise 1 on page 22, ask children to describe how they would count with the number blocks. Point out how they replace one more than nine units with one of the next larger unit. Ask them to explain why they would never need ten units, ten rods, or ten layers. Discuss the use of the abacus in exercise 2 similarly, asking children to explain why no more than nine beads are needed to show base-ten numerals. Also, discuss the meaning of the beads in one column compared to those in another. Point out that a bead in the tens' place is ten times as great as a bead in the ones' place, that a bead in the hundreds' place is ten times as great as a bead in the tens' place, and so on.

Using the Ideas

1. Give the number for each of these.
   - A 5 blocks, 3 layers, 2 rods, 4 units 5324
   - B 7 units, 4 layers, 9 blocks, 3 rods 9437
   - C 6 rods, 1 block, 7 units, 5 layers 1567
   - D 12 layers 1200

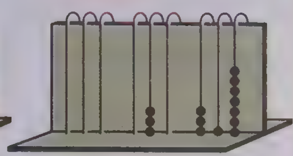
2. Give the number shown by each abacus.

A 314

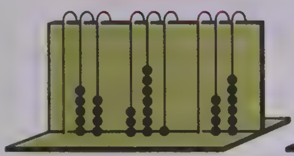


B 4114

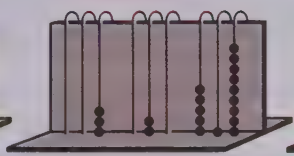
C 30 317



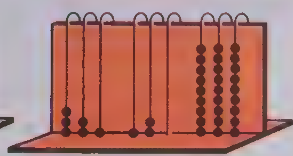
D 54 371 046



E 3 020 519



F 321 120 999



3. Solve the equations.
   - A $2756 = 2000 + 700 + v + 6$ 50
   - B $3428 = 3000 + 400 + 20 + n$ 8
   - C $1206 = 1000 + a + 6$ 200
   - D $9257 = m + 200 + 50 + 7$ 9000
   - E $6328 = 6000 + 300 + s + 8$ 20
   - F $4005 = t + 5$ 4000
   - G $7065 = 7000 + y + 5$ 60
   - H $8920 = 8000 + b + 20$ 900

think

My name is shown
By one abacus bead.
But to write my name,
Six digits you will need.

WHO AM I?
100 000

More practice, page A-1, Set 1

Follow-up
Encourage children to do research regarding the abacus. The Chinese abacus, called a *suan-pan*, with its use of 5 and 2 beads on each wire, might be very interesting to some. The Japanese version of the abacus, called a *soroban*, uses 5 units and a 5-counter on each wire. Encourage those interested to report on either of these and, if possible, to show how it is used.

Duplicator Masters, page 2
Workbook, page 5
Skill Masters, page 2

Using the Exercises
Assign the exercises on page 23 as independent work. If some children find exercise 3 difficult, encourage them to discuss the solutions with one or two classmates. When the children have finished, check their answers. Relate the place-value meaning of digits in exercises 1 and 2 to the equation form used in exercise 3. Note that this exercise set stresses not only the importance of grouping by tens but also the importance of the agreement that the meaning of a digit depends on its position in the symbol.

Assignments (page 23)
Minimum: 1–2, oral. Average: 1–2.
Maximum: 1–3.

PAGES 24–25

Objective

Given a base-ten numeral for a number less than 1 billion, the child will be able to give the place-value name of any digit in the numeral.

Preparation

Since the number of objects required for providing a demonstration of very large numbers and the place-value ideas associated with these numbers is prohibitive, we have used a Base-Ten Machine, for which the children can imagine lights on and off to represent certain large numbers. Since this machine operates in much the same way as the abacus, one good method of preparation for this lesson is to review the working of the abacus.

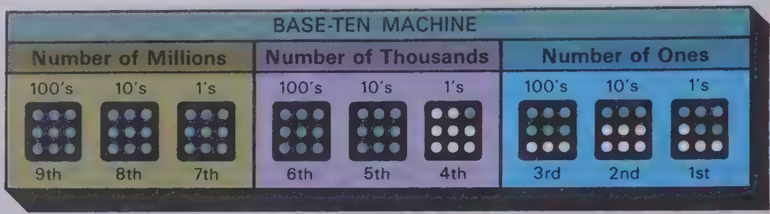
Investigation

Have the children study the Base-Ten Machine and record the number it is showing. Give ample opportunity for discussion of the machine. Have the children describe what lights would come on to signal other numbers. For example, the lights necessary to signal 27 435 would be 2 lights in the 5th place, 7 lights in the 4th place, 4 lights in the 3rd place, 3 lights in the 2nd place and 5 lights in the 1st place. Reverse this procedure by specifying the number of lights showing in each place and having the children name or write the numeral for the lights.

What are the place-value names?

Investigating the Ideas

A card placed in the Base-Ten Machine below makes the lights show a certain number.



? Can you tell how the Base-Ten Machine works?

Record the numeral that it shows. 8364

Discussing the Ideas

1. We use **place value** when we write symbols for whole numbers. Counting from right to left on the Base-Ten Machine, we call the 1st place the **ones'** place, the 2nd place the **tens'** place, and the 3rd place the **hundreds'** place. What is the 4th place called? The thousands' place
2. The 5th place is called the **ten thousands'** place. Think carefully, then decide what the 6th, 7th, 8th, and 9th places are called. 6th: hundred thousands' 7th: millions' 8th: ten millions' 9th: hundred millions'
3. In what way are the abacus and the Base-Ten Machine alike? See Discussion.
4. Each sentence below tells something about the numeral 35 746. Read and complete each sentence.
   - A The 3 is in the **ten thousands'** place.
   - B The 5 is in the ____ ? ____ place. thousands'
   - C The 7 is in the ____ ? ____ place. hundreds'
   - D The 4 is in the ____ ? ____ place. tens'
   - E The 6 is in the ____ ? ____ place. ones'

Discussion

Help the children see how the answers to exercise 2 follow the place-value pattern. That is, once they know that the fourth, fifth, and sixth places represent the number of thousands, it is easy enough for them to decide what the name of each of these places is. In the same way, they can name the seventh, eighth, and ninth places which represent millions, and so on.

We would like to have the children observe that in discussion exercise 3, the Base-Ten Machine and abacus are alike in the sense that for a given position they show a particular digit. For example, if the abacus shows zero beads in the seventh position, the machine would show this by having no lights on in the seventh panel.

Using the Ideas

1. Give the number the Base-Ten Machine was signalled to remember.

A 762


B 4203


C 513 090


D 518 417 417


2. For 683 547 201, tell what digit is in each of these places.
   - A thousands' 7
   - B millions' 3
   - C ten millions' 8
   - D hundred thousands' 5
   - E hundred millions' 6
   - F ten thousands' 4

3. Write a numeral that has 7 in the ten thousands' place. Sample answer: 170 000

4. Write a numeral that has 6 in the millions' place, 4 in the hundred thousands' place, 7 in the ten thousands' place, and zero in all other places. 6 470 000

5. Write a numeral that has 328 thousands, 496 millions, and 507. Now list the names of the first 9 places and give the digit that is in each place. See Answers, T.E. page 25.

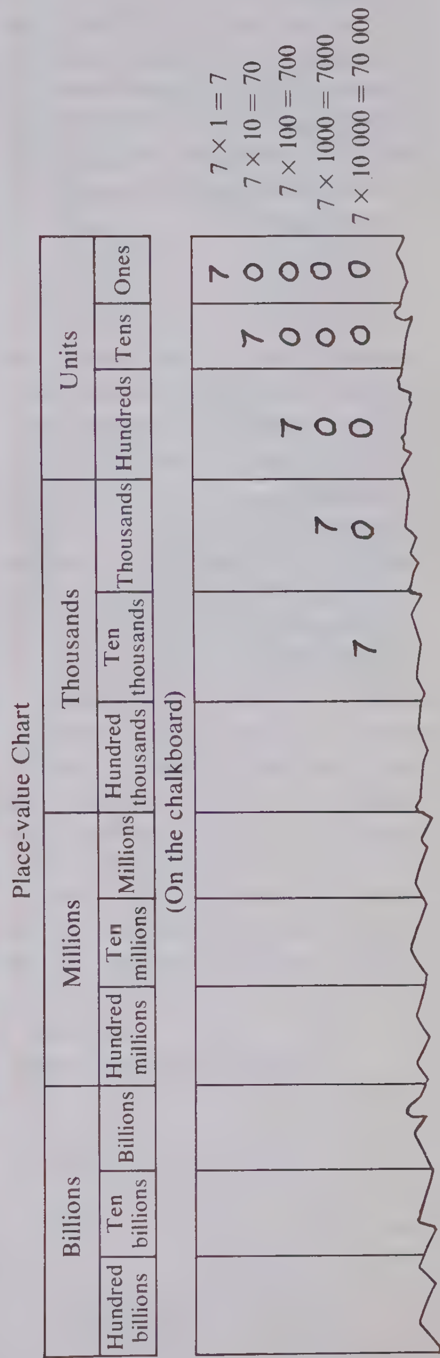
think

I could mean five million,
Or even fifty billion.
My value isn't known
Until my place is shown.

More practice, page A-1, Set 2

25

Follow-up
Make a demonstration-size period chart to help children see and understand place value, periods, and names for large numbers. Label 20-by-120-cm tagboard as shown.



Using the Exercises
Assign the exercises on page 25 according to the interests and needs of your children. You may wish to have exercises 1 and 2 presented orally.

When the children have finished, allow time for discussion and checking paper. Reemphasize the names of the various places.

The *Think* problem, which should be fairly easy for most of the children, continues to emphasize place-value concepts. The fact that the digit 5 could designate five million or fifty billion is an important part of the lesson.

Assignments (page 25)
Minimum: 1–2. Average: 1–4.
Maximum: 1–5.

Answers, exercise 5, page 25
496 328 507
ones – 7 tens – 0 hundreds – 5
thousands – 8 ten thousands – 2
hundred thousands – 3
millions – 6 ten millions – 9
hundred millions – 4

Workbook, page 6

PAGES 26–27

Objective

Given numerals for large numbers, the child will be able to read them by using the names of the periods.

Preparation

To prepare for this lesson, you might write 4-, 5-, and 6-digit numerals on the chalkboard and ask children to give the place value of several of the digits. For example, write the following numerals:

54 397	754 693
810 502	21 002
76 328	319 256

Then ask questions such as, "What digit in which numeral represents 5 ten thousands?" "Name a digit which represents a number of ten thousands."

Such a review should be brisk and brief, serving both as a warm-up activity and as a review of concepts essential to an understanding of place value.

Do you know number names for larger numbers?

Discussing the Ideas

1. Can you read the number in the sentence below?

The oceans of the earth cover 356 877 107 square kilometres.

Follow the directions below to read the number.

First read: **3 5 6** (million) 8 7 7 1 0 7

Then read: 3 5 6 **8 7 7** (thousand) 1 0 7

Then read: 3 5 6 8 7 7 **1 0 7**

2. Read each of these numbers. See Discussion.

A 426 753 127	E 127 240 316	I 17 286 000
B 306 526 319	F 5 103 286	J 80 000 275
C 345 281 407	G 175 024 156	K 100 000 000
D 36 258 342	H 100 300 346	L 65 070 002

3. We read the number in example A as:
six hundred fifty-three billion,
two hundred sixty-one million,
five hundred four thousand,
eight hundred ninety-two.

A: **653** (billion) 261 504 892

B: **492** (trillion) 765 931 407 682

Read the number in example B.
Four hundred ninety-two trillion, seven hundred sixty-five billion, nine hundred thirty-one million, four hundred seven thousand, six hundred eighty-two.

4. Read the numbers in these statements.
   - A Estimated world population in 2000 A.D.: 6 000 000 000. Six billion.
   - B Recent estimate of earth's age: 4 950 000 000 years. Four billion, nine hundred fifty-million
   - C Distance light travels in one year: 9 405 594 460 800 km.

Nine trillion, four hundred five billion five hundred ninety-four million, four hundred sixty thousand, eight hundred

26

Discussion

One of the main concerns of this lesson is to develop the children's ability to read the names for large numbers. Although the term *period* is not introduced until page 27 of the text, you might use it during the discussion.

Encourage volunteers to read the number of square kilometres given in the text. When several have read it, point out the directions which show the grouping of the digits into millions, thousands, and ones (even though we do not say "122 ones"). Explain that the digits of numerals are grouped by threes and that each group of three digits is called a *period.* The three digits in a period are labelled with the name of the period. Thus, we read the number of hundreds, tens, and ones within a period and then give the name of the period. For example, the number in exercise 2A is read, "four hundred twenty-six million, seven hundred fifty-three thousand, one hundred twenty-seven." Also point out that the spaces aid us in reading these numerals by separating the digits into the periods, or groups of three.

It is important for children to be given the opportunity to practice reading numbers orally. Provide additional examples, if necessary.

Using the Ideas

1. For large numbers, digits are grouped by threes. Each group is called a **period.** The names of some of the periods are given below. Give the missing word for each sentence.

Quintillions	Quadrillions	Trillions	Billions	Millions	Thousands	
6 4 3	5 0 1	0 7 6	8 3 0	2 7 5	1 6 3	7 8 5

A The 830 tells how many __?__. Answer: billions
B The 643 tells how many __?__. quintillions
C The 275 tells how many __?__. millions
D The 501 tells how many __?__. quadrillions
E The 076 tells how many __?__. trillions

2. For each number write an equation as in the example.
Example: 3654 = 3000 + 600 + 50 + 4

A 3653	D 26 385	G 163 827	J 1 265 837
B 4821	E 90 371	H 704 346	K 6 860 831
C 7260	F 84 027	I 762 005	L 23 456 029

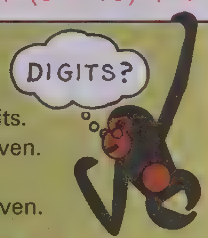
See Answers, T.E. page 27.

3. Sometimes the multiplication sign is used to show place value. For each number write an equation as in the example.
Example: $3654 = (3 \times 1000) + (6 \times 100) + (5 \times 10) + 4$

A 2845	D 27 643
B 6734	E 148 296
C 9258	F 7 268 489

See Answers, T.E. page 27.

think



Count my digits.
You'll find eleven.
All are zeros
Except one seven.

WHO AM I?
70 000 000 000

4. Write the ordinary numeral for each part.

357 A $300 + 50 + 7$
5438 B $5000 + 400 + 30 + 8$
463 C $(4 \times 100) + (6 \times 10) + 3$
7652 D $(7 \times 1000) + (6 \times 100) + (5 \times 10) + 2$

More practice, page A-2, Set 3

27

Follow-up
Allow the children to suggest some numerals and challenge volunteers to write them on the chalkboard, using commas to show the periods. Space information such as the following will provide many numerals to read and write.

Destination in Space	Minimum Distance from Earth
Moon	363 299
Mercury	77 210 000
Venus	38 220 000
Mars	54 490 000
Jupiter	588 410 000
Saturn	1 197 430 000
Uranus	2 586 230 000
Neptune	4 310 930 000
Pluto	4 290 930 000

Also, you may want to provide the class with additional names for very large numbers. These can be found in *The Lore of Large Numbers*, by Philip J. Davis (New York: Random House and L. W. Singer and Company, 1961), page 23.

You might also refer the children to pages 20–25 in Dr. Edward Kasner's book *Mathematics and the Imagination* (New York: Simon and Schuster, 1940). On those pages, Dr. Kasner says that his nephew named a very large number (1 followed by 100 zeros) a googol, and an even larger number (1 followed by a googol of zeros) a googolplex.

Answers, exercise 3, page 27

3.A $2845 = (2 \times 1000) + (8 \times 100) + (4 \times 10) + 5$
B $6734 = (6 \times 1000) + (7 \times 100) + (3 \times 10) + 4$
C $9258 = (9 \times 1000) + (2 \times 100) + (5 \times 10) + 8$
D $27\,643 = (2 \times 10\,000) + (7 \times 1000) + (6 \times 100) + (4 \times 10) + 3$
E $148\,296 = (1 \times 100\,000) + (4 \times 10\,000) + (8 \times 1000) + (2 \times 100) + (9 \times 10) + 6$
F $7\,268\,489 = (7 \times 1\,000\,000) + (2 \times 100\,000) + (6 \times 10\,000) + (8 \times 1000) + (4 \times 100) + (8 \times 10) + 9$

Using the Exercises
Much of the material on page 27 could serve as a basis for further discussion. For example, you might use the chart in exercise 1 to emphasize that numerals for large numbers are not hard to read.

Exercises 2 and 3 stress the meaning of place value by giving children practice with simple expanded notation. Provide other examples to make sure children can expand numerals using the notation in exercise 3.

Answers, exercise 2, page 27

2. A $3000 + 600 + 50 + 3$
B $4000 + 800 + 20 + 1$
C $7000 + 200 + 60$
D $20\,000 + 6000 + 300 + 80 + 5$
E $90\,000 + 300 + 70 + 1$
F $80\,000 + 4000 + 27$
G $100\,000 + 60\,000 + 3000 + 800 + 20 + 7$
H $700\,000 + 4000 + 300 + 40 + 6$
I $700\,000 + 60\,000 + 2000 + 5$
J $1\,000\,000 + 200\,000 + 60\,000 + 5000 + 800 + 30 + 7$
K $6\,000\,000 + 800\,000 + 60\,000 + 800 + 30 + 1$
L $20\,000\,000 + 3\,000\,000 + 400\,000 + 50\,000 + 6000 + 20 + 9$

Assignments (page 27)
Minimum: 1, oral; 2.
Average: 1–3. Maximum: 1–4.

Duplicator Masters, page 3
Workbook, page 7
Skill Masters, page 3

PAGES 28–29

Objective

Given the numerals for two different numbers, the child will be able to compare the numbers and place the proper sign (> or <) between the numerals for the two numbers.

Preparation

Although it would be appropriate to begin immediately with the investigation, you might provide children with practice in reading large numbers which you display on the chalkboard or overhead projector. Keep most of the numbers in the millions and thousands; higher periods need not be emphasized.

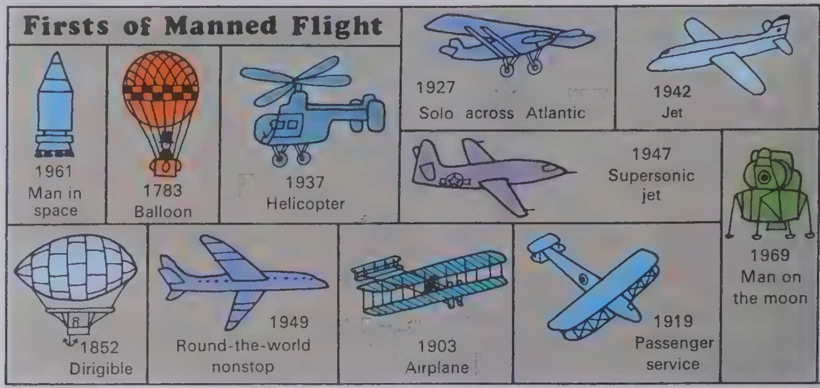
Investigation

Have available suitable reference materials, such as almanacs, encyclopedias, and history books, for children to use in looking for dates of historical significance. Encourage them to look up more than one other date. You might wish to point out that all the dates in the text, when ordered correctly, will show the progression of man's air and space flight history. You might also suggest to the children that they show the order and relationship of these dates by making a time line. Remind them that in a time line the same distance should be used to show a specific number of years.

Encourage the children to illustrate the various events. Long rolls of butcher paper are suitable for such a project. Children might also illustrate their chart if they choose to use this method of showing the dates in order.

Can you compare the "sizes" of numbers?

Investigating the Ideas



Look up at least one more date that interests you.

? Can you make a chart that lists your dates and those above in order? 1783, 1852, 1903, 1919, 1927, 1937, 1942, 1947, 1949, 1961, 1969

Discussing the Ideas

1. Which date used in the Investigation was longest ago? 1783 (unless student chooses an earlier date)
2. Which date is most recent? 1969 (unless student chooses a more recent date)
3. Part of these two numerals is covered. Can you tell which one names the greater number? Explain.

   ▭ 7 5 2
   ▭ 9 8 4

   No, because digits in periods greater than hundreds' are not known.
4. Can you tell which of these is greater if you know they have the same number of digits? Yes

   4 8 6 ▭
   4 7 8 ▭
5. Why can't you be sure which of these named the greater number before the paper was torn?

   1 6 4 7
   2 8 3

   Because the number of digits in each numeral is not known.

28

Discussion

As you work through exercises 1 and 2, you might also have the children calculate how long ago 1783 was. Lead them to observe that progress in the 1900's has been much more rapid than in the two previous centuries. As you work through exercises 3, 4, and 5, review the meaning of the terms and symbols *greater than* (>) and *less than* (<).

You might use a few other exercises to stress both the mathematical meaning of greater than and less than and the particular meaning of each place-value position. For example, write the following exercises on the chalkboard and ask children to supply the correct term, greater than or less than.

36 is 10 ___ 26
785 is 200 ___ 985
8 327 is 1000 ___ 9 327
367 468 is 10 000 ___ 357 468
6 493 684 is 400 000 ___ 6 093 684

Using the Ideas

1. Write the word (greater or less) that should go in each blank.
   - A Since 95 is __?__ than 52, we write $95 > 52$. greater
   - B Since 48 is __?__ than 79, we write $48 < 79$. less
   - C Since 256 is __?__ than 254, we write $256 > 254$. greater
   - D Since 714 is __?__ than 734, we write $714 < 734$. less
2. Give the correct sign ($<$ or $>$) for each ●.

A 36 ● 26 $>$	E 653 ● 597 $>$	I 15 286 ● 15 317 $<$
B 78 ● 74 $>$	F 6328 ● 6348 $<$	J 162 834 ● 162 097 $>$
C 326 ● 356 $<$	G 6428 ● 6228 $>$	K 9 762 483 ● 7 651 642 $>$
D 785 ● 985 $<$	H 9761 ● 9716 $>$	L 5 400 122 ● 5 399 876 $>$

3. Give the correct sign ($=$ or $\neq$) for each ●.
   - A 3624 ● 3000 + 600 + 20 + 4 $=$
   - B 72 437 ● 70 000 + 2000 + 400 + 40 + 7 $\neq$
   - C 632 847 ● 600 000 + 30 000 + 2800 + 47 $=$
   - D 56 482 ● 50 000 + 6000 + 410 + 70 + 2 $=$
   - E 734 ● $(7 \times 100) + (3 \times 10) + 4$ $=$
   - F 9284 ● $(9 \times 1000) + (8 \times 100) + (2 \times 10) + 4$ $\neq$
4. Give the number 100 000 greater than 54. 100 054
5. Give the number 10 000 000 greater than 54 365 847. 64 365 847
6. Give the number 1 000 000 less than
   - A 1 000 038 38
   - B 1 467 896 467 896
7.
   - A Give the largest 3-digit number that has the digits 4 and 7. 974
   - B Give the smallest 4-digit number that has only one 0 digit. 1011
   - C Give the largest 6-digit number with no two digits alike. 987 654

think

Give the largest number that uses all the digits once.

Give the smallest such number.

More practice, page A-2, Set 4

Mathematics

We define $a > b$ as follows:

> a is greater than b if and only if $a - b$ is a whole number c other than zero. That is, there exists a whole number c ($c \neq 0$), such that $a = b + c$.

Most children understand this definition intuitively. Inequalities and their symbols are utilized in this section to further clarify the place-value concepts.

Follow-up

If the children did not use a time line in the investigation, you might suggest that they do so as a follow-up activity. Or, they may make other time lines showing dates of other related events, such as important dates in Canadian history, or dates of inventions related to all forms of transportation, etc.

Duplicator Masters, page 4
Workbook, page 8
Skill Masters, page 4

Using the Exercises

After you have sufficiently reviewed the symbols $>$, $<$, $=$, and $\neq$, assign the exercises on page 29 as independent work. When the children finish, use some examples to stress the role of place value in comparing numbers. For example, in exercise 2F, it is not until the tens' place is examined that the inequality can be recognized.

Most children will be able to answer exercise 7 and the *Think* problem if they are encouraged to use a trial-and-error method. If time permits, you might have children read some of the large numbers on this page as an oral activity.

Assignments (page 29)

Minimum: 1–3, oral.
Average: 1–2, oral; 3–4.
Maximum: 1–2, oral; 3–7.

PAGES 30–31

Objective

The child will make estimations using large numbers such as thousands, millions, and billions.

Preparation

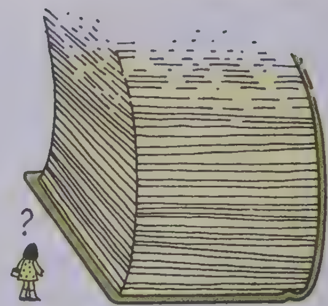
Before beginning the investigation, you might spend a minute asking the children to explain what "guessing" about something means. Make sure they realize the difference between "guessing" and "figuring out" a problem. Stress that a guess requires reasonable thinking but no computation, whereas "figuring out" a problem usually means performing some computation.

Investigation

Encourage the children to share with each other the guesses they made for the investigation and to discuss ways of figuring out how tall the 1 000 000 pages would be. Using the hint that is given, some may think: "In 1 000 000 pages there are 1000 pages, 1000 times. Since each of these 1000 pages is about 5 cm high, 1 000 000 would be about 5000 cm high." It would be helpful to suggest that the children relate 5000 cm to something they know. For example, 5000 cm is 50 metres, which is an approximate height of some twelve-storey buildings. You might suggest that children measure the height of their classroom (floor to ceiling) and compare this height to that of the 1 000 000 pages.

Investigating the Ideas

A book with 1000 pages would be about 5 centimetres thick.



Guess how tall your mathematics book would be if it had 1 000 000 pages.

About 5000 cm (or about 50 metres)

? Can you **figure out** about how tall your book would be if it had 1,000,000 pages? See Investigation.

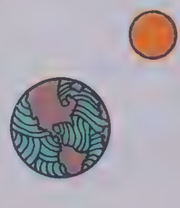
Discussing the Ideas

1. How large is each of these numbers? First guess the number of digits. Then check your guess. See Discussion.

A

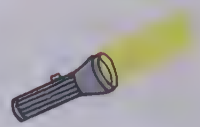
Population of the capital of your province

B



Distance in kilometres to the sun

C



Speed of light in kilometres per second

2. Find a use for at least one number that has more than 4 digits. Report your findings to the class. Answers will vary.

30

Discussion

It would be possible to use the discussion exercises as supplemental investigation questions and ask children to explore them without class discussion. However, if sufficient reference materials are not available, you may choose to write the numbers for exercise 1 on the chalkboard after you have given the children time to make their guesses.

Population of the provincial capital —(variable)

Distance to sun — 148 179 240 km (average)

Speed of light — 300 000 km per second

Using the Ideas

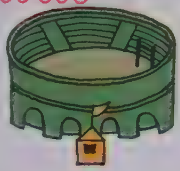
1. Match each picture with one of these large numbers.

1 000

100 000

1 000 000

A 100 000



Number of seats in a large stadium

B 1000

Number of pages in a thick telephone book

C 1 000 000

Number of grains of sand in a cup

2. Which is more,
   - A 100 thousands or a million?
   - B a thousand thousands or 10 millions?
   - C a thousand hundreds or a million?
   - D a billion or 100 millions?

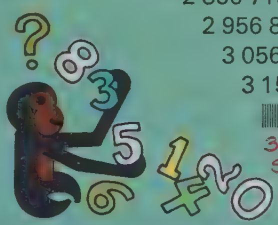
3. Write the number that is
   - A a thousand thousands. 1 000 000
   - B a thousand hundreds. 100 000
   - C ten millions. 10 000 000
   - D a thousand more than a million. 1 001 000
   - E a hundred millions. 100 000 000
   - F a thousand millions. 1 000 000 000
   - G a million more than a million. 2 000 000
   - H a million more than a billion. 1 001 000 000

think

Find the next number in this sequence.

2 856 719
2 956 819
3 056 919
3 157 019
3 257 119

See also, Using the Exercises.



31

Follow-up

Encourage children to make a chart showing at least one use for each power of ten. Some might have to do considerable research to find appropriate numbers. Those listed below are given as samples; do not present them to the children until they have utilized every opportunity to find their own.

- 10 — 10 pennies equal one dime
- 100 — 100 centimetres in one metre
- 1 000 — 1000 pages in a large telephone book
- 10 000 — the earth's diameter is about 12 740 kilometres
- 100 000 — Rose Bowl Stadium in Pasadena, Calif., holds about 100 000 people; in 1970, 253 183 people attended the 5 World Series baseball games.
- 1 000 000 — 1 square kilometre equals 1 000 000 square metres
- 10 000 000 — 21 377 000 people live in Canada (1971)
- 100 000 000 — About 200 000 000 people live in the United States (1970)

Using the Exercises

Assign the exercises on page 31 as independent work. Once children establish that a thick telephone book has about 1000 pages, they should be able to match the remaining numbers without much difficulty. In exercises 2 and 3, have children write down the numbers that they compare and identify. Remind them that the space can help them in reading and writing large numbers such as these.

Since the *Think* problem is quite challenging, you might wish to point out to the children that, in the sequence, the hundred thousands' place and the hundreds' place increase by one each time. Thus, to get successive numbers after the first, the children can keep adding 100,100. Make sure that the more able children are given an opportunity to discover this pattern before the correct answer is given.

Assignments (page 31)

Minimum: 1–2. Average: 1–3. Maximum: 1–3.

PAGES 32–33

Objectives

Given a set of fifteen or fewer objects, the child will be able to write a base-four numeral for the number of objects.

Given a two-digit number in base four, the child will be able to give its base-ten equivalent.

Preparation

To prepare for this lesson, review with the children the idea of grouping objects by tens and then writing numerals for the number of tens and the number left over. For example, ask them to write the numerals for the following:

3 tens and 6
8 tens and 2
5 tens and 0
4 tens and 9

Point out that, since the grouping is by ten, we do not write the symbol for ten (10) in either the tens' or the ones' position. For example, if we had 3 tens and 10, we would write 4 tens and 0. A form of this rule will be an important part of the investigation.

Investigation

It is important that you help children understand what they are being asked in the investigation, but you should try to do so without preempting their opportunity for discovery. The goal of the investigation is for children to take the fourteen pencils and group them in various ways, such as in groups of five, seven, or three. They are to record their groupings as shown in the example. The rule requires that the number which shows how many groups or singles never equal or exceed the number in the group. For example, 4 threes and 2 is not acceptable because 4 is greater than three, the number in the group. Some acceptable responses are as follows:

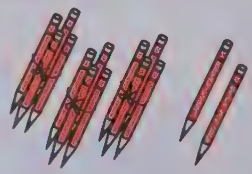
2 sevens and 0	1 ten and 4
1 eleven and 3	2 sixes and 2
2 fives and 4	1 eight and 6

● *Let's explore different bases for numerals.*

Investigating the Ideas

FOURTEEN PENCILS

Grouped by fours



3 fours and 2

Grouping Rule

These numbers are less than the group size.

?	How many other ways can you find to group fourteen pencils?	Record only the groupings that follow the grouping rule.

See Investigation.

Discussing the Ideas

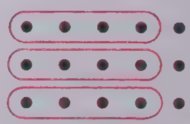
1. Study the chart below. Then explain how to write a numeral for some of the grouping you did in the Investigation. See Discussion.

NUMBER	GROUPING	NUMERAL
fourteen	3 fours and 2 (base four)	$32_{(4)}$

2. A How many groups of four dots are in this set? 3

   B How many dots are left over? 3

   C What numeral in base four tells how many dots are in the set? $33_{(4)}$



Grouping by fours

3. Explain how to write the base-five and base-six numerals for the set of dots. $30_{(5)}$ $23_{(6)}$

32

Discussion

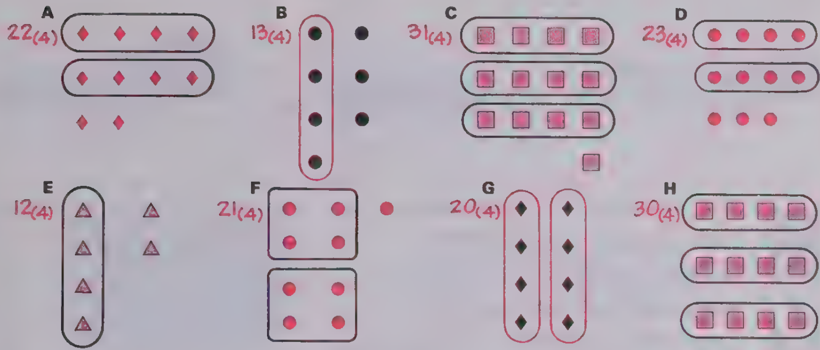
As you discuss exercise 1, present a few more examples until some of the children are able to explain the meaning of these numerals. For example, write other base numerals for fourteen:

2 fives and 4 (base five) $24_{(5)}$
2 sixes and 2 (base six) $22_{(6)}$
1 eight and 6 (base eight) $16_{(8)}$

It is essential that the children realize which digit shows the number of groups of the base number you have. Thus, the "2" in $24_{(5)}$ represents 2 groups of five, while the "2" in the same position in $22_{(6)}$ represents 2 groups of six. Use exercises 2 and 3 to discuss how different groups are represented by digits according to the base indicated by the subscript (the subscript is the small base number written in parentheses to the side and slightly below the numeral digits). A numeral in a base other than ten is read by using the names of each digit. Thus, "$32_{(4)}$" is read "three, two, base four" and "$24_{(5)}$" is read "two, four, base five."

Using the Ideas

1. Write the **base-four** numeral for the number of each set. (Example: For 2 fours and 3, write $23_{(4)}$.)



2. Write a base-four numeral for the number eleven. It may help to draw and group eleven dots. $23_{(4)}$

3. Continue the counting in base four.

$0_{(4)}$	$1_{(4)}$	$2_{(4)}$	$3_{(4)}$	$10_{(4)}$	$11_{(4)}$	$12_{(4)}$	$13_{(4)}$	$20_{(4)}$	$21_{(4)}$	$22_{(4)}$	$23_{(4)}$	$30_{(4)}$	$31_{(4)}$	$32_{(4)}$

★ 4. Study the example. Then complete each sentence. (Example: $32_{(4)}$ means 3 fours and 2 or 14 in base ten.)

- **A** $23_{(4)}$ means 2 fours and 3 or __?__ in base ten. 11
- **B** $12_{(4)}$ means 1 four and ___ or __?__ in base ten. 6
- **C** $31_{(4)}$ means ___ fours and 1 or __?__ in base ten. 13
- **D** $30_{(4)}$ means ___ fours and ___ or __?__ in base ten. 12
- **E** $21_{(4)}$ means ___ fours and ___ or __?__ in base ten. 9
- **F** $20_{(4)}$ means ___ fours and ___ or __?__ in base ten. 8
- **G** $10_{(4)}$ means ___ fours and ___ or __?__ in base ten. 4
- **H** $13_{(4)}$ means ___ fours and ___ or __?__ in base ten. 7

33

Using the Exercises

Depending on the abilities of the children, assign the exercises on page 33 either as independent work or as group work. Note that all of these exercises deal with base-four numerals. Exercise 3 gives children an opportunity to count in base four. It may take a while for some children to realize that $10_{(4)}$ is not "ten," and that $12_{(4)}$ is not "twelve." Reading these numerals properly (for example, "one, zero, base four") will help develop proper concepts of place value in whatever base the child is working.

Resources for Active Learning

Mathematics in Modules, WN18, Addison-Wesley.

Nuffield Project: *Computation and Structure* 2, "Place Value," pp. 70–81, Wiley.

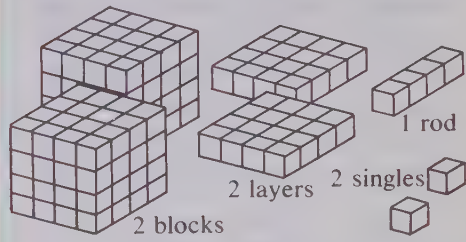
Assignments (page 33)

Minimum: 1–2. Average: 1–3.
Maximum: 1–4.

Mathematics

In our base-ten numeration system, we group objects by tens and use place value to write numerals for numbers. This grouping by tens is probably related to the fact that man has ten fingers. An interesting game for students is to use groupings other than ten and write other numerals for whole numbers.

Suppose we agree to group by fours rather than tens, as illustrated below.



According to the new place-value agreement, we would write 2212 for the number of cubes in the figure. Instead of representing the number two thousand two hundred twelve, the numeral 2212 now represents 2 sixty-fours, 2 sixteens, 1 four, and 2. To indicate that the numeral represents base-four groupings, let us agree to write it as $2212_{(4)}$. Of course, we do not bother to use the subscript to indicate the grouping for base ten; it is understood that the usual grouping is by tens.

Follow-up

Children may explore base four by using sugar cubes to build rods, layers, and blocks, according to the following equivalencies:

4 single cubes → 1 rod (of 4 cubes)
4 rods → 1 layer (of 4 rods or 16 cubes)
4 layers → 1 block (of 4 layers, 16 rods, or 64 cubes)

Other children may enjoy practicing grouping and writing numbers of items in base-four numeration, such as 12 erasers, 13 pencils, 8 sheets of paper, 9 pins.

Workbook, page 9

PAGES 34–35

Objective

Given simple addition and multiplication problems with base-four numerals, the child will be able to give the sums or products in base-four notation.

Preparation

Materials

counters (15 per child or per group)

Review with the children the writing of various 2-digit base-four numerals and their meanings. For example, write $13_{(4)}$ and ask children to name the base-ten numeral it represents. (They should respond "$13_{(4)}$ is 1 four and 3, which is 7.") Use several examples of this type:

$23_{(4)}$	2 fours and 3 or 11
$10_{(4)}$	1 four and 0 or 4
$20_{(4)}$	2 fours and 0 or 8
$31_{(4)}$	3 fours and 1 or 13

Investigation

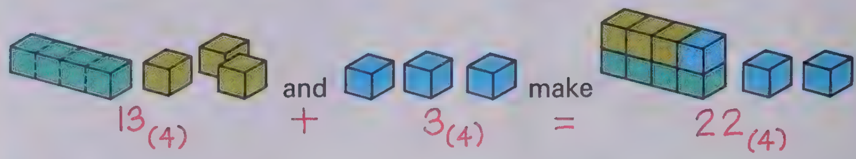
In this investigation, children are given an opportunity to concretely add and regroup in base four. Let the children attack the problems in the investigation in whatever way they choose. Some may immediately arrange their counters into two groups of 4, and 3. When they add the 2 more counters to their original arrangement, they must regroup the counters. If children give $25_{(4)}$ as the answer, remind them that, just as we would never use an "eleven" in the ones' place in base ten, so also we do not use a "five" in the ones' place in base four; rather, we regroup to form an additional group of four so that $23_{(4)} + 2_{(4)} = 31_{(4)}$. Some children may need to have other examples suggested before they make up their own; for example:

$30_{(4)} + 3_{(4)}$, $12_{(4)} + 13_{(4)}$,
$22_{(4)} + 2_{(4)}$, $21_{(4)} + 3_{(4)}$,
$13_{(4)} + 3_{(4)}$, $11_{(4)} + 20_{(4)}$,
$10_{(4)} + 13_{(4)}$, $3_{(4)} + 23_{(4)}$

Can base four be used in computing?

Investigating the Ideas

Study the diagram below.

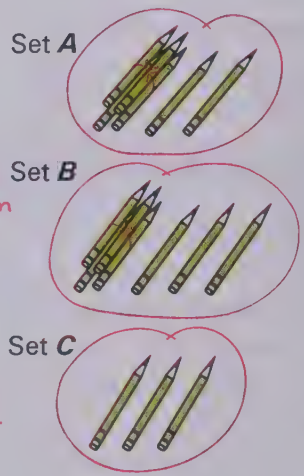


Can you use a set of fifteen or fewer counters to help you write and solve some more base-four problems? See Investigation.

$3_{(4)} + 3_{(4)} = n_{(4)}$
$10_{(4)} + 11_{(4)} = n_{(4)}$
$11_{(4)} + 2_{(4)} = n_{(4)}$

Discussing the Ideas

See Discussion.



1. A Explain how you can use sets *A* and *B* to "prove" this equation is correct.

   $12_{(4)} + 13_{(4)} = 31_{(4)}$

   Combine and regroup sets A and B as 3 sets of four and 1 more, which is written as $31_{(4)}$.

   B Use sets *A* and *C* to help you "prove" this equation is correct.

   $12_{(4)} - 3_{(4)} = 3_{(4)}$

   Regroup set A as 0 fours and 6, and subtract set C. Result is 3.

2. How could you think of Set *B* twice to "prove" this equation is correct?

   $2_{(4)} \times 13_{(4)} = 32_{(4)}$

   Doubling set B would give 2 groups of four and 6 more, which can be regrouped as 3 groups of 4 and 2 more, or $32_{(4)}$.

34

Discussion

As you discuss the investigation problems and exercise 1, stress the fact that every four counters must be thought of as a group. Thus, $3_{(4)} + 2_{(4)}$ counters must be regrouped from five single counters to 1 group of four and 1 single. Point out the similarity with adding numbers in base ten, such as $6 + 5$, which in base ten must be regrouped from eleven single counters to one group of ten counters and 1 single.

While discussing exercise 2, recall with the children that 2 times a number can also be found by thinking of adding that number to itself. Note that, although exercises 1 and 2 deal with both addition and multiplication, there is no intention of having children master algorithmic skills in base four. The entire lesson should be an enjoyable investigation based on an understanding of place value rather than algorithmic processes.

Using the Ideas

1. Copy and complete the addition and multiplication tables for base four.

+	$0_{(4)}$	$1_{(4)}$	$2_{(4)}$	$3_{(4)}$
$0_{(4)}$	$0_{(4)}$	$1_{(4)}$	$2_{(4)}$	$3_{(4)}$
$1_{(4)}$	$1_{(4)}$	$2_{(4)}$	$3_{(4)}$	$10_{(4)}$
$2_{(4)}$	$2_{(4)}$	$3_{(4)}$	$10_{(4)}$	$11_{(4)}$
$3_{(4)}$	$3_{(4)}$	$10_{(4)}$	$11_{(4)}$	$12_{(4)}$

×	$0_{(4)}$	$1_{(4)}$	$2_{(4)}$	$3_{(4)}$
$0_{(4)}$	$0_{(4)}$	$0_{(4)}$	$0_{(4)}$	$0_{(4)}$
$1_{(4)}$	$0_{(4)}$	$1_{(4)}$	$2_{(4)}$	$3_{(4)}$
$2_{(4)}$	$0_{(4)}$	$2_{(4)}$	$10_{(4)}$	$12_{(4)}$
$3_{(4)}$	$0_{(4)}$	$3_{(4)}$	$12_{(4)}$	$21_{(4)}$

2. Find the sums.

A $12_{(4)} + 1_{(4)} = 13_{(4)}$ B $12_{(4)} + 2_{(4)} = 20_{(4)}$ C $22_{(4)} + 1_{(4)} = 23_{(4)}$ D $22_{(4)} + 2_{(4)} = 30_{(4)}$ E $21_{(4)} + 3_{(4)} = 30_{(4)}$

3. Can you find and correct the mistake on Jim's paper?
#6 should be $32_{(4)}$.

Multiplication Base Four — Jim

1. $2_{(4)} \times 2_{(4)} = 10_{(4)}$ 2. $3_{(4)} \times 2_{(4)} = 12_{(4)}$ 3. $3_{(4)} \times 3_{(4)} = 21_{(4)}$

4. $12_{(4)} \times 2_{(4)} = 30_{(4)}$ 5. $10_{(4)} \times 3_{(4)} = 30_{(4)}$ 6. $13_{(4)} \times 2_{(4)} = 26_{(4)}$

4. Find these products.

A $3_{(4)} \times 2_{(4)} = 12_{(4)}$ B $11_{(4)} \times 3_{(4)} = 33_{(4)}$ C $12_{(4)} \times 2_{(4)} = 30_{(4)}$

D $11_{(4)} \times 2_{(4)} = 22_{(4)}$ E $23_{(4)} \times 1_{(4)} = 23_{(4)}$ F $13_{(4)} \times 2_{(4)} = 32_{(4)}$

5. Thinking about addition will help you find these differences.

A $3_{(4)} - 2_{(4)} = 1_{(4)}$ B $10_{(4)} - 2_{(4)} = 2_{(4)}$ C $11_{(4)} - 2_{(4)} = 3_{(4)}$ D $12_{(4)} - 2_{(4)} = 10_{(4)}$ E $20_{(4)} - 2_{(4)} = 12_{(4)}$ F $31_{(4)} - 2_{(4)} = 23_{(4)}$

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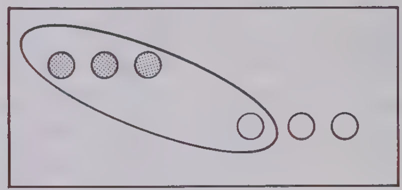
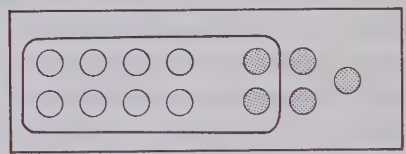
Using the Exercises
Encourage the children to complete the tables independently or in small groups. Circulate throughout the room to be sure that the tables are correct. The remaining exercises may be approached in a variety of ways. Some children may choose to use counters and concretely work through each problem, expressing their answers in base four. Others may choose to rely heavily upon the addition and multiplication tables they completed in exercise 1. Others may be capable enough to think through the regrouping process in base four. Encourage creativity and a variety of solutions. Although some children will want to work independently, small-group co-operation in trying to solve such problems will often stimulate original thinking and aid children in attacking such problems.

Assignments (page 35)
Minimum: 1–2. Average: 1–4.
Maximum: 1–5.

Mathematics
It is interesting and instructive to work with a base other than ten. It is not easy for adults to understand why children have difficulty with a problem such as $8 + 5 = 13$. On the other hand, how easy is it for someone new to base-four numeration to understand that $3 + 3 = 12_{(4)}$? To investigate this problem consider the sets in the following figure. Two sets of three are regrouped into one set of four and two more by a ring drawn around a set of four. Hence, $3 + 3 = 12_{(4)}$ (1 four and 2).



This technique of grouping sets is the very one we use to introduce children to sums such as $8 + 5$. For example, we present them with a set of 8 and a set of 5, as illustrated. We then have them ring a set of ten, and from this they see 1 ten and 3, or 13.



Observe that while we need ten digits for base ten, we need only four digits for base four. The usual base-ten digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Here we use 0, 1, 2, and 3 for base-four digits.

From this discussion, you can see that it is extremely important for children to have a firm understanding of place-value concepts. This reason, perhaps more than any other, justifies the time spent in studying number bases other than base ten.

Duplicator Masters, page 5
Skill Masters, page 5

PAGES 36–37

Objective

Given a table of Roman, Greek, and Egyptian numerals corresponding to the usual Hindu-Arabic numerals, the child will be able to give the numerals for numbers in each of the three numeration systems, and conversely.

Preparation

To prepare for this lesson, you might exhibit 13 objects and ask the children to write both the base-ten numeral and the base-four numeral for the number of objects shown. When the children have written 13 and $31_{(4)}$, point out that the digits 1 and 3 appear in both numerals. Ask if anyone knows another way of writing numerals without using the ordinary Hindu-Arabic digits (or their written names). Some children are likely to recall Roman numerals, which are presented in this lesson. It is unlikely that any will be familiar with Greek or Egyptian numerals.

Investigation

Allow time for the children to study the three kinds of numerals presented in the table. It would be best not to discuss how numerals are formed in the three systems until the children have had an opportunity to select some number and attempt to form the numeral for it in each of the three systems. Children might then place the numerals for their numbers on the chalkboard; this should bring about a lively discussion and lead smoothly into the discussion exercises on the lower part of the page.

Although you need not use the names of the Greek symbols with the children, they are given here for your convenience:

1 – alpha	10 – iota
2 – beta	20 – kappa
3 – gamma	30 – lambda
4 – delta	40 – mu
5 – epsilon	50 – nu
6 – sigma (bau)	60 – xi
7 – zeta	70 – omicron
8 – eta	80 – pi
9 – theta	90 – koppa
100 – rho	

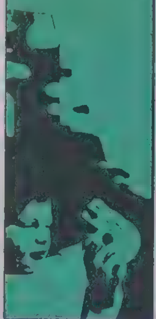
● *Let's explore some old numerals.*

Investigating the Ideas

This table shows three kinds of numerals used long ago. Roman numerals are still used today. Study the table and the examples.

EGYPTIAN	GREEK	ROMAN
1 = 𓏺 10 = 𓎆 100 = 𓍢 1000 = 𓆼 1 000 000 = 𓁨	1 = α 7 = ζ 40 = μ 2 = β 8 = η 50 = ν 3 = γ 9 = θ 60 = ξ 4 = δ 10 = ι 70 = ο 5 = ϵ 20 = κ 80 = π 6 = ϛ 30 = λ 90 = ϙ 100 = ρ	1 = I 5 = V 10 = X 50 = L 100 = C 500 = D 1000 = M
126 = 𓍢𓎆𓎆𓏺𓏺𓏺𓏺𓏺𓏺	126 = ρκϛ	126 = CXXVI

? Can you choose another number and write the numeral for it in each of the three systems? Choice of numbers will vary. See Investigation.



Discussing the Ideas

1. Which numeration system would be harder to learn, the Egyptian or the Greek system? Greek Why? It has more symbols.
2. Does the Egyptian system have place value like our system? No
3. The Roman system **does not** use place value. When two Roman symbols are placed side by side, we **add** or **subtract** to determine the number represented. For example, when the smaller number is represented on the right, we add.

   VI means 5 + 1, or 6.

   When the smaller number is represented on the left, we subtract.

   IV means 5 − 1, or 4.

   Explain what number each of these Roman numerals shows.

A IX 9	C XL 40	E CD 400	G MC 1100	I DIX 509
B XI 11	D LX 60	F DC 600	H CM 900	J MCM 1900

36

Discussion

An important point to stress in the discussion is that none of the three ancient numerations systems illustrated in the investigation used place value as does our decimal system. The Egyptian system is an *additive* system. That is, each unit symbol of each order was repeated as many times as necessary to represent the given number. Thus 32 was represented as

𓎆𓎆𓎆𓏺𓏺

and 234 was represented as

𓍢𓍢𓎆𓎆𓎆𓏺𓏺𓏺𓏺.

The order in which the symbols were written made no difference.

Roman numerals present no difficulty except, perhaps, the rules for adding and subtracting with regard to smaller and larger numbers represented by the various adjacent Roman numerals.

The Greek numeration system, which employed the letters of the Greek alphabet as numerals, was also an additive system. None of the three systems had a separate symbol for zero.

Using the Ideas

1. Write the Egyptian numeral for each of the following.
   A 14 B 23 C 172 D 1974 E 1 002 653
2. Give the ordinary numeral for each Egyptian numeral.
   A 132 B 1 001 111 C 1 002 323
3. Write the Greek numeral for each of the following.
   A 32 λβ B 58 υη C 181 ρπα D 95 ϙε E 199 ρϙθ
4. Give the ordinary numeral for each Greek numeral.
   A πγ 83 B ρλδ 134 C ρπε 185 D ϙη 98 E ρξϛ 166
5. Write a Roman numeral for each of the following.
   A 12 XII B 24 XXIV C 29 XXIX D 58 LVIII E 149 CXLIX F 3624 MMMDCXXIV
6. Give the ordinary numeral for each Roman numeral.
   A XIII 13 B IX 9 C XIX 19 D LIV 54 E CDIX 409
7. Give the Roman numerals for the numbers
   A 1 through 20. I, II, III, IV, V, VI, VII, VIII, IX, X, XI, XII, XIII, XIV, XV, XVI, XVII, XVIII, XIX, XX
   B 10 through 100 (counting by tens). X, XX, XXX, XL, L, LX, LXX, LXXX, XC, C
   C 100 through 1000 (counting by hundreds). C, CC, CCC, CD, D, DC, DCC, DCCC, CM, M

think

With ten toothpicks, "write" as many Roman-numeral equations as you can.

See Using the Exercises.

Using the Exercises

Assign the exercises on page 37 as independent work. You may wish to place the main emphasis on the Roman numeral system. If so, assign exercises 5, 6, and 7.

The *Think* problem can be used to provide some recreational mathematics with Roman numerals. There are many different solutions that can be found. If exactly 10 toothpicks must be used in each equation, the problem is harder. Some possible solutions follow.

V + V = X	I + IV = V
X + I = XI	I + V = VI
L + I = LI	I + IX = X
I + II = III	

Resources for Active Learning

Applied Mathematics Cards, "Roman Numerals," Group 2/2, Schofield and Sims. (Available from Mafex Associates, Willowdale, Ont.)

Nuffield Project: *Computation and Structure* 3, pp. 10–11, Wiley.

Assignments (page 37)

Minimum: 5–7. Average: 1–2, 5–7.
Maximum: 1–7.

Mathematics

The seven symbols of the Roman numeral system have come down to us through the centuries. There is evidence that early Roman numerals used only an additive principle. Thus, 4 was IIII, and 44 was XXXXIIII. The "subtractive principle" evolved much later. The usual rules for the "subtractive principle" are as follows:

I is subtracted only from V or X when it immediately precedes either.
X is subtracted only from L or C.
C is subtracted only from D or M.

However, if agreed upon, other subtractive rules could be used. For example, if we agree that I can be subtracted from L, then 49 in Roman numerals might be written as IL or as XLIX.

Follow-up

Encourage those interested to do further work on number bases or to do research on other number systems. Some references may be found in the Books to Explore section on pages A30–A32 of the text. Other references follow.

Adler, Irving and Ruth, *Numbers Old and New*. (New York: John Day, 1960). This book explains the counting methods of Australian natives and Mayan Indians, and the fractions used by Egyptians and Greeks. (Available from Longmans)

Simon, Leonard, *The Day the Numbers Disappeared.* (New York: Whittlesey House, 1963). The author compares Egyptian, Greek, and Roman numerals. (Available from McGraw-Hill Ryerson)

Smith, George O., *Mathematics: The Language of Science*. (New York: G. P. Putnam's Sons, 1961). Interesting stories in this book tell how Roman businessmen struggled with Roman numerals and how the Babylonians discovered place value. (Available from Longmans)

PAGES 38–39

Objective

The child will demonstrate his ability to work with the concepts presented in this chapter.

Preparation

Review with the children any topics which seemed particularly troublesome during the study of the chapter. It would also be helpful to write some numerals of 4, 5, 6, and 7 digits on the chalkboard, and ask the children to show the expanded notation for each. Also, you might write pairs of numbers on the chalkboard and ask children to insert a greater than or less than symbol between each pair to form a correct inequality.

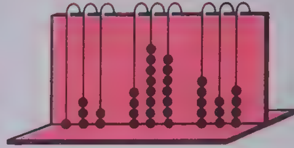
Reviewing the Ideas

1. Give the numeral for each picture.

A 2546

B 43 904

C 132 487 534



2. Solve these equations.

A $3458 = 3000 + 400 + t + 8$ 50

B $7391 = 7000 + 300 + 90 + s$ 1

C $1509 = 1000 + y + 9$ 500

D $6854 = z + 800 + 50 + 4$ 6000

E $8006 = m + 6$ 8000

F $4472 = 4000 + x + 70 + 2$ 400

3. For each number write an equation as in the example.
(Example: $2734 = 2000 + 700 + 30 + 4$) See Answers, T.E. page 38.

A 6218 C 3975 E 84 721 G 2 796 458

B 7466 D 9218 F 76 097 H 34 681 075

4. Give the correct sign (< or >) for each ◍.

A 57 ◍ 47 >

B 74 ◍ 75 <

C 342 ◍ 362 <

D 324 ◍ 326 <

E 236 ◍ 234 >

F 623 ◍ 432 >

G 651 ◍ 703 <

H 8426 ◍ 8326 >

I 7521 ◍ 7512 >

J 9236 ◍ 8236 >

K 6421 ◍ 6399 >

L 7846 ◍ 5399 >

M 9696 ◍ 9710 <

N 646 387 ◍ 645 999 >

O 6 287 512 ◍ 6 300 000 <

5. Give the correct sign (<, =, or >) for each ◍.

A 28 260 ◍ 20 000 + 8000 + 200 + 60 =

B 45 024 ◍ 40 000 + 5000 + 200 + 40 <

C 675 800 ◍ 600 000 + 70 000 + 5000 + 80 >

38

Discussion

If your main emphasis in this chapter has been on base-ten numeration and place value, you may want to limit the review to the exercises on page 38. The exercises on page 39 review base-four numerals and Roman numerals.

After children have finished working the exercises, check their work and allow time for questions.

In working the *Think* problem, children must recognize that we consider a 4-digit numeral to be one that contains no zero in the fourth place; that is, the number of thousands cannot be zero for a 4-digit numeral.

Answers, exercise 3, page 38

3.A $6218 = 6000 + 200 + 10 + 8$

B $7466 = 7000 + 400 + 60 + 6$

C $3975 = 3000 + 900 + 70 + 5$

D $9218 = 9000 + 200 + 10 + 8$

E $84\,721 = 80\,000 + 4000 + 700 + 20 + 1$

F $76\,097 = 70\,000 + 6000 + 90 + 7$

G $2\,796\,458 = 2\,000\,000 + 700\,000 + 90\,000 + 6000 + 400 + 50 + 8$

H $34\,681\,075 = 30\,000\,000 + 4\,000\,000 + 600\,000 + 80\,000 + 1000 + 75$

6. Give the base-four numeral for each set.

A 13(4)

B 21(4)

C 30(4)

D 10(4)

7. Give the base-ten numeral for each exercise.

A $21_{(4)}$ 9 B $31_{(4)}$ 13 C $13_{(4)}$ 7 D $22_{(4)}$ 10 E $10_{(4)}$ 4 F $2_{(4)}$ 2

8. Write the Roman numeral for each of the following.

A 1000 M B 500 D C 90 XC D 3 III

9. Combine the Roman numerals from exercise 8 to write 1593.
MDXCIII

10. Write a Roman numeral for each of these.

A 243 C 1256 E 1974
B 594 D 2349 F 1492

A CCXLIII C MCCLVI E MCMLXXIV
B DXCIV D MMCCCXLIX F MCDXCII

11. Give the Roman numeral for each number in exercise 7.

A IX B XIII C VII
D X E IV F II

12. What is the largest 4-digit number that uses only the digits 3, 5, and 7? 7753

think

Find a four-digit number so that the number of hundreds plus the number of thousands is one, the number of tens is seven times the number of thousands, and the number of ones is five times the number of hundreds. 1070

You are invited to explore

ACTIVITY CARD 2
Page 334

Workbook, page 10

Pages 40–67

General Objectives

To provide work with equations and solutions
To review the basic operations
To provide review of basic facts of addition, subtraction, multiplication, and division
To develop some basic concepts of functions
To stress the inverse relationships between addition and subtraction and between multiplication and division
To review the basic principles for addition and multiplication
To provide experiences in working with the number line
To provide word-problem experiences

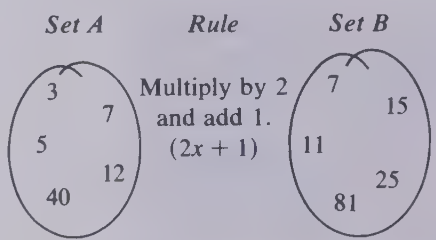
The initial pages of this chapter deal with the basic operations and relationships between them. The function machine is used to review the operations, and children are introduced to the use of n to designate any input number. Following the introductory review with operations and functions, particular relations between the operations are explored in more detail. Multiplication is related to repeated addition, and division is shown to be equivalent to repeated subtraction. This is followed by a study of the inverse relations: addition to subtraction, and multiplication to division.

Basic facts are then reviewed in a variety of ways. The function machine, equations to solve, and number-line games provide the children with practice of the facts in a way that replaces routine drill.

Finally, children are given an opportunity to study the basic principles for addition and multiplication and to apply them in solving equations. The concluding pages of the chapter constitute a review of the concepts treated throughout the chapter and a cumulative review.

Mathematics

The concept of a function is one of the most important ideas in mathematics, yet it can be presented on an intuitive basis very early in the child's experiences. Rather than present a precise mathematical definition of function, we give examples and point to some significant features.

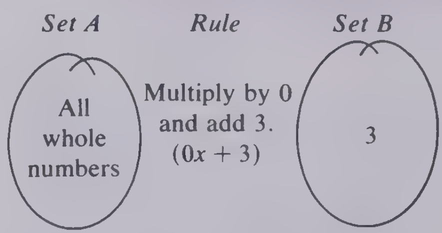


If we take a number from set A, say 7, and apply the rule, we get exactly one number in set B. Hence, we get this set of pairs:

(3,7), (7,15), (5,11), (12,25), (40,81).

One vital feature of these pairs is that for each first number there is only one second number.

Here is another example.



Some of the pairs of numbers are:

(17,3), (2,3), (1,3), (0,3), (285,3), . . .

Notice that, although every second number is 3, it remains true that, given any first number, we get *only one second number* (in this case, 3).

In summary, we have a set and a rule for each function. When we apply the rule to an element of the set, we get just *one* answer. Thus, for each function, we have a set of ordered pairs, no two of which have the same first number.

Since a large portion of this chapter deals with basic facts and since we rely upon the inverse relationship between operations to arrive at subtraction and division facts, one of the most important mathematical concepts for the chapter is this inverse relationship. The following definitions state the relationship for both subtraction and division.

Definition of subtraction:

> Let a, b, and c be whole numbers such that $a + b = c$. The number a is the difference $c - b$, and the number b is the difference $c - a$. In symbols,
> $$a = c - b, \; b = c - a.$$

Definition of division:

> If a, b, and c are whole numbers such that $b \neq 0$ and $a \times b = c$, then $a = c \div b$.

Notice that, in the definition of subtraction, we define an addend as a particular difference; and, in the definition of division, we define a factor as a particular quotient. Material such as that on page 52 is derived from this inverse relationship between the operations. That is, the children are asked to find differences by thinking about missing addends, and to find quotients by thinking about missing factors.

Teaching the Chapter

Materials

Centimetre rulers (1 per child)
Colored strips
Number line, for demonstration and for children's individual use (if available)
Slips of paper, approximately 20 by 9 cm
Strips of paper, approximately 40 by 4 cm

Vocabulary

add	parentheses
addend	plus
difference	product
divide	quotient
equals	repeated addition
factor	repeated subtraction
function	solution
minus	solve
multiply	subtract
number line	sum
operation	times

The first two or three lessons should indicate to you which children have difficulty understanding the meaning of any of the basic operations. Provide those children with materials such as sets of counters and paper cups and encourage them to use these materials to show several equations from the first few lessons. During the lessons which deal with the number-line games, it would be helpful to provide each child with a small number line. This, however, is not essential; you might find it sufficient to display a demonstration number line prominently. You might also prepare a number-line stack for various number-line games.

Lesson Schedule

Plan to cover the material in this chapter in about three weeks, though the time required for this material depends largely upon the background and abilities of your children. If your children have had a particularly strong or accelerated background, you might be able to cover this material in less than the recommended time. On the other hand, if your children are below average or have had a weak background in arithmetic, you may choose to allot more than the recommended time.

Evaluation of Progress

There are two criteria for evaluating children's achievement in this chapter: mastery of the basic addition, subtraction, multiplication, and division facts, and comprehension of the concepts presented. You can easily determine from their success with appropriate exercises whether or not the children know the basic facts. It is much more difficult to evaluate whether or not the children understand the more abstract concepts presented in this chapter. Specifically, do they understand the work with functions, the inverse relation between certain operations, the number-line activities, and so on? Such evaluation may best be made on a day-to-day basis rather than through use of a cumulative test. However, a sample evaluation instrument is provided in the form of a chapter review at the end of this chapter.

Much of the material in this chapter is included for the purpose of providing the children with necessary practice in basic facts. The function and number-line activities are intended to enliven what otherwise might be routine practice. Encourage as much discovery as possible while the children are working on these lessons.

Cumulative and chapter reviews on pages 66 and 67 should help you evaluate the children's progress. You may wish to use them as a guide in designing your own test.

Resources for Active Learning

General Activities

A Cloudburst, Vol. 3, Nos. 1115–1155, Midwest Publications

Franklin Series: *From Fingers to Computers,* "Napier's Rods," pp. 38–45, Lyons and Carnahan (Available from McGraw-Hill Ryerson)

Math Activity Cards, C5, Macmillan

Mathex: Numeration No. 7, "Experiments with Numbers," pp. 16–19 (pupil pages 13, 14), Encyclopaedia Britannica Publications Ltd.

Mathex: Operations and Problem Solving No. 8, pp. 1–6, Encyclopaedia Britannica Publications Ltd.

Modern Math Games . . ., "Math-O Activities" and others, pp. 5–9, 54, Fearon

Notes on Mathematics in Primary Schools, "Multiples," pp. 11–14; "An Arrow Game," pp. 94–102, Cambridge University Press (Available from Clarke-Irwin)

Nuffield Project: *Problems*—Red Set. No. 5, Wiley

SMSG: *Puzzle Problems and Games Project,* "Games with Addition and Multiplication Tables," pp. 97–109, Stanford University

Manipulative Devices

Cuisenaire Rods (Cuisenaire Co.)

"Invicta" Math Balance (Math Media; Selective Educational Equipment)

Papy Minicomputer (Macmillan)

SEE Calculator (Selective Educational Equipment)

Commercial Games

Equations (Creative Publications; Wff 'N Proof)

Heads Up (Creative Publications; Hammett; Math Media)

Imout (Imout)

Krypto (Creative Publications; Edmund Scientific)

Numble (Hammett)

Orbiting the Earth (Scott Foresman)

Playing Card Number Games—whole numbers (Heath)

Quinto (Hammett; Selective Educational Equipment)

Real Numbers Game (Wff 'N Proof)

Sum Times (Hammett; Selective Educational Equipment)

The Winning Touch (CCM School Materials; school supplier)

TUF (Creative Publications; Cuisenaire Co.; TUF)

Twin Choice (Holt, Rinehart and Winston)

PAGES 40–41

Objectives

The child will be able to express a given number by writing expressions using the symbols +, −, ×, ÷. The child will be able to recognize and write equations involving the four basic operations.

Preparation

Materials

slips of paper, approximately 20 by 9 cm

To introduce this chapter, you might simply write the symbols +, −, ×, ÷, = on the chalkboard and ask children what comes to their minds as they see them. Then explain the title of the chapter and the first lesson, and proceed to the investigation.

Investigation

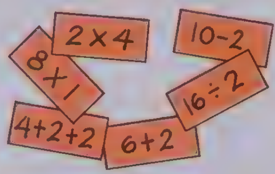
If you prefer, you might have the children simply list the expressions they can find that show 6. If children use an expression which contains a variety of symbols, accept it, but remind them to use parentheses. The expression inside the parentheses should be worked out first. However, the important thing to emphasize is that one number can be symbolized in many different ways.

3 Equations and Operations

Let's explore number symbols.

Investigating the Ideas

Each of these slips of paper shows a symbol for the number 8.



Use these digits ⟶ 1 4 2 6 3 9 8

and these signs. ⟶ + − × ÷

See Investigation.

? How many slips of paper can you make that show symbols for the number 6?

Sample answers:
4+2 1+2+3
8−2 9−3 6÷1
2×3 6×1
(Many others are possible.)

Discussing the Ideas

1. Study the chart below. Then give some equations by using the symbols you made that show 6. See Discussion.

We think:	13 is a symbol for the number thirteen. 8 + 5 is a symbol for the number thirteen.
We write an equation:	8 + 5 = 13

2. Give some symbols that might be covered in the figure.
Sample answers: 6+6, 8+4, 2×6, 3×4, 13−1, 24÷2

Discussion

This discussion section progresses from the idea of many symbols for a number to the idea of relating these symbols in an equation. Some equations the children might give for exercise 1 are 4 + 2 = 6, 1 + 2 + 3 = 8 − 2, 9 − 3 = 2 × 3, and so on.

As you discuss exercise 2, stress the idea that each symbol the children mention can be written with the visible symbols "= 12" in equation form. If necessary, point out the correct way to read the equal sign: "8 + 5 = 13" should be read, "Eight plus five *equals* thirteen." The concept in this lesson is fairly simple, so do not prolong discussion unnecessarily.

Using the Ideas

1. Each stack of six cards has symbols for just one number. Write what might be on the other 5 cards. Answers will vary.

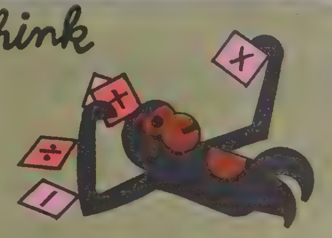
(Any symbol for ten) (Any symbol for four) (Any symbol for eight)

2. An equation for the top card in exercise 1A is $6 + 4 = 10$. Write an equation for each of your answers in exercise 1. Answers will vary.

3. Mark **T** (True) or **F** (False) for each statement.

A $4 + 3 = 12$ F	L $23 \div 1 = 23 \times 1$ T
B $3 \times 4 = 12$ T	M $(5 + 9) + 7 = 5 + (9 + 7)$ T
C $3 + 3 + 3 = 12$ F	N $(8 - 3) - 2 = 8 - (3 - 2)$ F
D $6 \times 6 = 12$ F	O $(4 \times 3) \times 2 = 4 \times (3 \times 2)$ T
E $6 \times 2 = 12$ T	P $3 \times 3 = (2 \times 4) + 1$ T
F $12 \div 1 = 12$ T	Q $5 \times 5 = (4 \times 6) + 1$ T
G $5 + 7 = 7 + 5$ T	R $87 + 5 = 85 + 7$ T
H $5 \times 7 = 7 \times 5$ T	
I $8 - 3 = 3 - 8$ F	
J $8 + 9 = 9 \times 8$ F	
K $7 \times 0 = 7 + 0$ F	

think



Give two different symbols for ninety. For each symbol use the digit "9" three times. You may use parentheses and any of the signs, +, ×, −, ÷, that you need. 99−9 ; (9×9)+9

★ 4. Give symbols for ten different numbers. In each symbol, use each of the three digits "3," "6," and "9" only once. You may use parentheses and one or more of the signs, +, ×, −, or ÷.

Examples: $(3 \times 6) \div 9$

Answers will vary. $(9 + 3) - 6$

Sample answers: (3×9)−6; (3+6)−9; (3+9)÷6; (6−3)+9; (9+6)÷3; etc.

41

Using the Exercises

Assign the exercises on page 41 as independent work. Note that an unlimited number of answers are possible for each part of exercise 1. Check exercise 3 carefully, asking children to give reasons for their answers. You might have those children who do exercise 4 check each other's work.

Some children may become frustrated as they try to find the two numerals for the *Think* problem, but urge them to try to discover the symbols independently. After ample time, let those who solve it explain it to the others.

Assignments (page 41)

Minimum: 1–3. Average: 1–3.
Maximum: 1–4.

Follow-up

Suggest several numbers for children to name in several different ways. Numbers which are multiples of many numbers would be very suitable, such as 20, 24, 36, 40, 50.

You might also suggest that children try to find as many numbers as they can which can be symbolized by using only 1, 3, and 5 in their expression; for example:

$1 + 3 + 5 = 9$	$(1 + 5) - 3 = 3$
$(5 - 3) - 1 = 1$	$(1 + 3) \times 5 = 20$
$(5 \times 3) + 1 = 16$	$(5 \times 3) - 1 = 14$

This same question may be suggested for any other combination of three single-digit numbers:
2, 4, 6; 7, 8, 9; 4, 5, 6; etc.

Resources for Active Learning

Developmental Math Cards,
"Number Sentences for 1000," I-9, Addison-Wesley.

PAGES 42–43

Objective

Given simple word problems, the child will be able to apply his understanding of the four basic operations to solve them.

Preparation

To prepare for this lesson, you might suggest two numerical items, such as 8 teams of 5 members each, and ask the children if they can think of a problem involving them. Then proceed to the investigation.

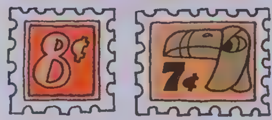
Investigation

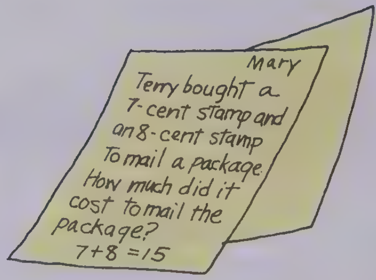
In this investigation, it is expected that the children will write a problem for each picture and write and solve an equation for each picture. The operations most obviously suggested by the pictures are, in order, multiplication ($3 \times 6 = 18$), subtraction ($12 - 2 = 10$), and division ($32 \div 4 = 8$). However, children may use any operation for any of the pictures, as long as it is appropriate to the problem they have written. In an investigation of this type, although each child is expected to write his own problem, free sharing of ideas will help stimulate ideas.

When do you add, subtract, multiply, or divide?

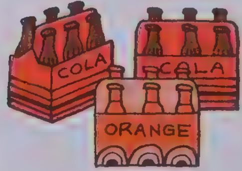
Investigating the Ideas

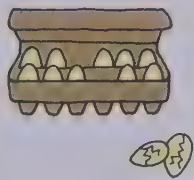
Mary wrote and solved this problem for the stamp picture below.

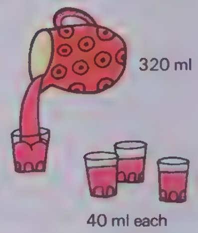




? Can you write and solve a problem for each picture below?
Answers may vary. See Investigation.







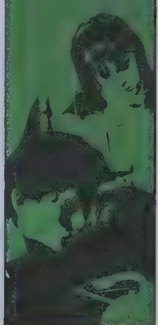
Discussing the Ideas

1. Make up a problem using one of the operations (+, −, ×, ÷) and tell it to the class. Answers will vary.
2. Suppose each ▥ covers a numeral. Which operation (+, −, ×, ÷) would you use to solve this problem? Subtraction

   ▥ boy scouts now.
   ▥ boy scouts needed for a full troop.
   How many more needed?

3. Make up a problem that uses two operations and tell it to the class. Answers will vary.

42



Discussion

Although this lesson appears to be built around problem solving, the main purpose of the lesson is to review the concrete meaning of the operations. Thus, when you discuss the problems the children have written, stress the concept of each operation, so that each operation is considered in a concrete, physical situation.

The purpose of discussion exercises 1 and 2 also is to stress the meaning of each operation. If children happen to repeat one or two operations while neglecting others, ask for problems specifically involving the neglected operations. If possible, have the children write one equation for the problems they suggest in exercise 3, using parentheses where necessary.

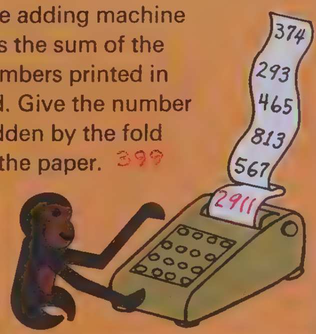
Using the Ideas

Use **A, S, M,** or **D** to tell which operation or operations (**A**ddition, **S**ubtraction, **M**ultiplication, **D**ivision) you would use to find the answers if numbers were given.

1. |||| girls' bikes. |||| boys' bikes. How many bikes? A
2. |||| girls in the choir. |||| moved away. How many girls in the choir now? S
3. Bob, |||| years old. Don, |||| years old. How much older is Bob than Don? S
4. ||||chocolate cookies in each box. |||| boxes. How many cookies? M
5. Coin collection. |||| coins in all. |||| on each page. How many pages? D
6. Hiked |||| km in the morning, |||| km in the afternoon. How many km? A
7. |||| skirts. |||| sweaters. How many different outfits? M
8. Butterfly collection. |||| butterflies. |||| boxes. How many in each box? D
9. Had |||| hockey cards. Gave |||| away. Bought ||||. How many cards now? S, A
10. |||| nickels. How many cents? M
11. |||| nickels. |||| dimes. How many cents? M, A
12. ||||large bottles of pop. ||||ml in each. |||| small bottles of pop. |||| ml in each. How many ml in all? M, A
13. Have |||| cents. How many ||||-cent stamps can you buy? D
14. |||| green marbles. |||| red marbles. Each boy gets |||| marbles. How many boys? A, D

think

The adding machine has the sum of the numbers printed in red. Give the number hidden by the fold in the paper. 399



More practice, page A-3, Set 5

43

Follow-up
Have children make up problems and write them on task cards to be filed for occasional practice in problem solving. They might use pictures from magazines or catalogues to suggest ideas and to illustrate the problems.

Resources for Active Learning
Developmental Math Cards, H[1]16, Addison-Wesley.

Duplicator Masters, page 6
Workbook, page 11

Using the Exercises
On page 43, read the directions with the children, and then instruct them to do the exercises. Some children may have difficulty determining the operation for exercise 7. This, of course, expresses the idea of product sets. With each skirt, the girl could wear one of her sweaters; hence, this type of problem involves the idea of multiplication.

When the children have finished these exercises, be sure to allow time for discussion of each exercise.

Assignments (page 43)
Minimum: 1–5, oral; 6–10.
Average: 1–10. Maximum: 1–14.

PAGES 44–45

Objectives

Given an equation using one of the four basic operations, the child will be able to rewrite the equation using the inverse operation.

The child will be able to restate a given repeated addition problem in terms of multiplication and to restate a given repeated subtraction problem in terms of division.

Preparation

Unless you prefer to spend a few minutes reviewing problem solving with one of the four basic operations, proceed to the investigation immediately.

Investigation

The key to this investigation is freedom of choice for the child. Encourage children to find as many equations as they can; for example:

$4+4+4=12$; $12-4-4-4=0$; $12 \div 4=3$; $12 \div 3=4$; $4 \times 3=12$.

For those who finish quickly, you might write on the chalkboard another situation which could be described by various equations, such as 4 rows of 6 chairs, or 7 teams with 5 members on each.

How are the operations related?

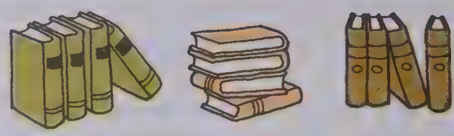
Investigating the Ideas

You might think of this picture as showing

$3 \times 4 = 12$

or

$4 + 4 + 4 = 12$.



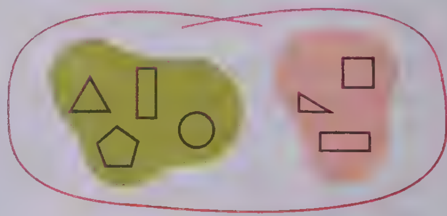
3 sets of 4

?

Can you use a set of 12 objects and show some other arrangements?

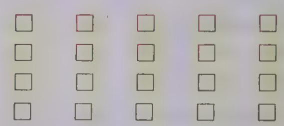
Write an addition, subtraction, multiplication, or division equation for each of your arrangements. Answers will vary. See Investigation.

Discussing the Ideas



1. Give the two addition and two subtraction equations suggested by these sets of figures. $4+3=7$; $3+4=7$; $7-3=4$; $7-4=3$

2. Give one addition, one multiplication, and one division equation for these sets. $4+4+4+4+4=20$
   $5 \times 4=20$
   $20 \div 5=4$ or $20 \div 4=5$



5 sets of 4

3. Rewrite each equation. Use an operation different from the one given. (Answer to A: $18 \div 6 = 3$) or $18 \div 3=6$
   - A $3 \times 6 = 18$
   - B $7 \times 3 = 21$ $21 \div 7=3$ or $21 \div 3=7$
   - C $2 + 2 + 2 + 2 = 8$ $4 \times 2=8$
   - D $6 \div 2 = 3$ $3 \times 2=6$
   - E $5 \times 6 = 30$ $30 \div 5=6$ or $30 \div 6=5$
   - F $8 + 8 + 8 = 24$ $3 \times 8=24$

   G
   $$\begin{array}{r} 24 \\ -6 \\ \hline 18 \\ -6 \\ \hline 12 \\ -6 \\ \hline 6 \\ -6 \\ \hline 0 \end{array}$$
   $24 \div 6=4$

   H
   $$\begin{array}{r} 15 \\ -5 \\ \hline 10 \\ -5 \\ \hline 5 \\ -5 \\ \hline 0 \end{array}$$
   $15 \div 5=3$

44

Discussion

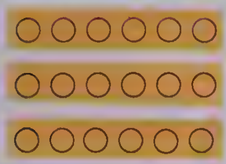
The main purpose of this lesson is to review the relationships between the operations without unduly belaboring these ideas. It would be helpful to first discuss the equations children thought of for the investigation. Be sure to relate $4 + 4 + 4 = 12$ to $3 \times 4 = 12$ and to relate $3 \times 4 = 12$ to $12 \div 3 = 4$. (The equations $3 \times 4 = 12$ and $12 = 3 \times 4$ are considered to be the same equation.)

In exercise 1, be sure the children see how the figures suggest the concepts of combining and of separating into groups. In exercise 2, children might also give a subtraction equation: $20-4-4-4-4-4=0$. Throughout the discussion, relate the equations to the physical situation suggested by the picture; this lesson presents a concrete view of relationships between operations. The children may need considerable discussion before finding the equations for parts G and H of exercise 3 ($24 \div 6 = 4$ and $15 \div 5 = 3$).

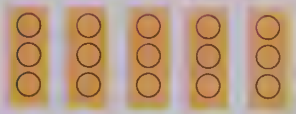
Using the Ideas

1. Write an addition and a multiplication equation for each set.

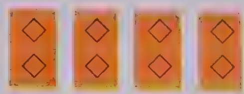
A 6+6+6=18; 3×6=18



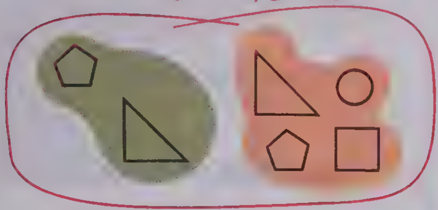
B 3+3+3+3+3=15; 5×3=15



2. Write one multiplication and one division equation for this set.
4×2=8; 8÷4=2 or 8÷2=4



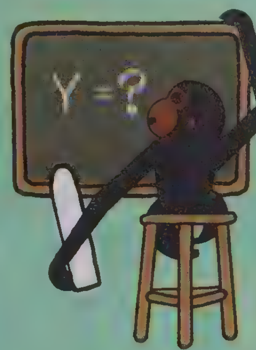
3. Write two addition and two subtraction equations for this set. 2+4=6; 4+2=6; 6−2=4; 6−4=2



4. Match each equation in the first column with an equation having the same solution in the second column.

A $7 - 3 = n$ 2	1 $18 \div 9 = n$
B $8 + 8 = n$ 7	2 $n + 3 = 7$
C $n \times 9 = 18$ 1	3 $13 - 7 = n$
D $15 - 8 = n$ 6	4 $25 \div 5 = n$
E $n \times 7 = 21$ 8	5 $3 \times 3 = n$
F $n + 7 = 13$ 3	6 $n + 8 = 15$
G $3 + 3 + 3 = n$ 5	7 $2 \times 8 = n$
H $n \times 5 = 25$ 4	8 $21 \div 7 = n$

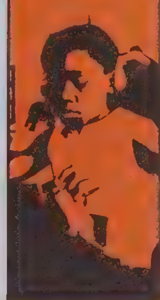
think



Give a number for y so that the equation $y \times y = y + y$ will be true.

Find another such number. 2, 0

More practice, page A-3, Set 6



Mathematics
It is important that children understand the *inverse relationship* between addition and subtraction as well as the inverse relationship between multiplication and division. Finding the difference of two numbers is equivalent to finding a missing addend in an addition problem. Similarly, finding the quotient of two numbers, can be shown to be equivalent to finding a missing factor in a multiplication problem. The two examples below illustrate these ideas.

This difference has been found **S A A** $9 - 6 = n$	when this addend has been found. **A A S** $n + 6 = 9$
This quotient has been found **P F F** $28 \div 4 = n$	when this factor has been found. **F F P** $n \times 4 = 28$

Follow-up
If children need more work with related equations, prepare a worksheet of exercises similar to exercise 4.

Match the following equations.

1. $3 \times 5 = n$	A $n \times 4 = 16$
2. $16 \div 4 = n$	B $n \div 8 = 2$
3. $7 \times n = 56$	C $3 \times 9 = n$
4. $2 \times 8 = n$	D $5 + 5 + 5 = n$
5. $9 + 9 + 9 = n$	E $56 \div n = 7$

Resources for Active Learning
Discovery, Section II, Unit 17/5,6, Encyclopaedia Britannica Educational Corp.

Duplicator Masters, page 7
Skill Masters, page 7

Using the Exercises
Assign the exercises on page 45 as independent work. When the children have finished, check their work and allow time for questions. Exercise 4 should be treated carefully inasmuch as children must have a fair grasp of the relationships in order to match the equations correctly.

Some children may have difficulty with the *Think* problem; however, when the answers 2 and 0 are given, most should understand that both these answers will make the equation true.

Assignments (page 45)
Minimum: 1–3. Average: 1–4.
Maximum: 1–4.

PAGES 46–47

Objective

Given an input number, n, and a simple function rule, the child will be able to apply the rule and find the output number.

Preparation

To prepare for this lesson, you might give children an opportunity to work with function rules orally by playing the "What's My Rule" game. Ask the children to give you a number; then apply a rule of your choice to that number; and respond to the children by giving the number resulting from application of your rule.

For example, if a child gives you the number 7 and you are using the rule $n + 2$, you would respond with 9. After several such exchanges of numbers, have the children try to guess what rule you were thinking of.

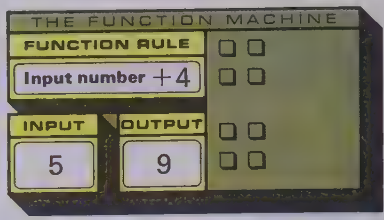
You may vary the game by having the children take turns thinking of a rule while the class guesses this rule by giving numbers. For this particular lesson, it is most desirable that the rule be related to simple addition facts.

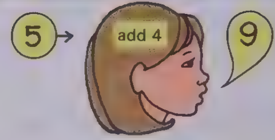
To give slower thinkers a chance to play the game, ask the children who believe they know your rule to cross their arms over their chests. Then when you call on them, they should ask you for an input number and respond with an output number according to the rule they think you are using. If they are correct, you can tell them so and continue to call on those who do not yet know the rule, until most children discover it.

- *How does the function machine work?*

Discussing the Ideas

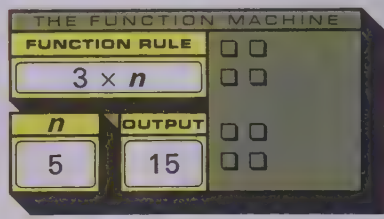
1. You can think of yourself as a function machine. Study the picture. Then use the rule to give the output for each of these input numbers.





A 3 7	E 0 4
B 6 10	F 9 13
C 8 12	G 12 16
D 10 14	H 18 22

2. In how many ways is this function machine different from the one above? Use this new function rule to give output numbers for each of these inputs.



A 2 6	E 0 0
B 4 12	F 6 18
C 7 21	G 1 3
D 3 9	H 10 30

3. Sometimes we write a function rule like this:

   Output number $= n - 2$

   We can use a table to record input and output numbers. Give the missing numbers.

Function Rule

$n - 2$

n Input number	Output number
12	10
5	3
7	A 5
4	B 2
10	C 8

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Discussion

As you work through discussion exercise 1, be sure the children see the correspondence between the picture of the child and the function machine: the child hears the number 5, the number 5 is put into the function machine; the child thinks "add 4," the machine applies the rule "input number plus 4"; the child says the number 9, the machine shows the number 9 as the output number.

In exercise 2, help the children see that n is used instead of the phrase "input number." Also, the function rule differs from that in exercise 1; thus, the output number is different even though both exercises began with the same input number, 5.

In exercise 3, the children are encouraged to think of the output number as the result of the function rule applied to n, the input number. The main purpose of this discussion is to review the operation and use of the function machine and function tables for children who studied from *Investigating School Mathematics* last year and to introduce the function machine to those who have not encountered it before.

Using the Ideas

For exercises 1 through 6, think about the function machine and complete the function tables.

1. Function Rule: $n + 6$

Input number	Output number
4	A 10
6	B 12
8	C 14
18	D 24
9	E 15

2. Function Rule: $n - 1$

Input number	Output number
3	A 2
8	B 7
9	C 8
19	D 18
29	E 28

3. Function Rule: $8 + n$

Input number	Output number
2	A 10
5	B 13
7	C 15
27	D 35
47	E 55

4. Function Rule: $2 \times n$

n	Output
3	A 6
7 B	14
6	C 12
10	D 20
6 E	12
20 F	40

5. Function Rule: $n + n + 1$

n	Output
4	A 9
7	B 15
9	C 19
8 D	17
21	E 43
50 F	101

6. Function Rule: A $n + 10$

n	Output
5	15
0	10
10	20
50	B 60
30 C	40
37	D 47

For exercises 7, 8, and 9, give the output for each input number, A, B, C, and D.

★ **7.** output $= \begin{cases} n + 10, \text{ if } n \text{ is even} \\ n + 5, \text{ if } n \text{ is odd} \end{cases}$ A 2 (12) B 13 (18) C 0 (10) D 20 (30)

★ **8.** output $= \begin{cases} 3 \times n, \text{ if } n < 10 \\ n + 100, \text{ otherwise} \end{cases}$ A 6 (18) B 30 (130) C 7 (21) D 100 (200)

★ **9.** output $= \begin{cases} n \times 10, \text{ if } n < 10 \\ n - 10, \text{ if } n > 9 \end{cases}$ A 7 (70) B 18 (8) C 4 (40) D 36 (26)

Mathematics
You will recall from the discussion of functions in the mathematics section for this chapter that one of the key properties of a function is that the function associates with each number of a given set exactly one number from another set. You may wish to reread that section in preparation for this lesson.

Follow-up
Encourage children to make up function rules of their own. Some capable children may enjoy making rules similar to those in the starred exercises, applying different rules for different conditions.

Resources for Active Learning
Inquiry in Mathematics via the Geoboard, "Linear Graphs," Geo-Cards 25/1–26/6, Walker. (Available from Fitzhenry and Whiteside)

Workbook, page 12

Using the Exercises
Assign the exercises on page 47 as independent work. Note that exercises 7, 8, and 9 are starred, but all the children might be able to do them if you help them realize that they must apply a different function rule depending on which condition each input number satisfies.

Assignments (page 47)
Minimum: 1–5. Average: 1–6.
Maximum: 1–9.

Objective

Given a multiplication or division problem, the child will be able to relate it to repeated addition or repeated subtraction, respectively.

Preparation

It would be appropriate to begin this lesson immediately with the investigation. However, if you prefer, use the "What's My Rule" game as suggested on page 46, stressing a review of basic multiplication facts.

Investigation

Direct the children first to study the "automatic 8 multiplier." If you have them work in groups, encourage them to explain to one another how they think this will help them find the quotients and products for A – E. Even if the children are working in groups, each child should make his own multiplier using a number other than 8. Then each child may write problems for his multiplier and have other children in his group try to solve them by using the multiplier.

As children work on activities such as this, circulate throughout the room, spot checking for computational accuracy on the multiplier and asking questions to evaluate children's understanding of what they are doing.

Let's explore multiplication and division.

Investigating the Ideas

AUTOMATIC 8 MULTIPLIER and DIVIDER

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
8 +	8 +	8 +	8 +	8 +	8 +	8 +	8 +	8 +	8 +	8 +	8 +	8 +	8 +	8 +	8 +
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128

Use the Automatic 8 Multiplier and Divider to find these products and quotients.

A 9×8 72 B 13×8 104 C $104 \div 8$ 13 D $120 \div 8$ 15 E 16×8 128

? Can you make and use an automatic multiplier and divider for a number other than 8? See Investigation.

Discussing the Ideas

1. Solve these equations. Use the 8 Multiplier.
   - A $7 \times 8 = n$ 56
   - B $17 \times 8 = y$ 136
   - C $8 \times 15 = a$ 120
   - D $8 \times 12 = t$ 96
   - E $14 \times 8 = s$ 112
   - F $96 \div 8 = q$ 12
   - G $136 \div 8 = m$ 17
   - H $128 \div 8 = b$ 16
   - I $88 \div 8 = u$ 11
2. How can you figure out the product 18×8 by looking at the 8 Multiplier? Sample answer: Find the product 16×8 and add 2 more eights.
3. The Automatic Multiplier uses **repeated addition** to find products and quotients. The example below uses **repeated subtraction** (subtracting one 6 at a time) to find quotients.
   $42 \div 6 = n \longrightarrow 42 - 6) - 6) - 6) - 6) - 6) - 6) - 6 = 0$ 7
   - A How many sixes were subtracted to get from 42 to 0?
   - B What is the quotient for $42 \div 6$? 7
   - C Subtract one 3 at a time to find $51 \div 3$. 17

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Discussion

The most important point treated in this lesson is the relation of repeated addition to multiplication and repeated subtraction to division.

Exercises 1 and 2 provide an opportunity for you to have several children explain how they use the multiplier and divider. Exercise 3 extends the discussion into consideration of repeated subtraction. Note that, even though the notation of one-sided parentheses is new, children should not be troubled if you point out that it simply means to subtract each six one at a time.

Using the Ideas

1. Use this Automatic 13 Multiplier to find the products and quotients.

1	2	3	4	5	6	7	8	9	10	11	12	13
13 +	13 +	13 +	13 +	13 +	13 +	13 +	13 +	13 +	13 +	13 +	13 +	13 +
13	26	39	52	65	78	91	104	117	130	143	156	169

A $5 \times 13 = r$ 65 E $13 \times 9 = s$ 117 I $91 \div 13 = t$ 7
B $8 \times 13 = f$ 104 F $13 \times 11 = d$ 143 J $143 \div 13 = y$ 11
C $13 \times 4 = n$ 52 G $117 \div 13 = b$ 9 K $104 \div 13 = w$ 8
D $12 \times 13 = a$ 156 H $52 \div 13 = m$ 4 L $156 \div 13 = c$ 12

2. Find each sum and product.

A $6 + 6 + 6 + 6 = n$ 24
$4 \times 6 = n$ 24

B $4 + 4 + 4 + 4 + 4 = r$ 20
$5 \times 4 = r$ 20

C $9 + 9 + 9 + 9 = t$ 36
$4 \times 9 = t$ 36

D $11 + 11 + 11 + 11 = m$ 44
$4 \times 11 = m$ 44

3. Write a division equation for each correct repeated subtraction. Two are incorrect.

A $30 - 6 - 6 - 6 - 6 - 6 = 0$
B $36 - 9 - 9 - 9 - 9 = 0$
C $48 - 8 - 8 - 8 - 8 - 8 = 0$
D $24 - 6 - 6 - 6 - 6 = 0$
E $28 - 7 - 7 - 7 - 7 - 7 = 0$
F $48 - 12 - 12 - 12 - 12 = 0$

A $30 \div 6 = 5$ D $24 \div 6 = 4$
B $36 \div 9 = 4$ E Incorrect
C Incorrect F $48 \div 12 = 4$

think

Pam told Sam, "If you give me a nickel, we will each have the same amount of money."

1. What is the smallest amount of money they could have started with? Pam, 0¢; Sam, 10¢
2. If they have 36¢ in all, how much money did Pam start with? 13¢

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Follow-up
Encourage children to make up problems for the multipliers and dividers they made in the investigation. They may then give these problems with their multiplier to another classmate in exchange for his. Remind the children that their problems must be solvable by means of their multiplier.

Duplicator Masters, page 8

Using the Exercises
Assign the exercises on page 49 as independent work. Although these exercises review some multiplication facts, the main emphasis is on understanding the relationship between the operations, as mentioned in the statement of the objective for this lesson. Allow time for discussion and checking papers when the children have finished the exercises.

Analysis of the *Think* problem should stimulate an interesting class discussion. Urge those students who solve it to share the method they used with the rest of the class.

Assignments (page 49)
Minimum: 1–2. Average: 1–3. Maximum: 1–3.

PAGES 50–51

Objective

Given an equation involving one operation, the child will be able to solve the equation by applying his understanding of the relationship between inverse operations.

Preparation

If you choose to begin with an oral practice session, use some mental chain games to review basic facts.

"Start with 6. . . Multiply by 7. . . Add 3. . . Divide by 5. . . Add 3. . . Divide by 2. . . What's my number?" (6)

"Start with 25. . . Divide by 5. . . Multiply by 5. . . Add zero. . . Divide by 1. . . What's my number?" (25)

"Start with 49. . . Divide by 7. . . Add 3. . . Multiply by 4. . . Divide by 8. . . Add 3. . . Multiply by 8. . . What's my number?" (64)

If you prefer, begin immediately with the investigation.

Investigation

The purpose of this investigation is to help children realize that subtraction is related to addition and that division is related to multiplication. Unless children understand that the terms (addends and sum) in a subtraction equation are the same as in an addition equation, and that the terms (factors and product) in a division equation are the same as in a multiplication equation, they will have difficulty finding equations to write for the numbers 6 and 4. Do not give close guidance; let the children study the examples and figure out the directions by themselves. You might, however, tell them that there are eight possible equations.

$6 + 4 = 10$	$6 \times 4 = 24$
$4 + 6 = 10$	$4 \times 6 = 24$
$10 - 6 = 4$	$24 \div 4 = 6$
$10 - 4 = 6$	$24 \div 6 = 4$

How are the operations related?

Investigating the Ideas

You can think of Addends and a Sum in both addition and subtraction.

A A S	S A A
$2 + 3 = 5$	$5 - 3 = 2$

You can think of Factors and a Product in both multiplication and division.

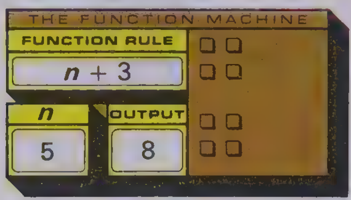
F F P	P F F
$2 \times 3 = 6$	$6 \div 3 = 2$

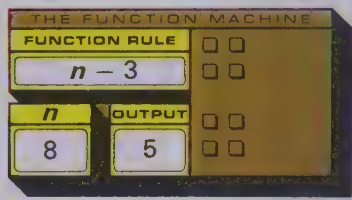
? How many different equations can you write using 6 and 4 when both are addends **or** both are factors?

See Investigation.

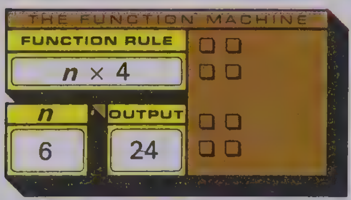
Discussing the Ideas

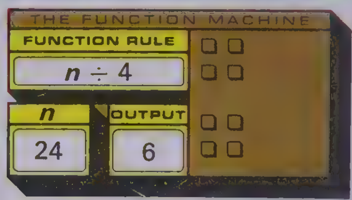
1. The two function machines help you see how addition and subtraction are related. Which numbers are addends and which is the sum? 5 and 3 are addends; 8 is the sum.

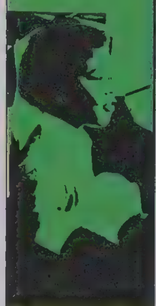




2. These two function machines help you see how multiplication and division are related. Which numbers are factors and which is the product? 4 and 6 are factors; 24 is the product.







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Discussion

In discussion exercise 1, point out to the children that for the first function machine they are to think about adding 3 to a number and in the second function machine they are to think about subtracting 3 from a number. Therefore, by considering the output number from the first machine as the input number in the second machine, they are assured that each time they will get an output in the second machine that is the same as the input in the first machine. The general concept emphasized here is that if we start with a number, add to it, and then subtract that same addend from the sum, we will return to the number with which we started.

Develop exercise 2 similarly. You might present other examples which emphasize the inverse relationships between addition and subtraction and between multiplication and division.

Using the Ideas

1. Find the missing addend in the addition equation. Then write the subtraction equation with the correct difference.

A $n + 7 = 12$ 5 C $r + 8 = 15$ 7 E $b + 5 = 13$ 8 G $t + 18 = 24$ 6
$12 - 7 = n$ 5 $15 - 8 = r$ 7 $13 - 5 = b$ 8 $24 - 18 = t$ 6

B $a + 6 = 13$ 7 D $s + 9 = 14$ 5 F $n + 9 = 17$ 8 H $q + 27 = 36$ 9
$13 - 6 = a$ 7 $14 - 9 = s$ 5 $17 - 9 = n$ 8 $36 - 27 = q$ 9

2. Find the missing factor in the multiplication equation. Then write the division equation with the correct quotient.

A $n \times 4 = 32$ 8 C $n \times 5 = 45$ 9 E $n \times 3 = 27$ 9 G $a \times 6 = 54$ 9
$32 \div 4 = n$ 8 $45 \div 5 = n$ 9 $27 \div 3 = n$ 9 $54 \div 6 = a$ 9

B $t \times 6 = 42$ 7 D $n \times 8 = 40$ 5 F $p \times 8 = 8$ 1 H $b \times 8 = 48$ 6
$42 \div 6 = t$ 7 $40 \div 8 = n$ 5 $8 \div 8 = p$ 1 $48 \div 8 = b$ 6

3. In **multiplication** we find the product (P) of the factors (F). ⟶ $\overset{F}{6} \times \overset{F}{4} = \overset{P}{n}$
In **division** we find one of the factors of the product. ⟶ $\overset{P}{24} \div \overset{F}{4} = \overset{F}{n}$ $\overset{P}{24} \div \overset{F}{6} = \overset{F}{n}$
Find the missing product or factor.

A $\overset{F}{6} \times \overset{F}{9} = \overset{P}{n}$ 54   D $\overset{F}{n} \times \overset{F}{6} = \overset{P}{42}$ 7
B $\overset{P}{54} \div \overset{F}{9} = \overset{F}{n}$ 6   E $\overset{P}{42} \div \overset{F}{6} = \overset{F}{n}$ 7
C $\overset{P}{54} \div \overset{F}{n} = \overset{F}{9}$ 6   F $\overset{P}{42} \div \overset{F}{n} = \overset{F}{6}$ 7

think

In a magic square the row, column, and diagonal sums are all the same. Give the missing numbers so that the square will be a magic square.

19	5	6	16
8	14		11
	10	9	
7		18	4

4. Which equation has no solution, which has many, and which has just one?

Many A $n \times 0 = 0$ C $n \times 0 = 7$
one B $n \times 5 = 0$ None

5. Use exercise 4 to explain why we **do not divide by zero.**
See Using the Exercises.

More practice, page A-4, Set 7

Mathematics

We can think of the subtraction operation as an operation that "undoes" addition and of the division operation as an operation that "undoes" multiplication. These relationships can be seen by the following examples:

Start	Add 3	Subtract 3	End
5 →	5 + 3 →	8 − 3 →	5
2 →	2 + 3 →	5 − 3 →	2

Start	Multiply by 4	Divide by 4	End
6 →	6 × 4 →	24 ÷ 4 →	6
3 →	3 × 4 →	12 ÷ 4 →	3

In the addition example, the net effect of both operations is the same as if no operation had been performed. Thus, we say that subtraction of a given number is the *inverse* of the addition of that number. Similarly, in the multiplication example, the net effect of multiplying and then dividing by the same number is the same as if no operation had been performed. Thus, we call multiplication and division by the same number *inverse* operations. (Note that, since we do not divide by zero, we do not speak of the "inverse" of multiplication by zero.)

Follow-up

You might suggest that children make up magic squares of their own. Or you might give them one partially completed. A few examples follow.

8	1	6
3	5	7
4	9	2

(15)

16	2	12
6	10	14
8	18	4

(30)

16	2	3	13
5	11	10	8
9	7	6	12
4	14	15	1

(34)

Duplicator Masters, page 9
Workbook, page 13
Skill Masters, page 9

Using the Exercises

Assign the exercises on page 51 as independent work. When children finish, discuss exercise 4 in particular. Use equations A and C to stress the fact that since $n \times 0 = 0$ can be solved by any number and since there is no number for which $n \times 0 = 7$, both $0 \div 0$ and $7 \div 0$ are not defined; hence, we never divide by zero.

In the *Think* problem, most children will see that the first row and one diagonal of the magic square are complete, so that the sum, 46, is the total they must find for each row, column, or diagonal. Some children may complete each column by finding subtotals of three known addends and subtracting this sum from 46. Others may compare the pattern of each row of numerals with a completed row. For example, to compare row 2 (8, 14, blank, 11) with row 1 (19, 5, 6, 16), they might think, "Since 14 is 2 less than 16, and 8 is 2 more than 6, these two pairs are equal. However, 11 is 8 less than 19, so the number I need to know must be 8 more than 5. Thus, the missing number for this row is 13."

Assignments (page 51)
Minimum: 1–3. Average: 1–3.
Maximum: 1–4.

PAGES 52–53

Objective

The child will review basic facts by studying sets of multiples and by participating in timed exercises.

Preparation

Materials

colored strips; centimetre ruler

The nature of the investigation makes it appropriate to omit formal preparation and begin immediately with the investigation.

Investigation

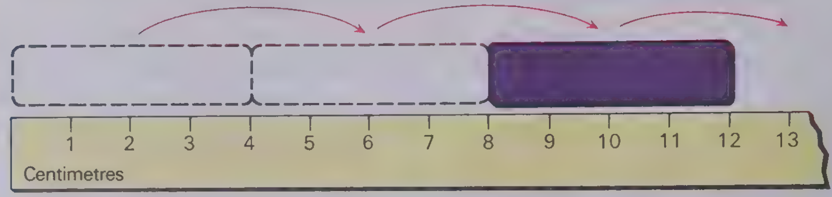
If children associate the pictured purple strip with the set of numbers, they may more quickly realize that all the numbers in the set are multiples of four. You might find it necessary to point out that this use of the 4-strip has something to do with what is "special" about the numbers in the set, but let children figure out among themselves the property of the set.

After children realize that the special property of the set is that it contains multiples of the 4-strip, some may be able to give the multiples for the 5-strip, 6-strip, and 7-strip without the aid of the ruler. Others will need to pattern their work with the other strips after the model of the 4-strip and the centimetre ruler.

Throughout this investigation, encourage children to try to figure things out without your guidance. If some children quickly identify the other sets of multiples, you might suggest that they do the same for the 8-strip and the 9-strip. Or, you might have them list the basic facts which generate each set of multiples (1×4, 2×4, 3×4, etc.).

Do you know your facts?

Investigating the Ideas

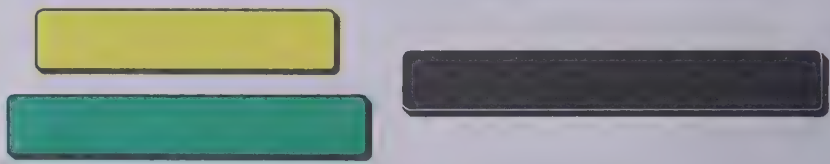


Can you tell what is "special" about this set of numbers?

8 0 12 16 32 4 28 24 36 20

All are multiples of 4.

? Can you make a "special" set of numbers for each of these strips? See Investigation.



Discussing the Ideas

1. Give the "special" sets of numbers for 8 and 9.
   {0,8,16,24,32,40,48,56,64,72} {0,9,18,27,36,45,54,63,72,81}
2. It is important for you to be able to recall addition, subtraction, multiplication, and division facts quickly. Try these.

A $6 + 5$ 11	G $15 - 7$ 8	M 6×7 42	S $42 \div 6$ 7
B $7 + 5$ 12	H $14 - 7$ 7	N 5×8 40	T $54 \div 9$ 6
C $9 + 8$ 17	I $17 - 9$ 8	O 6×9 54	U $48 \div 8$ 6
D $8 + 7$ 15	J $16 - 8$ 8	P 8×6 48	V $36 \div 4$ 9
E $8 + 8$ 16	K $15 - 9$ 6	Q 9×5 45	W $72 \div 8$ 9
F $6 + 9$ 15	L $13 - 6$ 7	R 8×9 72	X $49 \div 7$ 7

52

Discussion

Show how each set of multiples is related to the basic multiplication facts. Emphasize the importance of knowing these and the addition facts for quick recall. Encourage them to suggest ideas or patterns they may use to help them remember the facts. Also, discuss alternative approaches that can be used if they forget one of the facts they need. For example, if a child knows that $7 \times 8 = 56$, but forgets 8×8, he can simply add $8 + 56$ to get 64.

Use this and subsequent lessons to determine if some children still have not mastered the basic facts. To help children gain this mastery, have the class (or group) study a "fact for the week." For example, display five or six of the most commonly missed facts in a prominent place in the classroom. Choose one of these to be the "fact for the week" and highlight it in some manner. During the week children should study this one fact in particular, and at the end of the week remove this fact from display. Thus, children will have the facts they need visible to them, but they will also be working to memorize at least one fact a week.

Using the Ideas

1. Can you find these sums in $1\frac{1}{2}$ minutes?

A $3+9$ 12	G $4+6$ 10	M $7+9$ 16	S $7+4$ 11
B $9+3$ 12	H $9+5$ 14	N $5+1$ 6	T $2+3$ 5
C $7+7$ 14	I $8+2$ 10	O $6+8$ 14	U $9+8$ 17
D $9+1$ 10	J $4+0$ 4	P $8+3$ 11	V $8+5$ 13
E $8+6$ 14	K $5+9$ 14	Q $5+4$ 9	W $8+7$ 15
F $2+8$ 10	L $2+9$ 11	R $5+7$ 12	X $7+3$ 10

2. Try these in 2 minutes.

A $12-7$ 5	G $9-6$ 3	M $15-8$ 7	S $16-8$ 8
B $11-6$ 5	H $13-6$ 7	N $16-7$ 9	T $8-6$ 2
C $12-5$ 7	I $16-9$ 7	O $15-6$ 9	U $12-4$ 8
D $7-2$ 5	J $11-5$ 6	P $9-8$ 1	V $17-9$ 8
E $13-7$ 6	K $13-9$ 4	Q $12-8$ 4	W $14-8$ 6
F $12-6$ 6	L $6-4$ 2	R $14-5$ 9	X $5-3$ 2

3. How many of these products can you find in 2 minutes?

A 3×4 12	G 0×3 0	M 9×4 36	S 9×8 72
B 6×6 36	H 7×9 63	N 8×8 64	T 8×5 40
C 3×2 6	I 4×9 36	O 9×6 54	U 6×3 18
D 7×6 42	J 1×5 5	P 7×4 28	V 3×5 15
E 2×9 18	K 5×6 30	Q 3×8 24	W 8×7 56
F 7×7 49	L 5×8 40	R 8×2 16	X 8×9 72

4. Find as many of these as you can in 3 minutes.

A $12\div3$ 4	G $63\div9$ 7	M $27\div9$ 3	S $18\div6$ 3
B $20\div4$ 5	H $18\div2$ 9	N $49\div7$ 7	T $5\div5$ 1
C $24\div4$ 6	I $64\div8$ 8	O $24\div3$ 8	U $30\div5$ 6
D $24\div8$ 3	J $15\div5$ 3	P $28\div4$ 7	V $42\div6$ 7
E $35\div5$ 7	K $30\div6$ 5	Q $21\div7$ 3	W $48\div6$ 8
F $9\div1$ 9	L $36\div6$ 6	R $56\div8$ 7	X $32\div8$ 4

More practice, page A-4, Set 8

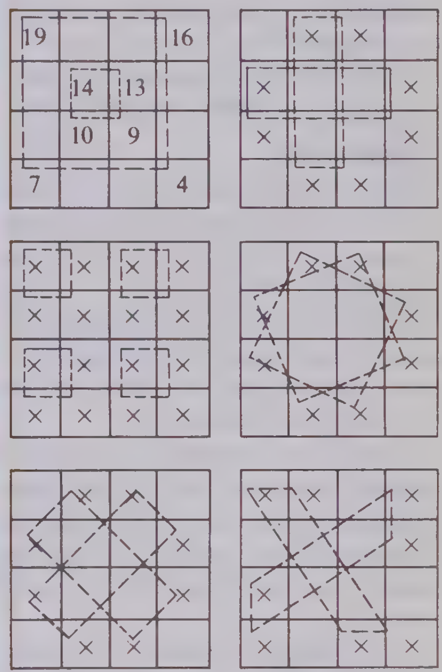
Using the Exercises

These timed exercises might be handled in a variety of ways. You might have each child time himself if an appropriate clock is visible; or you might have a child in the class time a group of children; or you yourself might time everyone together. In any event, try to use a light touch, so that children do not feel pressured; you might make a kind of game out of working and timing and checking these exercises. Help children recognize their weak points so that they can make a conscious effort to learn those facts they still cannot quickly recall.

Assignments (page 53)
Minimum: 1–4. Average: 1–4.
Maximum: 1–4.

Follow-up

If children enjoyed working with the magic squares suggested on page 51, duplicate a worksheet of magic squares such as the one below. Ask the children to find patterns of four numbers (other than the patterns given for the rows, columns, and diagonals), so that each pattern has the magic sum 46. In most of the examples, the numbers are excluded so that the patterns are clear, but the children will have to experiment with the actual numbers to discover the patterns.



The sum of any four numbers marked X and joined by a dashed line is the magic number.

Duplicator Masters, pages 10–11
Skill Masters, pages 10-11

PAGES 54–55

Objective

Given the function machine chart, the child will be able to discover the pattern and state the function rule or apply a given function rule.

Preparation

Materials

colored strips (optional)

If you choose, you might want to spend a few minutes reviewing basic facts. The "What's My Rule" game would be appropriate for this lesson. (See page 46.) Another brisk game is the "Buzz" game (described in the follow-up section on page 17).

If you decide to use such a preparation, remember to keep it brief.

Investigation

In this investigation, children need not use their strips unless they choose to. Familiarity with the strips will enable many to think directly in terms of the number the strip represents. However, those who want to use the strips should be encouraged to do so.

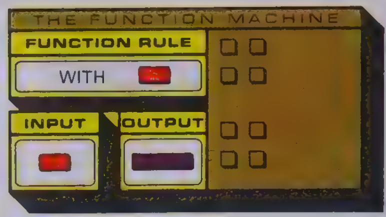
When you introduce the investigation, point out how the function table problems should be read. For instance, the sample illustrated would be read, "Red with red gives purple."

You might let those who finish quickly make up their own problems using a function rule you suggest.

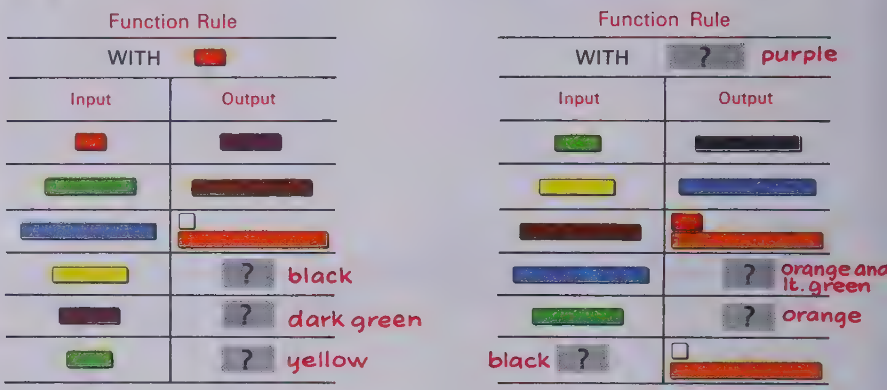
Can you find the function rule?

Investigating the Ideas

This is a special function machine that uses the strips instead of numbers.



Can you use your strips to help you find the missing strips in the tables?



Discussing the Ideas

1. Can you find the function rule for each of these tables? See Discussion.

Function Rule	?
n	Output
256	2
1475	4
5000	0
12 763	7
46	0

Function Rule	? $n \times n$ or n^2
n	Output
2	4
4	16
1	1
7	49
5	25

2. Invent your own function rule. List a set of input and output numbers and see if some of your classmates can find your rule. Answers will vary.

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Discussion

Although this lesson is intended to review some basic facts, it is also intended to be a recreational lesson showing a variety of uses of the function rule. For example, using the strips in the function machine is a variation of the usual addition rules. In discussion exercise 1, the first table contains a function rule which does not use one of the four basic operations. (Children might explain the rule as, "Give the digit in the hundreds' place for each n." Note that although each output number may not be unique, there is only one output number for each n.) The second table is the familiar squaring function, though children may express the rule simply as, "Multiply each n by itself."

Using the Ideas

Find the missing numbers and function rules.

1. Function Rule: $n + 7$

n	Output
9	A 16
6	B 13
7	C 14
5 D	12
8 E	15

2. Function Rule: $n + 5$

n	Output
4	A 9
6	B 11
8	C 13
20 D	25
40 E	45

3. Function Rule: $n \times 7$

n	Output
8	A 56
6	B 42
5	C 35
4 D	28
9 E	63

4. Function Rule: A $n \times 6$

n	Output
5	30
1	6
4	24
6	B 36
8 C	48
9 D	54

5. Function Rule: A $(n \times 5) + 2$ or $5n + 2$

n	Output
6	32
3	17
9	47
5	27
8	B 42
4	C 22

6. Function Rule: A $(n \times 3) + 1$ or $3n + 1$

n	Output
2	7
3	10
5	16
1	4
6	B 19
10	C 31

7. Function Rule: A $(n \times n) - 1$

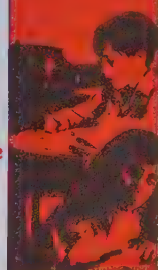
n	Output
2	3
4	15
1	0
7	48
5	B 24
6	C 35

★ **8.** Function Rule: A Give next higher multiple of 10

n	Output
7	10
12	20
39	40
24	30
16	B 20
87	C 90

★ **9.** Function Rule: A With lt. green

n	Output
●	●
●	●
●	●
●	●
●	B black
●	C orange



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Using the Exercises

Have the children work independently on the exercises on page 55. If some children give the missing numbers but cannot state the function rule used to obtain them, accept their work as correct. Notice that exercises 8 and 9 are starred. The rule for exercise 8 is challenging: "Give the next higher multiple of ten." The colors in exercise 9 relate numerically to the colors of the strips.

Assignments (page 55)
Minimum: 1–5. Average: 1–7.
Maximum: 1–9.

Follow-up

Encourage children to continue making function rules and function tables of their own and then exchange their work to be completed by others.

Resources for Active Learning

Discovery, Section II, Unit 21/4, Encyclopaedia Britannica Educational Corp.

Inquiry in Mathematics via the Geoboard, "Functions. . ." Geo-Cards 7/1–8/3, Walker. (Available from Fitzhenry and Whiteside)

Maths Mini-lab, "The Shuttle Game," Cards 73–74, Selective Educational Equipment.

Nuffield Project: *Problems*—Red Set, No. 13, Wiley.

SMSG: *Puzzle Problems and Games Project,* "Linear Function Games," pp. 79–95, Stanford University.

Workbook, page 14

PAGES 56–57

Objective

Given equations using one or two of the four basic operations and having one unknown, the child will be able to solve the equation.

Preparation

Use a favorite game to review basic facts. Or, you might use the function tables made by the children in the previous lesson and have them identify or apply rules.

Solving Equations

1. Write an equation for each problem. Solve the equation.

A Jeff scored 9 points in the first half of the game. He scored 7 points in the last half. How many points did he score? $9+7=16$

B Sally bought 4 hamsters. Each of her hamsters had 8 baby hamsters. How many hamsters does Sally have now? $4\times8=32$

2. To give the number for n in an equation such as $n + 3 = 7$, we write $n = 4$. Solve the equations. The solution is given for exercise A.

A $n + 2 = 9$ $n = 7$	E $8 + 9 = n$ 17	J $32 = n \times 4$ 8	O $n + 9 = 16$ 7
	F $8 \times 9 = n$ 72	K $32 = n + 4$ 28	P $n \times 7 = 63$ 9
B $n \times 3 = 6$ 2	G $n \times 6 = 24$ 4	L $n \times 6 = 54$ 9	Q $40 = 8 \times n$ 5
C $18 = n + 9$ 9	H $6 + n = 20$ 14	M $n + 9 = 14$ 5	R $49 = 7 \times n$ 7
D $5 \times n = 20$ 4	I $n + 8 = 24$ 16	N $6 \times n = 48$ 8	S $15 = 9 + n$ 6

3. We often use other letters in equations in the same way we have been using the letter n. Solve the equations. The solution is given for exercise A.

A $a \times 4 = 36$ $a = 9$	E $t \times 7 = 42$ 6	J $54 = s \times 9$ 6	O $8 + q = 12$ 4
	F $y + 7 = 16$ 9	K $r + 6 = 15$ 9	P $6 \times r = 6$ 1
B $n + 7 = 13$ 6	G $64 = 8 \times n$ 8	L $9 \times 0 = n$ 0	Q $n \times 6 = 0$ 0
C $9 + c = 20$ 11	H $14 = 6 + x$ 8	M $0 \times 7 = n$ 0	R $7 + s = 7$ 0
D $8 \times 7 = s$ 56	I $b + 7 = 13$ 6	N $9 + 0 = y$ 9	S $7 \times t = 28$ 4

4. Solve the equations.

A $5 + (8 + 7) = m$ 20	F $5 + (3 \times 4) = r$ 17	K $(3 \times 4) + (5 \times 4) = q$ 32
B $(5 + 8) + 7 = m$ 20	G $(5 + 3) \times 4 = s$ 32	L $8 \times 4 = s$ 32
C $(9 + 7) + 8 = x$ 24	H $(5 + 4) \times 3 = a$ 27	M $(5 \times 6) + (4 \times 6) = n$ 54
D $9 + (7 + 8) = x$ 24	I $3 \times (5 + 2) = t$ 21	N $9 \times 6 = t$ 54
E $(9 + 8) + 7 = q$ 24	J $(3 \times 5) + 2 = y$ 17	O $5 \times 0 = a + 0$ 0

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Discussion

These exercises are intended to be used for independent practice and review. You may, however, choose to use some of them as a basis for discussion, particularly exercise 6. In an exercise such as 6H, the children must first find 6×2 and then think, "What number adds to 12 to give 20?" One good way to help the children think about exercises 6O through U is to have them cover up part of each exercise. For example, in part O, the children cover up $n \times 4$ and then think, "What number plus 3 equals 23?" They will, of course, see that this is 20. Now, they look again at $n \times 4$ and think, "What number times 4 is 20?" Thus, they should arrive at 5 as the solution.

Do not, however, attempt to prescribe particular rules for solving these equations. Treat them primarily as a discovery activity. As the children solve each equation, it may be helpful to have them write out the whole equation with the solution in place of the variable. For example, if a child chooses the answer 7 for exercise 6R and then writes the equation $(7 \times 8) + 3 = 51$, he should observe that his solution is incorrect. This method may help him find the correct answer, 6.

Encourage all the children to try

5. In her rock collection, Sue had 6 boxes with 5 rocks in each box. She found 3 more rocks. How many rocks did she have in all? Write an equation and solve it. $(6 \times 5) + 3 = 33$

6. Solve the equations.

A $(4 \times 6) + 7 = n$ 31	H $(6 \times 2) + q = 20$ 8	O $(n \times 4) + 3 = 23$ 5
B $(3 \times 7) + 8 = y$ 29	I $(3 \times 8) + t = 30$ 6	P $(y \times 6) + 5 = 41$ 6
C $(8 \times 3) + 9 = a$ 33	J $(4 \times 7) + r = 34$ 6	Q $(r \times 9) + 5 = 50$ 5
D $(6 \times 3) + 6 = c$ 24	K $(3 \times 9) + n = 35$ 8	R $(t \times 8) + 3 = 51$ 6
E $(9 \times 3) + 6 = d$ 33	L $(6 \times 7) + b = 42$ 0	S $(n \times 7) + 6 = 62$ 8
F $(8 \times 2) + 8 = s$ 24	M $(8 \times 9) + a = 81$ 9	T $(b \times 6) + 5 = 59$ 9
G $(7 \times 6) + 9 = t$ 51	N $(9 \times 6) + x = 60$ 6	U $(c \times 7) + 8 = 64$ 8

7. The same letter is used more than once in the equations below. As in exercise A, give the number that will make the equation true.

A $n + n = 16$ C $y + y = 18$ 9 F $5 + n + n = 9$ 2 H $r \times r \times r = 27$ 3
$n = 8$ D $y \times y = 81$ 9 G $x + x + 7 = 15$ 4 I $s + s + s = 12$ 4
B $a \times a = 16$ 4 E $t + t = 64$ 32

think

Give the number for *a* and the number for *b* in each exercise. The pair of numbers you give must serve as both addends and factors to give the correct sum and product.
For exercise A, you could write:
$a = 4, b = 2$

	Sum			Product
A	6	a 4	2 b	8
B	9	a 4	5 b	20
C	12	a 6	6 b	36
D	13	a 6	7 b	42
E	15	a 7	8 b	56
F	17	a 8	9 b	72

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the *Think* problem. Then let volunteers explain how they found their answers.

Assignments (pages 56–57)
Minimum: 1–21. Average: 1–5.
Maximum: 1–7.

Workbook, page 15

PAGES 58–59

Objective

The child will practice basic number facts by using jumps on a number line.

Preparation

It would be appropriate to begin immediately with the investigation. However, if you choose, hold a short drill of the basic number facts.

Investigation

After studying the sample, which uses jumps of 3 and would land on goals 24, 30, or 36, children should select *one* goal and figure out the jumps the grasshopper might make. For example, if the goal 30 is chosen, the grasshopper might make the following jumps:

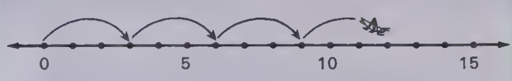
1 jump of 30	2 jumps of 15
30 jumps of 1	15 jumps of 2
5 jumps of 6	3 jumps of 10
6 jumps of 5	10 jumps of 3

Encourage children to work independently on this investigation. Give guidance to those who need it, and ask children if they are sure they have found *all* possible jumps the grasshopper might use.

Let's explore jumps on the number line.

Investigating the Ideas

If the grasshopper keeps jumping by 3's, which of the goals will he reach? 24, 30, 36



? Can you select one of the goals and find **all** the ways the grasshopper can get there if, on any one trip, his jumps are all the same? (No fractions.) See Investigation.

Discussing the Ideas

1. Give a multiplication equation for each "trip" you found in the Investigation. Sample answers: $8 \times 3 = 24$, $10 \times 3 = 30$, $12 \times 3 = 36$

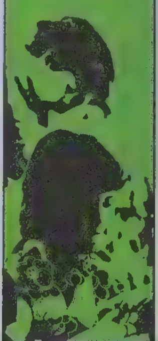
2. Study the new symbols. Then give the landing point for each part.

A 2 ③→ means: Start at 2. Jump 3 units to the **right.** 5

B 9 ←⑤ means: Start at 9. Jump 5 units to the **left.** 4

C 6 ③→ ←④ means: Start at 6. Jump 3 units to the **right;** then jump 4 units to the **left.** 5

D 12 ←⑥ ←⑥ means: Start at 12. Jump 6 units to the **left;** then jump 6 more units to the **left.** 0



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Discussion

For discussion exercise 1, have several children write their multiplication equations on the chalkboard, listing together those which relate to the same goal. Use these to review the use of the number line for multiplication. Discuss exercise 2 carefully so that children understand the meaning of the symbols. Help them see that these exercises (A–D) are just simple jumps on the number line and, hence, involve little more than finding the sum or difference of two numbers.

Using the Ideas

1. Give the landing point for each exercise.

A 2 ④→ 6	F 5 ⑥→ 11	K 16 ⑧→ 24	P 6 ←⑥ 0	U 48 ⑤→ 53
B 1 ⑤→ 6	G 7 ⑥→ 13	L 7 ⑨→ 16	Q 16 ←⑨ 7	V 13 ←⑤ 8
C 5 ①→ 6	H 8 ⑦→ 15	M 17 ⑨→ 26	R 26 ←⑨ 17	W 37 ⑥→ 43
D 4 ⑤→ 9	I 18 ⑦→ 25	N 9 ←① 8	S 17 ←⑧ 9	X 13 ←⑥ 7
E 5 ⑤→ 10	J 6 ⑧→ 14	O 8 ←④ 4	T 37 ←⑧ 29	Y 53 ←⑥ 47

2. Give the landing point for each exercise.

A 2 ①→ ③→ 6	E 6 ④→ ←① 9	I 6 ④→ ④→ ←⑧ 6
B 7 ③→ ⑤→ 15	F 5 ⑦→ ←⑥ 6	J 8 ←⑧ ⑧→ ⑧→ 16
C 8 ①→ ←① 8	G 12 ←⑨ ③→ 6	K 13 ④→ ⑥→ ←⑨ 14
D 9 ←③ ③→ 9	H 20 ⑧→ ⑧→ 36	L 17 ⑧→ ←⑥ ←⑨ 10

3. Give the landing point for each exercise.

A 0 ④→ ④→ ④→ ④→ ④→ 20	E 0 ⑦→ ⑦→ ⑦→ ⑦→ ⑦→ ⑦→ ⑦→ 49
B 0 ③→ ③→ ③→ ③→ ③→ ③→ ③→ 21	F 0 ⑧→ ⑧→ ⑧→ ⑧→ ⑧→ ⑧→ 48
C 0 ⑤→ ⑤→ ⑤→ ⑤→ ⑤→ ⑤→ ⑤→ ⑤→ 40	G 0 ⑨→ ⑨→ ⑨→ ⑨→ ⑨→ ⑨→ ⑨→ 63
D 0 ⑥→ ⑥→ ⑥→ ⑥→ ⑥→ ⑥→ ⑥→ ⑥→ ⑥→ 54	H 0 ③→ ③→ ③→ ③→ ③→ ③→ 18

4. Give the missing numbers.

A ▮ ⑤→: lands at 14 9	G ▮ ←⑦: lands at 6 13
B ▮ ⑧→: lands at 15 7	H ▮ ←⑨: lands at 5 14
C ▮ ⑨→: lands at 18 9	I ▮ ⑦→ ⑥→: lands at 16 3
D ▮ ⑦→: lands at 13 6	J ▮ ②→ ⑤→: lands at 15 8
E ▮ ⑥→: lands at 15 9	K ▮ ←⑦ ⑦→: lands at 54 54
F ▮ ⑨→: lands at 16 7	L ▮ ⑨→ ←⑨: lands at 73 73

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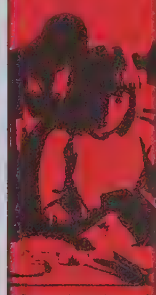


Using the Exercises

Before you have the children do the exercises on page 59, you might discuss exercises 2 and 3. Help the children realize that, when the exercise has two or more jumps indicated, a shortcut can usually be found. For example, for exercise 2F, since they jump 7 to the right and 6 to the left, they can think of this as one jump of 1 to the right. In some exercises, such as 2J, the first two jumps nullify each other, so they need think only of the third jump, or 8 + 8. In exercise 3, children should recognize the relationship of the repeated jumps to multiplication. Children may think they need to draw a number line to solve exercise 4, but treat this exercise primarily as a discovery activity and say as little as possible about how to work these problems.

Assignments (page 59)
Minimum: 1–2, oral.
Average: 1–3. Maximum: 1–4.

PAGES 60–61

Objective

Given a "number-line stack," the child will practice basic number facts by solving number-line puzzles.

Preparation

Unless you want to review multiplication facts, such as the 7's, 8's and 9's, begin immediately with the investigation.

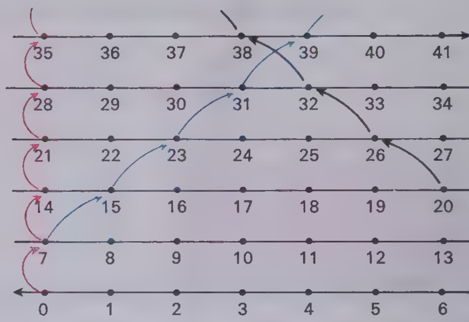
Investigation

Although children might work on this investigation independently, group work will give them an opportunity to discuss and share their ideas. Note that the number line is stacked by sevens; that is, if you begin at 0 and go directly up, you will find the multiples of 7. If you begin at 3 and go directly up, each jump will be a jump of 7. Consequently, each diagonal jump will be a jump of 8 or 6. To answer the investigation question, children not only have to figure out how to use the number-line stack, but must also be able to imagine additional rows. Circulate through the room and guide children when necessary, but try to have them do as much on their own as possible.

● *Can jumps be made on a "number-line stack"?*

Investigating the Ideas

Think of a number line that is stacked up in pieces. Imagine as many rows as you need.



? Can you figure out where the 6th jump will land for each color?

Red, 42; blue, 55; black, 56

Discussing the Ideas

1. Can you find an easy rule to tell where you will be for each type of jump in the Investigation? See Investigation.

2. Can you find the missing landing points and explain those given?

	Symbol	Start	Land
A	4 ①	4	5
B	34 ①	34	33
C	0 ①	0	7
D	8 ①	8	1
E	10 ①	10	18
F	30 ①	30	24
G	20 ①	20	12
H	4 ①	4	10

	Symbol	Start	Land
	18 ②	18	20
	0 ③	0	21
I	28 ③	28	7
	0 ⑤	0	40
J	18 ②	18	6
K	8 ② ③	8	31
	20 ③	20	23
★ L	14 ②	14	12
	13 ②	13	29
★ M	28 ①	28	20

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Discussion

Have children explain their results from the investigation. Discuss the meaning of each of the directional symbols thoroughly. Point out for the children that they should continue to think in terms of the stack pictured for the investigation. Then have children answer and explain how they found their answer to the parts of discussion exercise 2.

One objective for this type of exercise is to get the children to discover that they can think about multiplication for some of these number-line jumps. For example, on this number line, to jump to the upper right is equivalent to adding eight; hence, two jumps to the upper right would be equivalent to adding 2 × 8, or 16. Three jumps straight up would be equivalent to adding 7 three times, or 21.

Using the Ideas

1. Using the number-line stack in the Investigation, give the landing point for each exercise.

A 0 ① 7   D 0 ⑤ 35   G 0 ⑩ 70   J 1 ⑤ 36   M 4 ② 18   P 42 ② 28

B 0 ② 14   E 0 ⑥ 42   H 0 ⑧ 56   K 1 ⑨ 64   N 4 ⑥ 46   Q 42 ③ 21

C 0 ④ 28   F 0 ⑨ 63   I 1 ② 15   L 1 ⑦ 50   O 4 ⑨ 67   R 42 ⑥ 0

2. Give the landing point for each exercise.

A 24 ⑤ 59   D 59 ② 45   G 3 ③· ⑥ 48   J 3 ②· ② ① 12

B 60 ⑤ 95   E 28 ③ 7   H 12 ④ ④ 12   K 30 ·① ② 15

C 100 ⑧ 156   F 63 ⑨ 0   I 40 ⑤ ⑥ 47   L 30 ② ·① 15

★ **3.** Give the landing point for each exercise.

A 0 ⑤ 40   D 20 ① 28   G 4 ⑤ 44   J 6 ⑥ 42   M 7 ⑥· 13

B 0 ⑥ 48   E 4 ② 20   H 1 ⑧ 65   K 7 ① ·① 13   N 0 ② 12

C 27 ①· 28   F 4 ③ 28   I 6 ⑤ 36   L 7 ① 13   O 0 ⑧ 48

Use a number-line stack like this for exercises 4 and 5.

8 9 10 11 12 13 14 15

0 1 2 3 4 5 6 7

★ **4.** Give the landing point for each exercise.

A 0 ① 8   C 0 ⑧ 64   E 0 ① 9   G 12 ① ① ① ① 12

B 0 ⑥ 48   D 24 ③ 0   F 0 ⑦ 63   H 12 ·① ② ②· ② ·① 12

★ **5.** Give the missing numbers.

A 16 ▥ : lands at 0 2   D 72 ▥ : lands at 0 9   G 18 ▥ : lands at 0 2

B 32 ▥ : lands at 0 4   E 57 ▥ : lands at 1 7   H 63 ▥ : lands at 0 7

C 48 ▥ : lands at 0 6   F 67 ▥ : lands at 3 8   I 49 ▥ : lands at 7 6

61

Using the Exercises

After reminding the children that for exercises 1, 2, and 3 they should continue to think of the stack pictured for the investigation, have them do the exercises. When they have finished, allow time for checking papers and discussion.

Some of these exercises will be quite difficult for those children who have not yet discovered the multiplicity of ideas involved in these exercises. For example, in exercise 2A, starting at 24 and taking 5 jumps up may be quite difficult for a child who has not discovered that 5 jumps up is the same as 5 sevens, or 35, and hence the answer is 59. The child who has not grasped this multiplicity concept will have to visualize pieces of the number line until he reaches 78. You are encouraged to provide extra help for slower children, at least to the extent of suggesting to them the necessary ideas which they have not been able to discover for themselves. You should, however, avoid taking initiative away from those children who are able to make discoveries on their own.

Assignments (page 61)
Minimum: 1–2, oral.
Average: 1–2. Maximum: 1–5.

Objective

Given multiplication and addition equations, the child will understand that he can solve them because of certain basic principles.

Preparation

To stimulate interest in this lesson, you might write some equations such as the following on the chalkboard.

$$3 + 2 = 2 + 3$$
$$4 + (6 + 5) = (4 + 6) + 5$$
$$2 \times 45 = (2 \times 40) + (2 \times 5)$$
$$9 \times 1 = 9$$
$$8 \times 0 = 0$$

The purpose is to prepare children for a study of the basic principles. The equations are true because they are based on the principles studied in this lesson.

If children recall some of the principles studied in previous years, they might name the order, or grouping, principle. However, use these equations primarily to lead into the material in the text.

Basic Principles

In this lesson you will review some of the basic principles for addition and multiplication. Special names for the principles are given.
For each exercise, copy the equations and give the number for ***n***.

1. 0 principle — When you choose a whole number and **add 0,** the sum is the number you chose.

A $37 + 0 = n$ 37 B $n + 0 = 86$ 86 C $13 + n = 13$ 0 D $n + 0 = 0$ 0

2. 1 principle — When you choose a whole number and **multiply by 1,** the product is the number you chose.

A $53 \times 1 = n$ 53 B $64 \times n = 64$ 1 C $n \times 1 = 89$ 89 D $n \times 1 = 0$ 0

3. Commutative principle, + (Order principle, +) — When you add, you can **change the order** of the addends and the sum is the same.

A $9 + 7 = n + 9$ 7 B $n + 93 = 93 + 68$ 68 C $537 + n = 86 + 537$ 86

4. Commutative principle, × (Order principle, ×) — When you multiply, you can **change the order** of the factors and the product is the same.

A $5 \times n = 7 \times 5$ 7 B $49 \times 51 = n \times 49$ 51 C $90 \times 70 = 70 \times n$ 90

5. Associative principle, + (Grouping principle, +) — When you add, you can **change the grouping** and get the same sum.

A $(5 + 2) + 3 = 5 + (2 + n)$ 3 B $87 + (n + 93) = (87 + 34) + 93$ 34

6. Associative principle, × (Grouping principle, ×) — When you multiply, you can **change the grouping** and get the same product.

A $(n \times 3) \times 4 = 2 \times (3 \times 4)$ 2 B $34 \times (8 \times 7) = (34 \times 8) \times n$ 7

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Discussion

Although these exercises are indicated for independent study, you might use them as a basis for discussion. In particular, you might want to discuss the new terminology with the children before they begin to work on their own. You may prefer to continue using the names *order* and *grouping;* however, even if you do, introduce the words *commutative* and *associative* as alternate terms. One of the principal justifications for the inclusion of the associative and commutative principles in the elementary curriculum is the relationship between these principles and the generalization that we can rearrange terms in an addition or multiplication equation in any way that is convenient.

Assignments (page 62)

Minimum: Parts A, B of 1–6.
Average: 1–6. Maximum: 1–6.

Using the Principles

1. Use part I and one of the basic principles to help you complete part II. Then tell which principles you used.

		I		II	
A	Since	937+685=1622,	we know that	685+937=*n*.	1622 (C,+)
B	Since	(18+36)+37=91,	we know that	18+(36+37)=*n*.	91 (A,+)
C	Since	6×(9×37)=1998,	we know that	(6×9)×37=*n*.	1998 (A,×)
D	Since	39×8=312,	we know that	*n*=8×39.	312 (C,×)
E	Since	17×1=17,	we know that	44×1=*n*.	44 (1)
F	Since	43+(39+27)=109,	we know that	(43+*n*)+27=109.	39 (A,+)
G	Since	559+0=559,	we know that	*n*=747+0.	747 (0)
H	Since	534+876=1410,	we know that	(534+876)×1=*n*.	1410 (1)

2. Think about part I. Then use part I and any principles you need to complete part II.

	I		II	
A	23+17=40	→	23+64+17=40+*n*	64
B	16+14=30	→	14+51+16=*n*+51	30
C	25+40=65	→	25+6+40=*n*	71
D	39+47+35=121	→	35+39+47=*n*	121
E	7×5×9×2=630	→	5×2×7×9=*n*	630
F	10×6=60 and 8×5=40	→	10×8×6×5=60×*n*	40
G	18×9=162	→	(18×9)×1=*n*	162
H	17+18=35 and 24+36=60	→	18+36+17+24=*n*	95
I	4×2×10×10=8×100	→	4×10×2×10=*n*	800

63

Mathematics

For all whole numbers a, b, and c:

Addition	Multiplication
1. $a+b=b+a$ 2. $(a+b)+c$ $=a+(b+c)$ 3. $a+0=a$	1. $a\times b=b\times a$ 2. $(a\times b)\times c$ $=a\times(b\times c)$ 3. $a\times 0=0$, $a\times 1=a$
1. Order (commutative) principle 2. Grouping (associative) principle 3. Zero and one principles	

In elementary mathematics, the most important consequences of the order and grouping principles are the generalizations which arise from them: the order and grouping principles together allow us to rearrange addends in addition and factors in multiplication.

Once these generalizations are drawn and children have learned to rearrange addends and factors freely, there is little need to continue stressing the order and grouping principles separately. However, since it is important that children understand that the rearranging is possible because of the order and grouping principles, we will continue to restate this fact from time to time.

Workbook, page 16

Using the Exercises

Exercise 1 on page 63 requires not only that children complete the equations in the second part of each section, but also that they name the basic principle applied in each case. Many children would benefit from working these exercises with one or two classmates. A discussion of their choices would also be helpful when they have completed them. Note that the distributive principle is not included among these exercises. It will be treated in the next lesson.

Assignments (page 63)

Minimum: 1–2, oral.

Average: 1–2. Maximum: 1–2.

PAGES 64–65

Objective

Given addition and multiplication equations, the child will solve them by applying the basic principles (page 64), or by using his understanding of place value (page 65).

Preparation

To prepare for this lesson, you might name a principle and ask the children to give an example for it. For instance, mention the zero principle and a child might state or write on the chalkboard an equation such as $183 \times 0 = 0$; or mention the commutative principle for addition, and a child might respond with an equation like $73 + 42 = 42 + 73$.

Investigation

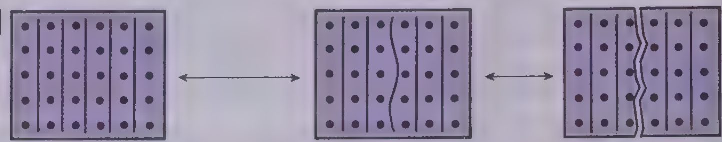
In this investigation, children examine an example of the distributive, or multiplication-addition, principle. Explain that two applications of the principle are given for a particular multiplication fact. Then they are challenged to show an example of their own. Observe with them that they need only one multiplication fact, or phrase, but that they should draw two illustrations of it. Help them realize that either the first factor or the second might be separated into subsets. You might suggest a multiplication problem to help some children begin.

● *Do you understand the multiplication-addition principle?*

Investigating the Ideas

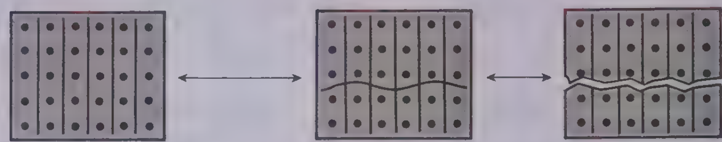
Example A shows how you can think about "breaking apart" the first factor before multiplying. Example B shows how you can think about "breaking apart" the second factor before multiplying.

A



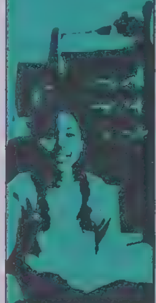
$6 \times 5 = (3 + 3) \times 5 = (3 \times 5) + (3 \times 5)$

B



$6 \times 5 = 6 \times (3 + 2) = (6 \times 3) + (6 \times 2)$

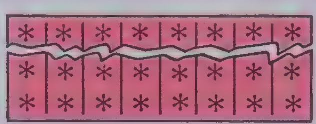
? Can you draw a set of dots for a multiplication problem?	Show pictures and equations for your problem like the ones above. Answers will vary. See Investigation.



Discussing the Ideas

1. The principle illustrated above is called the **multiplication-addition** principle or the distributive principle. Explain how you can use this principle to help you find the product 6×9 if you know 6×5 and 6×4.
Sample answer: Add 6x5 and 6x4 to find 6x9.
2. How can you use the figures to help you solve the equations?

A



$8 \times (1 + 2) = (8 \times 1) + (8 \times n)$ 2

The figure shows that 8x3 is 8 ones and 8 twos, so n=2.

B

$6 \times 4 = (n \times 4) + (4 \times 4)$ 2

The figure shows that 6x4 is 2 fours and 4 fours, so n=2.

64

Discussion

Encourage children to discuss the illustrations they made in the investigation. The use of an overhead projector would be helpful, if available. Then develop the examples given in discussion exercise 1. Stress that this principle "distributes" the multiplication operation over the addition combination:

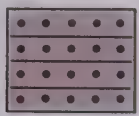
$$\begin{aligned} 6 \times 9 &= 6 \times (5 + 4) \\ &= (6 \times 5) + (6 \times 4) \end{aligned}$$

Also, observe with them that figures may be broken apart in various ways, as shown in exercise 2.

Using the Ideas

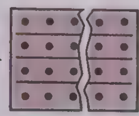
1. Solve the equations.

A



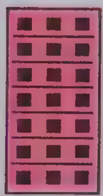
$4 \times 5 = a$ 20

B



$(4 \times 3) + (4 \times 2) = b$ 20

C



$7 \times 3 = c$ 21

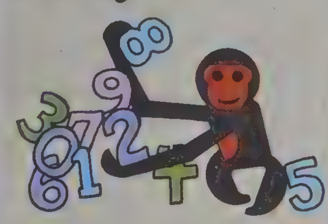
D

$(2 \times 3) + (5 \times 3) = d$ 21

Solve the equations.

2. $5 \times 8 = (n + 3) \times 8$ 2
3. $4 \times 7 = (2 + n) \times 7$ 2
4. $6 \times 8 = (3 \times 8) + (n \times 8)$ 3
5. $8 \times 7 = (n \times 7) + (3 \times 7)$ 5
6. $6 \times 9 = (n \times 9) + (1 \times 9)$ 5
7. $7 \times 7 = (5 \times 7) + (n \times 7)$ 2
8. $9 \times 8 = (n \times 8) + (4 \times 8)$ 5
9. $5 \times 12 = n \times (10 + 2)$ 5
10. $3 \times 14 = 3 \times (n + 4)$ 10
11. $2 \times 13 = 2 \times (10 + n)$ 3
12. $6 \times 11 = (6 \times n) + (6 \times 1)$ 10
13. $4 \times 16 = (n \times 10) + (4 \times 6)$ 4
14. $7 \times 15 = (7 \times 10) + (7 \times n)$ 5
15. $8 \times 17 = (8 \times n) + (8 \times 7)$ 10
16. $7 \times 23 = (7 \times n) + (7 \times 3)$ 20

think



A number pair are we.
Our product, 54.
We differ by just 3.
We'll tell you nothing more.

WHO ARE WE?

6, 9

More practice, page A-5, Set 10

Using the Exercises

You might choose to use exercise 1 on page 65 as a basis for discussion. Point out to the children that the distributive principle gives them an easy way to think about finding products.

When children have completed the remaining exercises, you might also point out how their understanding of place value can be applied with this principle to simplify problems with 2-digit numerals, as in the last 8 equations. For example:

$$7 \times 23 = (7 \times 20) + (7 \times 3)$$

Encourage those who solve the *Think* problem to explain it to the whole class.

Assignments (page 65)

Minimum: 1–8. Average: 1–12.
Maximum: 1–16.

Mathematics

An important generalization from the commutative and associative principles is that these two principles, together, allow us to rearrange addends (or factors) in any way that is convenient.

Since three numbers can be ordered six ways and since each of these orderings can be grouped two ways, three addends such as a, b, and c can be ordered and grouped in 12 ways:

$(a+b)+c$	$c+(b+a)$
$a+(b+c)$	$(c+b)+a$
$a+(c+b)$	$(b+c)+a$
$(a+c)+b$	$b+(c+a)$
$(c+a)+b$	$b+(a+c)$
$c+(a+b)$	$(b+a)+c$

We can proceed down this list and verify the equality of these expressions by alternately applying the grouping and order principles for addition. Even without this complete list, we can prove the equality of any two of these expressions by using the order and grouping principles. To show that

$$c+(b+a)=b+(c+a),$$

we offer two different proofs.

First proof:

$$\begin{aligned} c+(b+a) &= (c+b)+a \quad \text{GP} \\ &= (b+c)+a \quad \text{OP} \\ &= b+(c+a) \quad \text{GP} \end{aligned}$$

Second proof:

$$\begin{aligned} c+(b+a) &= (b+a)+c \quad \text{OP} \\ &= b+(a+c) \quad \text{GP} \\ &= b+(c+a) \quad \text{OP} \end{aligned}$$

Another important idea in this lesson is that the grouping principle allows the omission of parentheses from expressions involving the sum of three numbers. Since we can group these numbers in any convenient way, the expression will be just as meaningful without parentheses. Of course, this leads to the additional step of rearranging the numbers in the absence of parentheses.

Follow-up

As a review, provide a list of large numerals and have children practice reading them.

Duplicator Masters, page 12
Skill Masters, page 12

PAGES 66–67

Objectives

The child will demonstrate his ability to work with the concepts presented in this chapter.

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

Any game like "What's My Rule" would be suitable for a short review of basic facts.

Reviewing the ideas

1. Solve the equations.

A $x + 9 = 16$ 7 E $8 + n = 17$ 9 I $5 \times b = 5 \times 3$ 3 M $42 \div 7 = r$ 6
B $16 - 9 = n$ 7 F $17 - x = 8$ 9 J $8 \times 7 = r \times 8$ 7 N $8 \times 0 = q$ 0
C $13 = 5 + p$ 8 G $54 - 54 = n$ 0 K $17 \times 1 = b$ 17 O $t \times 9 = 0$ 0
D $13 - 5 = a$ 8 H $4 \times n = 0$ 0 L $n \times 7 = 7$ 1 P $0 \div 9 = s$ 0

2. Solve the equations.

A $7 + 7 + 7 + 7 + 7 = n$ 35
$5 \times 7 = n$ 35

B $30 - 5 - 5 - 5 - 5 - 5 - 5 = n$ 0
$30 \div 5 = n$ 6

3. Solve.

A $8 + 7$ 15 E $15 - 9$ 6 I 5×9 45 M 7×8 56 Q 9×8 72 U $42 \div 7$ 6
B $6 + 5$ 11 F $14 - 6$ 8 J 7×6 42 N 9×6 54 R 7×7 49 V $63 \div 9$ 7
C $9 + 4$ 13 G 7×4 28 K 9×9 81 O 8×8 64 S $40 \div 5$ 8 W $48 \div 6$ 8
D $12 - 8$ 4 H 8×6 48 L 5×7 35 P 6×5 30 T $32 \div 8$ 4 X $56 \div 8$ 7

4. Find the missing output numbers.

Function Rule: $(2 \times n) + 1$

n	Output
0	A 1
2	B 5
3	C 7
5	D 11

Function Rule: $(n \times 8) + 5$

n	Output
3	E 29
5	F 45
7	G 61
9	H 77

Function Rule: $(4 \times n) + (3 \times n)$

n	Output
5	I 35
0	J 0
8	K 56

Function Rule: $(n \times n) - n$

n	Output
4	L 12
8	M 56
10	N 90

think

For this table, give four different function rules, each of which uses only one of the signs, +, −, ÷, ×.

$n + 0$ $n \times 1$
$n - 0$ $n \div 1$

Function Rule: ?

n	Output
1	1
2	2
3	3
4	4
5	5
⋮	⋮

Discussion

Whether you use page 66 as a class review or as a test page, it is important that the children be given an opportunity to discuss these exercises once they have completed the page. Discuss in particular any topic which caused difficulty for the children.

The *Think* problem should prove to be a stimulating activity for many. The problem does point up a significant mathematical idea: that there may be many different function rules for a single set of ordered pairs. That is, $n + 0$ gives exactly the same set of ordered pairs as does $n \times 1$ or $n \div 1$ or $n - 0$.

Keeping in Touch with Addition, Place value, Multiplication, Sets

1. Solve the equations.

A $10 \times 10 = n$ 100
B $(10 \times 10) \times 10 = n$ 1000
C $(10 \times 10 \times 10) \times 10 = n$ 10 000
D $(10 \times 10) \times (10 \times 10) = n$ 10 000

2. Find the products.

A $5 \times (10 \times 10) = n$ 500
B $8 \times (10 \times 10 \times 10) = n$ 8000
C $7 \times (10 \times 10) = n$ 700
D $6 \times (10 \times 10 \times 10) = n$ 6000
E $8 \times (10 \times 10 \times 10 \times 10) = n$ 80 000
F $9 \times (10 \times 10 \times 10 \times 10) = n$ 90 000
G $18 \times (10 \times 10) = n$ 1800
H $23 \times (10 \times 10 \times 10) = n$ 23 000

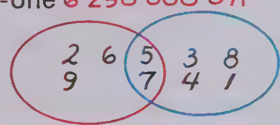
3. Solve the equations.

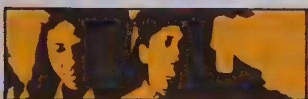
A $638 = (6 \times n) + (3 \times 10) + 8$ 100
B $375 = (3 \times 10 \times 10) + (n \times 10) + 5$ 7
C $5671 = (5 \times n) + (6 \times 100) + (7 \times 10) + 1$ 1000
D $54\,965 = (n \times 1000) + (9 \times 100) + (6 \times 10) + 5$ 54
E $39\,732 = (39 \times 1000) + (7 \times n) + (3 \times 10) + 2$ 100
F $5472 = (5 \times 10 \times 10 \times 10) + (4 \times n) + (7 \times 10) + 2$ 100

4. Write the numerals for the numbers.

A six thousand three hundred thirty-seven 6337
B fifty-four thousand five hundred twenty 54 520
C seven thousand thirty-four 7034
D eighty thousand ninety 80 090
E three million nine thousand five 3 009 005
F six billion two hundred thirty million seventy-one 6 230 000 071

5. Which number is inside the red loop, inside the blue loop, and larger than 6? 7





You are invited to explore

ACTIVITY CARD 3 Page 334

Using the Exercises

Assign the review exercises on page 67 as independent work. Use any exercises you choose as a basis for discussion. Note that exercise 5 reviews the type of reasoning problems studied in Chapter 1. You might encourage any children who have been using attribute games to describe these activities to others.

Follow-up/"Combo"

A game similar to Bingo will help the children review multiplication facts. The object of the game is to cover five products in any row, column, or diagonal as the caller shows the facts on flash cards and calls them out. Winners call "Combo" and verify the products from the caller's master sheet.

To make the game, cut 10-by-12-centimetre cards and rule 30 2-cm boxes on them. Fill in "Combo," "Free," and products at random (see illustration). The players should have cork or paper discs to cover products with. (*Note:* Since speed is essential to the success of this game, it may be best for the teacher to be caller.) If the children mark another grid on the back with the digits 0 to 9 using a color different from the first side, the card can be used to practice finding differences and quotients for the basic combinations as well as for finding products.

C	O	M	B	O
40	72	48	21	12
18	25	32	81	20
49	42	FREE	16	45
28	35	56	24	9
36	27	15	30	54

Also, you might continue to review large numerals by giving children worksheets like the one below.

> Give the numeral for each of the following expressions:
> 1. $(6 \times 1000) + (7 \times 100) + (4 \times 10) + 8 = n$
> 2. $(5 \times 100) + (3 \times 10) + 9 = n$
> 3. $(4 \times 100) + 2 = n$
> 4. twenty thousand, four hundred thirty-one
> 5. eight hundred fifty-five thousand, seven hundred three
> 6. two thousand six hundred forty
> 7. forty million, three hundred five thousand, eight hundred thirty-two

Workbook, page 18 (Use with text page 66.)

Pages 68–95

General Objectives

To introduce some basic concepts of geometry
To introduce some standard geometric notation
To introduce the concept of congruence for segments, angles, and triangles
To review concepts of length and the measurement process
To introduce angle measure using radians and degrees
To review the concepts of perimeter and area
To provide experience in finding the areas of rectangles, parallelograms, and triangles

The opening lesson of this chapter is devoted to helping the children understand the relationship between the physical world and the ideas of geometry. The material is designed to help the children understand that we only *think about* the points, lines, rays, and segments of geometry, whereas we can actually touch or see the things in the physical world which remind us of these geometric ideas.

Although some attention is given to separating the physical world from mathematical geometry, it should be noted that the geometry we treat in this chapter is primarily geometry of the physical world, or "drawing board" geometry.

After the introduction, the following topics of plane geometry are presented: (1) segments, rays, and lines; (2) congruent segments, angles, and triangles; (3) comparison of angles; (4) parallel and intersecting lines; (5) triangles and symmetry; and (6) polygonal shapes.

Concepts of measurement are integrated with the geometric topics. These concepts include: (1) the arbitrary selection of units, unit segments, and unit angles; (2) angle measure with a protractor; (3) perimeter of polygons; and (4) area of rectangles, parallelograms, and triangles.

Mathematics

Many parallels could be drawn between the study of geometry and the study of arithmetic. In both disciplines, we begin with certain undefined elements, make assumptions about these elements, and then proceed—through deduction and definition—to construct the mathematical system. At the elementary school level, many areas of mathematics are treated intuitively, and this is the case with geometry. For example, we begin in arithmetic with the concept of number, which is basically an undefined concept; certain basic principles are established for these undefined elements—numbers; and then the structure is built from these numbers and basic principles. At every stage, this is kept on an intuitive level and at a level which is compatible with the age and maturity of the children.

Much the same type of development is used in geometry. We begin the study of geometry with certain undefined elements—points and lines. We then develop the children's intuition for points and lines, and thus make plausible certain basic principles concerning points and lines.

Since geometry is so closely related to objects in the physical world with which children are familiar, the structure of geometry might be less obvious than that of arithmetic, which is presented more abstractly. However, in this chapter, there are certain areas that illustrate some of the structure of the mathematical system. For example, the ideas of congruence bring out a certain element of the structure of the study of geometry. In many developments of geometry, the idea of congruence is taken basically as an undefined concept relating to segments and angles. We then need to make assumptions about this undefined relationship. Having made such assumptions, we can proceed to define such things as comparative sizes of segments or of angles and continue to build the structure from this undefined relationship and the assumptions about it.

However, in this chapter we present a view that appeals more to children's intuition. We do not specifically state what it means for one segment to be longer than another; we merely provide experiences for the children which build their intuition for the ideas of longer and shorter and larger and smaller. Once this intuition is developed, we define the idea of congruence between two segments in terms of this undefined relationship, *longer* and *shorter*. We say that two segments are congruent to one another if neither is longer than the other. Of course, we could say that the two segments are congruent if they are the same size, but it should be recognized that "same size" is also an intuitive concept. This same procedure is used to develop the idea of the congruence of two angles.

Also, several techniques for angle measure are suggested. We use these techniques to give the children a general understanding of the basic ideas involved in angle measure. Next, we introduce the protractor as a device for precisely measuring a given angle. The importance of understanding is continually stressed as the children actually construct a protractor, using a unit angle.

Congruent triangles are then studied. Although children are not required to measure angles of a triangle, they learn that two triangles are congruent if their angles and

segments can be matched so that the angles and segments of one triangle are congruent to the corresponding angles and segments of the other.

Finally, we explore concepts of perimeter and area for various plane figures.

Teaching the Chapter

Materials

- Cardboard
- Centimetre ruler
- Chalkboard compass
- Compass, screw type, if possible
- Envelopes (to store tangrams)
- Geoboards
- Graph paper
- Metre stick
- Protractor
- Ruler
- Scissors
- String (minimum of 3 metres for every 4 children)
- Tagboard
- Tracing paper

Vocabulary

angle	metre stick
area	parallel lines
base (of a parallelogram)	parallelogram
	perimeter
bisect	perpendicular lines
centimetre	
circumference	plane
compass	point
congruent	polygon
construction	protractor
degree	quadrilateral
formula	radian
height (of a parallelogram)	ray
	rectangle
length	segment
line	unit
measure	width

You will probably find it useful to have a compass and metre stick to use for chalkboard illustrations. In addition, each child should have his own compass. It is also essential for each child to have a centimetre ruler. Cardboard backing is recommended to keep the children from marring their desk tops while using the compasses. It is not absolutely necessary for each child to have a protractor, but one should be available to refer to for accuracy, after the children make their own.

The vocabulary list for this chapter is quite long. Many of the words given will already be familiar to the children since they have occurred in measurement lessons in earlier parts of the text or in geometry and measurement lessons in previous books.

Many of the other words in the vocabulary list are vital to the study of geometry, and an attempt should be made to see that the children understand their meaning and try to remember them, although not to the point of rote memorization.

Lesson Schedule

Plan to spend about two-and-one-half to three weeks on this chapter. Of course, you will want to adjust your schedule according to the needs, abilities, and interests of your children, and perhaps allot some extra time to capitalize on any special interests.

Evaluation of Progress

In evaluating the children's achievement for this chapter, you should avoid exaggerating the importance of the mechanical skills involved in construction; the greater stress should be upon children's understanding of a few general ideas. The children should understand the general concept of congruence for segments, angles, and triangles. Certainly, they should also be familiar with some of the notation for segments, rays, and angles.

The children should develop some basic understanding of measurement using unit segments for length and perimeter, unit squares for area, and unit angles for angle measure. Again, we emphasize that one of the primary objectives of this chapter is to give the children a general feeling for the study of geometry rather than to have them memorize many specific facts, types of notation, and definitions.

Resources for Active Learning

General Activities

Applied Mathematics Cards. Group 2/21, Schofield and Sims. (Available from Mafex Associates, Willowdale)

Experiments in Mathematics, Stage 1, "Paper Folding," pp. 18-19, Houghton Mifflin (Available from Thomas Nelson and Sons)

Experiments in Mathematics, Stage 1, p. 52, Houghton Mifflin [Optical illusions] (Available from Thomas Nelson and Sons)

Franklin Series: *Mathematics Around the Clock,* Lyons and Carnahan (Available from McGraw-Hill Ryerson)

Freedom to Learn, "Geometry—Properties of Shapes," pp. 149–152, Addison-Wesley

Maths Mini-lab, Cards 128–130, Selective Educational Equipment

Modern Math Games . . ., "How Many Polygons?" p. 57, Fearon

Nuffield Project: *Computation and Structure* 2, "Development of Weights and Measures," pp. 6–41, Wiley

Tangrams — 330 Puzzles, Dover Publications (Available from Musson Book Company)

Manipulative Devices

Geo Blocks (Selective Educational Equipment; McGraw-Hill Ryerson)

Geoboards (Addison-Wesley)

Geo Strips (Math Media; Selective Educational Equipment)

Mirror Cards (McGraw-Hill Ryerson)

Optical Illusion Kit (Edmund Scientific)

Pattern Blocks (Selective Educational Equipment; McGraw-Hill Ryerson)

Sigma Chips (Sigma, Scott Scientific)

Tangrams, cards and pieces (Selective Educational Equipment; McGraw-Hill Ryerson)

Commercial Games

Configurations (Hammet; Wff 'N Proof)

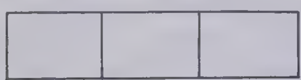
PAGES 68–69

Objective

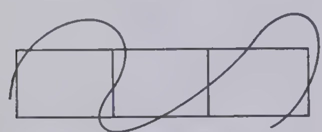
Given the names or figures for point, line, segment, and ray, the child will be able to identify the figures and use symbols to label them.

Preparation

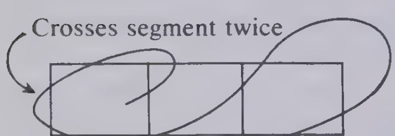
You might begin with the investigation immediately. Or, you might use a figure on the chalkboard to prepare for the exploration suggested in figure A. For example, draw the following figure:



Then draw a path through it, pointing out that each segment may only be crossed by this path once.



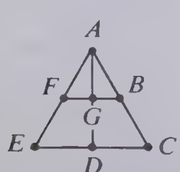
Acceptable



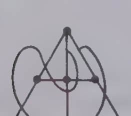
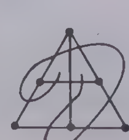
Unacceptable

Investigation

Make sure the children understand that the path they draw can cross each segment only once. Note that, for the purposes of this activity, a segment must be considered to continue only from one endpoint to the next endpoint. For example, in figure E, $\overline{AB}$ is distinct from $\overline{BC}$.



Although we may also speak of the segment $\overline{AC}$, in this investigation $\overline{AB}$ and $\overline{BC}$ should be thought of as two segments, both of which may be crossed once. There is more than one acceptable path for each figure, as shown below.

 or 

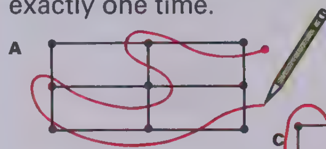
4 Geometry and Measurement I

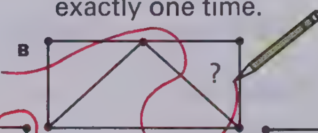
● *What are points, lines, and segments?*

Investigating the Ideas

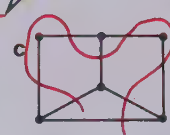
In figure **A**, a path can be drawn that passes through each segment of the figure exactly one time.

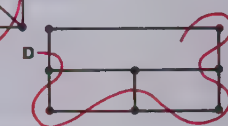
In figure **B**, **no** path can be found that will cross each segment exactly one time.



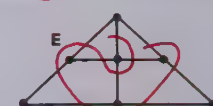


Copy figures **C**, **D**, and **E** on your paper.



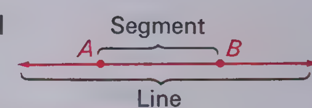


? Can you find a path that will cross each segment of each figure exactly one time?

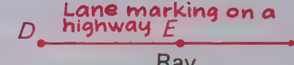


Discussing the Ideas

1. The figures in the Investigation show **points** and **segments**. Are points and segments parts of **lines**? Yes



2. How would you describe a segment? Sample answer: Two points of a line and all points of the line that lie between them.
3. Can you name some physical objects that remind you (Answers will vary.)
   A of points? Tip of pencil
   B of segments? Edge of ruler
   C of lines? Lane marking on a highway
4. This is one way we picture rays.



   A How is a ray different from a line? Ray has an endpoint.
   B How is it different from a segment? Ray has only one endpoint; segment has two.
   C Can you name some things that remind you of rays? Flashlight beam
5. Any flat surface suggests a plane. What are some objects that remind you of a plane? Sheet of paper, table top, etc.

Plane

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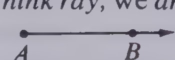
Discussion

One of the main purposes of the investigation is to open the geometry chapter with an enjoyable experience, related in part to the review of such basic geometric figures as point, segment, line, and ray.

Allow children to discuss and display the paths they found, but focus most of the discussion on the discussion exercises in the text. As you work through the exercises, help children understand that the objects and pictures presented in this lesson represent basic geometric ideas, just as numerals are symbols which represent the ideas of numbers. Thus we *see* a *headlight*, we *think ray*, we *draw*

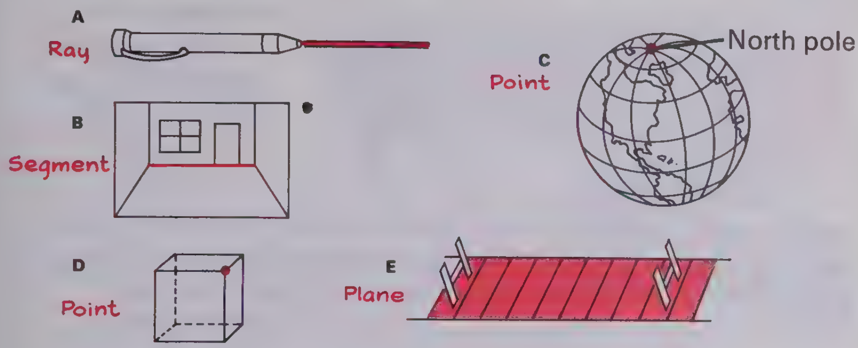


and we *use a symbol*, $\overrightarrow{AB}$.

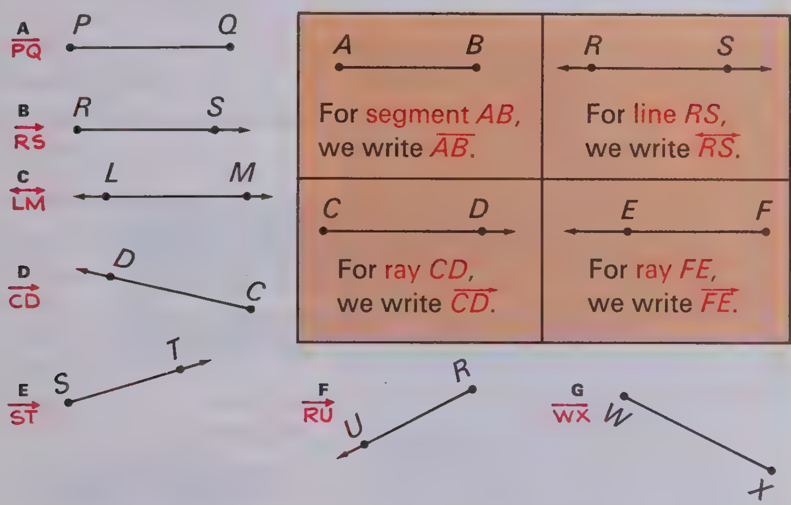
Also, explain that figures like triangles, squares, and angles are called *plane geometric figures* because all their points lie on a plane, that is, a flat surface. Figures such as cubes, cylinders, and spheres are called *space figures* because they do not lie in one plane. These space figures will be studied in a later chapter.

Using the Ideas

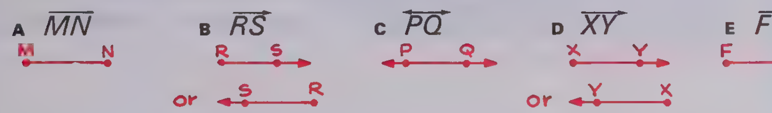
1. Give the geometric figure suggested by each picture.



2. Study each of the examples. Then give the symbol for each figure shown in exercises A through G.



3. Draw and label a picture for each symbol.



69

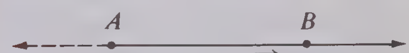
Mathematics

Some basic geometric figures are introduced in this lesson. The words *point, line,* and *plane* are usually taken as undefined words in formal developments of geometry. For children, such abstract concepts must be related to the physical world. Thus, a dot reminds us of a point, a stretched string suggests a part of a line, and a table top suggests a portion of a plane. Definitions of segment and ray introduced in this lesson are given below.

A segment consists of two points and *all* the points on a line between the two points.

Segment XY ($\overline{XY}$)

A ray is the union of one point, called the endpoint, and all the points on one side of that point on a line.



Ray AB ($\overrightarrow{AB}$)

Follow-up

Children may enjoy trying to draw figures comparable to those in the investigation and exploring the possibility of tracing a path which crosses each segment only once.

To give children an opportunity to observe objects which suggest basic geometric ideas, take the class on a walk around the school grounds. Ask them to point out any object which suggests a point (top of flagpole, door knob, corner of a box), a line (telephone wires, painted lines on playground), a ray (tetherball pole, flagpole), or a plane (floor, playground). When you return to the classroom, some children might like to make a chart showing the figure, the symbol, and some objects they saw which suggest it.

Resources for Active Learning

Experiments in Mathematics, Stage 3, "Networks and Routes," pp. 20-21, Houghton Mifflin (Available from Thomas Nelson and Sons)

Math Activity Cards, "Euler's Relation," D14, Macmillan.

Workbook, page 19

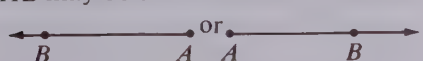
Using the Exercises

Before assigning the exercises on page 69 as independent work, you might choose to go over the symbols with the children.

$\overline{AB}$ – segment, $\overrightarrow{CD}$ – ray, $\overleftrightarrow{RS}$ – line

Note that the arrowhead in the symbol for a ray always points to the right. A ray directed to the left is indicated by writing the endpoint label first.

Thus, the figure symbolized by $\overrightarrow{AB}$ may be drawn



As you discuss this idea, you might point out that a line may be thought of as two rays in opposite directions from the same point.

Assignments (page 69)

Minimum: 1, oral; 2.

Average: 1, oral; 2–3.

Maximum: 1–3.

Objective

Given two segments, the child will be able to determine whether or not they are congruent.

Preparation

Materials

tracing paper; compass; cardboard

To prepare for the investigation, help the children observe that if one object is not shorter than another, then it is either the same size or longer. Draw one line segment on the chalkboard. Then tell the children that you are thinking about a segment that is *not* shorter than the one already drawn. Help them realize that there remain two possible segments: a segment of the same length and a segment longer than the one drawn. The objective is to have children realize that, given any two segments, the first may be shorter than the other, equal to the other, or longer than the other.

Investigation

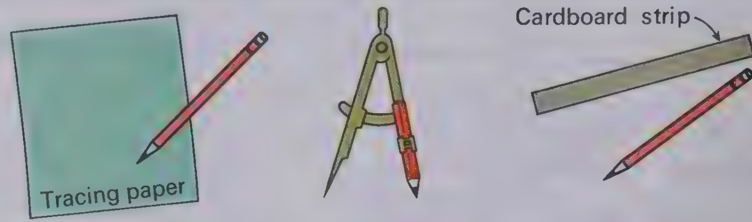
Although children may think that the compass is used primarily to construct circles, this skill is not pertinent to the use of the compass in this lesson. Encourage the children to use the materials suggested in any way they want.

Notice that a ruler is not suggested; the purpose is to make a nonmetric comparison of the segments. If children trace one segment, they can lift the paper and superimpose the drawn segment on the second one to compare length.

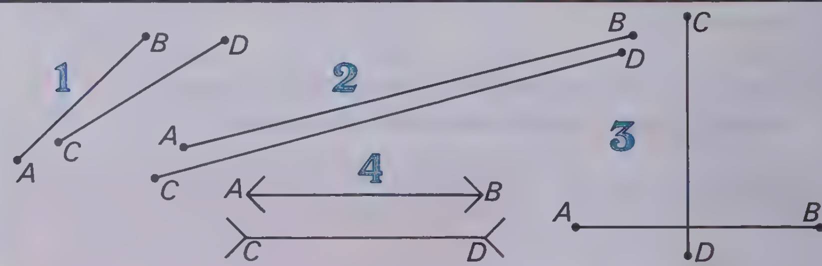
Children may be surprised to find that the pairs of segments in figures 3 and 4 have equal lengths.

When are two segments congruent?

Investigating the Ideas



? Can you find a way to use each of the devices shown above to tell which segment in each pair is longer (if one is)?
See Investigation.



Discussing the Ideas

Two segments are congruent to each other if their ends are equally far apart.

If $\overline{AB}$ is congruent to $\overline{CD}$, we write $\overline{AB} \cong \overline{CD}$.

1. A Explain how you used the devices to decide whether the segments above are congruent.
   B In which case above can we write $\overline{AB} \cong \overline{CD}$? Why?
   Case 3, because the segments have the same length.
2. How could you use each of the devices above to help you draw $\overline{EF}$ so that $\overline{EF} \cong \overline{PQ}$? P ———— Q
   See Discussion.

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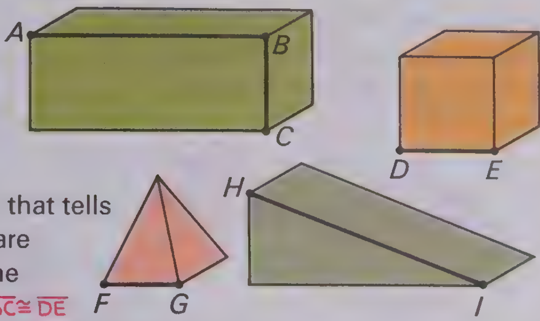
Discussion

Explain and discuss the meaning of the term *congruent*. To help the children understand the meaning of congruence, you might also speak of two segments as being congruent if they have the "same size."

However, encourage children to use the terminology given in the definition in the text. Use the symbol for congruence (≅) as you refer to congruent segments. The symbol is read, "is congruent to." In discussion exercise 2, remind children to read the symbols correctly. Direct children to use the three methods explored in the investigation to draw segment *EF*.

Using the Ideas

1. Each darkened segment is an edge of one of the figures below.
   - A Which darkened segment is longest? $\overline{HI}$
   - B Which of these segments is shortest? $\overline{FG}$
   - C Write a statement that tells which segments are congruent. Use the correct symbols. $\overline{BC} \cong \overline{DE}$



2. Use one of the devices shown in the Investigation to draw a segment that is congruent to each segment given below.

   A ———— B ————

   C ———— D ————

   Constructions will vary.

3. Draw a segment on your paper. Label it XY. Draw another segment PQ so that $\overline{XY} \cong \overline{PQ}$.
   Constructions will vary.

★ 4. Write a story about the picture below. Be sure to include some of the mathematical ideas you learned in this lesson.
   See Using the Exercises.

think

1. Start inside a triangle and draw a path to cross each side exactly once. Where does your path end, inside or outside? Outside
   Try this with a figure of
   A 4 sides. Inside B 5 sides. Outside C 6 sides. Inside
2. Would you be inside or outside if you did this with a figure of
   A 2384 sides? Inside B 6749 sides? Outside

Also see Solution, T.E. page 71.

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Follow-up

Children might enjoy making a larger "compass" from two tapered strips of heavy cardboard joined together near one end by a paper fastener. With this larger instrument they might compare various "segments" suggested by objects in the classroom, such as heights of books, lengths of window panes, edges of desk tops, and so on.

Solution, Think, page 71

The children should discover the pattern for part 2 of the *Think* by performing the crossings for the triangle and then trying the four-, five-, and six-sided figures as suggested in part 1 of the *Think*.

Starting inside a simple, closed, plane curve, an even number of crossings would leave them inside when they finish, while an odd number of crossings would leave them outside.

Workbook, page 20

Using the Exercises

Assign the exercises on page 71 as independent work, although you might choose to use exercise 1 as a basis for discussion. A sample story for exercise 4 might be something like the following.

"Long ago a primitive hunter caught a large fish. It was so large that he wanted to be able to tell people about it after he had eaten it. He found a stick and cut notches in it to show the length of his fish, as shown in the picture. Then he ate the fish.

"Later this hunter's friend caught a fish and boasted that his fish was just as long as the hunter's. But the hunter could prove him wrong or right. The hunter just had to compare this friend's fish to the distance between the notches in the stick."

Assignments (page 71)

Minimum: 1–2. Average: 1–3. Maximum: 1–4.

PAGES 72–73

Objective

Given a segment to measure, the child will be able to select a unit and use it to measure the segment.

Preparation

Materials

piece of string about 3 metres long (1 per child)

To prepare for this lesson, tell the children something about Goliath. The famous story of David and Goliath may be found in the Bible, in I Samuel 17. If children are not familiar with the story, tell them about David's feat of vanquishing the giant warrior with a rock hurled from his sling.

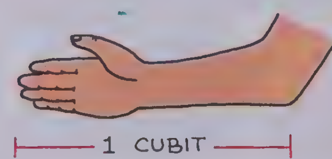
Investigation

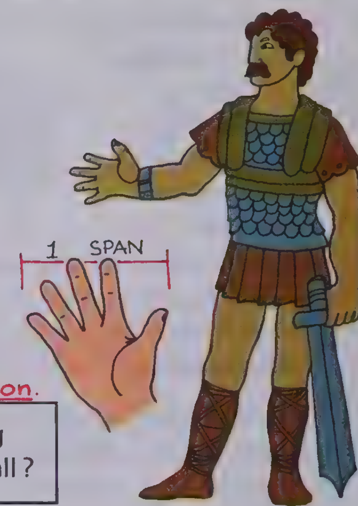
If you give every four children a roll of string at least 12 metres long, they will be able to help each other measure and cut out pieces of string which approximate Goliath's height. Goliath's height is considered to have been almost 3 metres, but since the children's cubit unit will be shorter than that of an adult, few will come very close to a measurement of 3 metres. Note that there is no mention of measuring the string in terms of metres, but you may want to have some children do so. However, each will be asked to compare his length of string with the pieces cut by his classmates.

How do you find the length of a segment?

Investigating the Ideas

Goliath, the Philistine giant, was reported to have had a measured height of 6 cubits and 1 span.



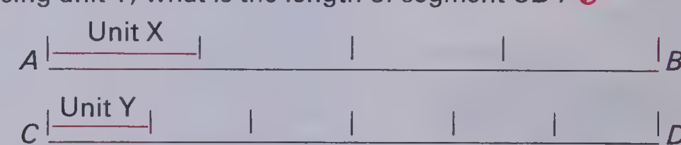


See Investigation.

? Can you cut a segment of string that is as long as Goliath was tall?

Discussing the Ideas

1. Is each classmate's string the same length? Why?
   No, because the sizes of classmates' arms and hands may vary.
2. Why is it difficult to find out how tall Goliath was?
   Because cubit and span lengths are variable.
3. If an adult completed the Investigation, would his string probably be longer or shorter than yours? Why? See Investigation and Discussion.
   Longer
4. We find the length of a segment or object by counting the number of unit segments it takes to "fill" the segment.
   A Using unit X, what is the length of segment AB? 4
   B Using unit Y, what is the length of segment CD? 6



   C The segments AB and CD are congruent. Explain why you got a larger number for the length of $\overline{CD}$ than for the length of $\overline{AB}$.
   Sample answer: A smaller unit of measure gives a larger number for the length of the object measured.

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Discussion

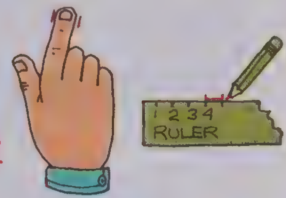
To discuss exercise 1, children must compare the lengths of string they cut in the investigation. Point out that the unit used to measure the string was the cubit (and the span) but that this unit depends upon the length of a person's forearm. Thus, as exercise 3 makes clear, the cubit used by a child may not be the same cubit used by an adult. Extend this idea to bring out the idea that the selection of a unit is arbitrary.

Exercise 4 develops one of the main ideas of the lesson, namely, that the "length" of an object depends on the unit used to measure the object. Help the children think about putting unit X on $\overline{AB}$ four times in order to find the length for $\overline{AB}$ (4 of the X units). Similarly, unit Y can be put down on $\overline{CD}$ six times, so its length is 6 of the Y units.

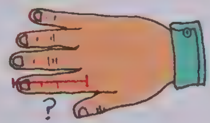
As you discuss exercise 4C, remind the children that two segments are congruent if their ends are equally far apart. Thus, the two segments are congruent even though their "lengths" differ. Of course, if both lengths were given in the same unit, they would be equal.

Using the Ideas

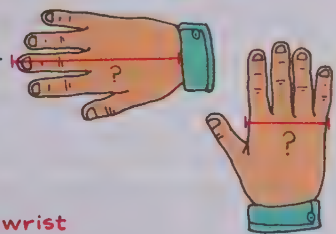
1. Mark a segment that is the width of your index finger. Using this segment as a unit, make a ruler along the edge of a sheet of paper that is at least 16 units long. Use this ruler to complete exercises 1A through 1E. Measure to the nearest unit. Answers will vary.



   A Find the length of your index finger.
   B Find the length and width of your mathematics book.
   C Find the length of your desk.
   D Find the length of your pencil.



   E Measure, on your hand, the distance shown in the picture.
   F Mark half units on the ruler you made. Measure to the nearest half unit, on your hand, the distances shown in the picture. Which distance is greater?



   Distance between fingertip and wrist

2. You can choose any unit you wish for measuring. You have used **centimetres** and **metres.**
   The red mark shows the length of a centimetre unit.

   centimetre (cm)

   Answers will vary.
   A Is the centimetre a longer or shorter unit than the one you used in exercise 1?
   B Without measuring, tell whether the length of your mathematics book, using the centimetre unit, would be more or less than the length you found in exercise 1B.
   C Use a centimetre ruler to check your answer to exercise 2B.
   D Give the length of your shoe to the nearest centimetre.

Using the Exercises
As the children work on exercise 1 on page 73, some may need help in constructing their rulers. Children with narrow fingers may have a very small unit and correspondingly greater numbers for the lengths, whereas other children may have a considerably larger unit and lesser numbers for the lengths. Point out to the children that, since their units vary in size, they cannot compare the numbers to tell whether or not the objects they are measuring are the same length.

When you check exercise 2, point out that the longer the unit, the lesser the number for the length of a given object, and the smaller the unit, the greater the number for the length.

Assignments (page 73)
Minimum: 1 A-D, 2. Average: 1–2. Maximum: 1–2.

Follow-up
Suggest that children use string and cubit units to measure their own height and that of some of their classmates. They might chart their heights and show the unit they used.

Also, suggest that children do research on the history of measurement, uncommon units, origin of the standard units, and any other related topics.

As another worthwhile follow-up activity, you might provide a list of objects and divide the class into two groups. Then ask one group to measure all the objects in red strips, and the other to measure the same objects in green strips. Have the children record the data and compare the measurements. This activity will help children reach the generalization that longer units give smaller numbers for the length of an object.

Workbook, page 21

PAGES 74–75

Objectives

The child will be able to identify parallel and intersecting lines.

The child will be able to name angles by using standard angle notation.

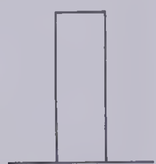
Preparation

Materials

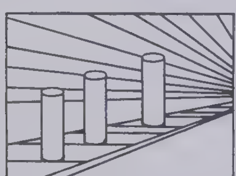
tracing paper

To interest children in this lesson, exhibit some optical illusions.

Which line is longer? (They both have the same length.)



Is the hat taller than it is wide? (No, height and width are equal.)



Are the cylinders the same size?

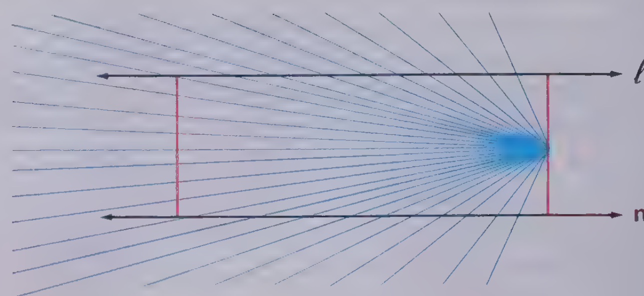
Investigation

Before the children begin the investigation, review the meaning of the term *intersect*. Make sure children realize that they should imagine the lines to be extended indefinitely before they judge whether or not the lines will meet.

Stress the importance of tracing the lines accurately. If the tracing is not done carefully, the children's investigation findings will be irrelevant to the lines in the text. After children trace the lines on the paper, encourage them to fold, cut, overlap their paper, to do whatever they think will help them find out if the lines are parallel. Some may discover that if they fold the paper so that the fold is perpendicular to the lines, the pair of lines will match up evenly. If children do not discover or consider the method they use to be valid, suggest that they test their method by using it to investigate two lines which very obviously are not parallel.

● *Let's explore parallel and intersecting lines.*

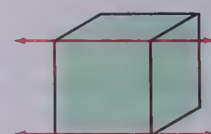
Investigating the Ideas



Do lines ℓ and **m** intersect? (Would the black lines meet on one side if extended?) No

? Can you figure out a way to use tracing paper to help you decide if the lines would meet? See Investigation.

Discussing the Ideas



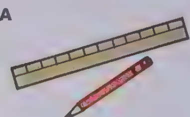
1. Two lines that are in the same plane and do not intersect are parallel to each other.

A Ann said railroad tracks remind her of parallel lines. What objects remind you of parallel lines? Sample answers: Sets of power transmission lines, double stripes on highways

B Explain what Jeff meant when he said, "Parallel lines are everywhere the same distance apart." See Discussion.

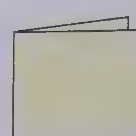
2. Can you draw parallel lines by using

A



a ruler and a pencil?

B

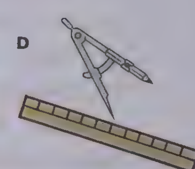


paper folding?

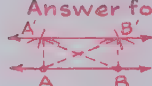
C

a line and a book?

D



a compass and a ruler?

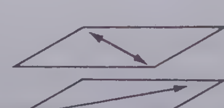


Answer for D: Use one compass length to draw arcs from points A and B. Then repeat procedure with another compass length to intersect arcs at A′ and B′. $\overleftrightarrow{A'B'} \parallel \overleftrightarrow{AB}$

Discussion

As you work through the discussion exercises, highlight the contrast between parallel lines and intersecting lines. You may choose to incorporate parts of the exercises on page 75 into this discussion.

In exercise 1, point out the phrase "in the same plane." Give examples of lines not in the same plane which do not intersect and help the children see that such lines are *not* parallel.



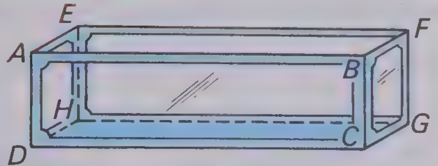
As children suggest objects which remind them of parallel lines, help them recall the qualification that the lines must be in the same plane.

Note that in exercise 1B Jeff is talking about the *perpendicular distance*. For example, the measurement must be made along a straight line from a point on one line to the point directly below it on the other line.

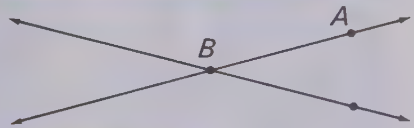
$\overline{AB} \parallel \overline{EF}$, $\overline{AB} \parallel \overline{HG}$, $\overline{AB} \parallel \overline{DC}$, $\overline{EF} \parallel \overline{HG}$, $\overline{EF} \parallel \overline{DC}$, $\overline{HG} \parallel \overline{DC}$,
$\overline{AE} \parallel \overline{BF}$, $\overline{AE} \parallel \overline{CG}$, $\overline{AE} \parallel \overline{DH}$, $\overline{BF} \parallel \overline{CG}$, $\overline{BF} \parallel \overline{DH}$, $\overline{CG} \parallel \overline{DH}$,
$\overline{AD} \parallel \overline{EH}$, $\overline{AD} \parallel \overline{FG}$, $\overline{AD} \parallel \overline{BC}$, $\overline{EH} \parallel \overline{FG}$, $\overline{EH} \parallel \overline{BC}$, $\overline{FG} \parallel \overline{BC}$

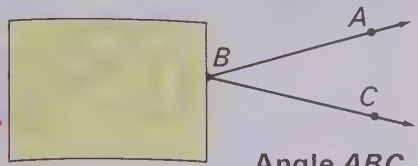
Using the Ideas

1. Name as many pairs of parallel edges as you can find in this picture of an aquarium.
   See answers above.



2. If we start with two intersecting lines and cover up "half" of the picture, an **angle** is formed by the two rays from the point.

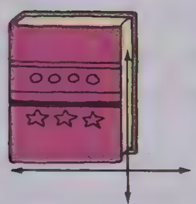




Angle ABC

   A Draw 5 different-sized angles. Constructions will vary.
   B Label one of your angles so that it could be called angle RST ($\angle RST$).
   C Label and name the other angles.

3. If two lines intersect like this,

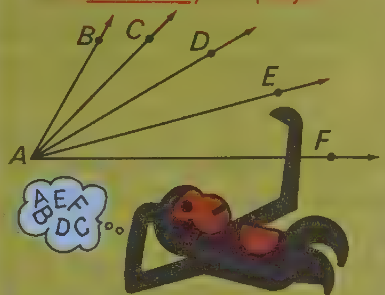


   they are said to be perpendicular to each other. Draw a pair of perpendicular lines.

★ 4. Find out what a parallelogram is and draw a picture of one.
   See Using the Exercises.

think

There are 10 different angles in the figure below. See how many of them you can name.
See Solution, T.E. page 75.



75

Mathematics

Two lines in the same plane that do not intersect are *parallel lines*.

Two nonintersecting lines which are not in the same plane are *skew lines*. (Any two intersecting lines, of necessity, are coplanar; that is, they are in the same plane.)

Two lines that intersect at right angles are *perpendicular lines*.

A *parallelogram* is a quadrilateral having two pairs of parallel sides.

An *angle* is the union of two rays from a single point.

Solution, Think, page 75

The 10 different angles which may be found:

$\angle BAC$	$\angle CAE$
$\angle BAD$	$\angle CAF$
$\angle BAE$	$\angle DAE$
$\angle BAF$	$\angle DAF$
$\angle CAD$	$\angle EAF$

Resources for Active Learning

Franklin Series: *Pencil and Paper Geometry,* "Intersecting Lines," pp. 44–47, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Inquiry in Mathematics via the Geoboard, "Parallel Line Segments," Geo-Cards 27/1-6, Walker. (Available from Fitzhenry and Whiteside)

Nuffield Project: *Shape and Size* 3, " . . . parallels and angles," pp. 9–10, Wiley.

Workbook, page 22

Using the Exercises

You may choose to discuss exercise 2 before assigning the exercises on page 75. Use the diagram in the text to explain the naming of angles. Note that the letter at the vertex is always written between the other two letters which are used to name the angle. Also, note that the angle consists of the two rays and their common point. It does not consist of the "space" that is the *interior* of the angle.

As children try starred exercise 4, some may recall that a parallelogram is a four-sided figure with opposite sides parallel. Others may refer to the glossary in the text.

Help the children organize their answers to the *Think* problem so that they can be sure to find 10 different angles. Angle *BAC* is the same angle as angle *CAB*. It may be necessary to reemphasize that when three points are used to describe an angle, the common endpoint of the rays (the vertex) is listed as the second point, or point "in the middle."

Assignments (page 75)

Minimum: 1, oral; 2–3.
Average: 1–3. Maximum 1–4.

PAGES 76–77

Objective

Given two angles, the child will be able to determine whether or not they are congruent.

Preparation

Materials

tracing paper; compass (1 per child)

To prepare for this lesson, you might remind the children that two *segments* are *congruent* if their endpoints are equally far apart. Explain that in this lesson they will investigate ways to determine whether two *angles* are *congruent*.

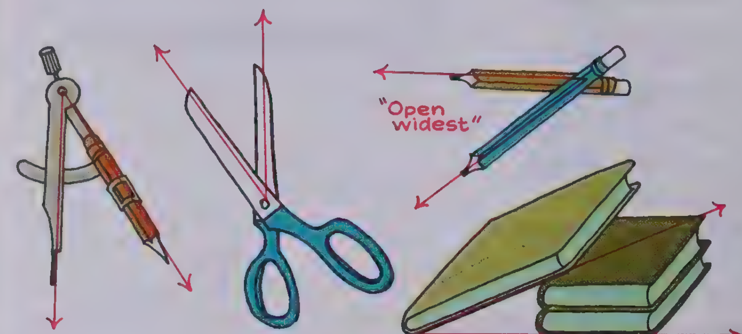
Investigation

Have tracing paper available, but do not distribute it until children decide they need it, or until it is necessary to suggest that it may help them. By tracing one angle, they can place it on top of another to compare the two. Some children may be familiar with use of a protractor; others may be able to use a compass to mark same-size arcs on each angle and measure the distance between these marks. You might find it necessary to remind some children that they are to compare how wide the angles "open," not how long the rays are.

● *When are two angles congruent?*

Investigating the Ideas

Each of these objects could remind you of an angle.



? Can you find a way to decide which "angle" is "open widest"? See Investigation.

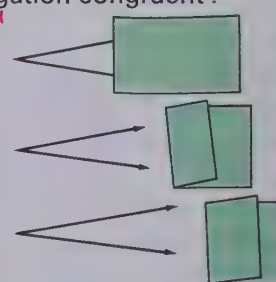
Discussing the Ideas

1. Explain how you decided which angle in the Investigation was "open widest" (largest) and which was smallest.
   Answers will vary.
2. Two angles are **congruent** if neither is larger than the other. Are any of the angles in the Investigation congruent?
   Yes, the angle of the compass and the angle of the scissors
3. Think about the angle in A. We might uncover the drawing and find B, or we might find C. Is the angle in C larger than the angle in B? Why or why not? Explain how you think we compare angles.
   All are congruent. See Discussion.

A

B

C



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Discussion

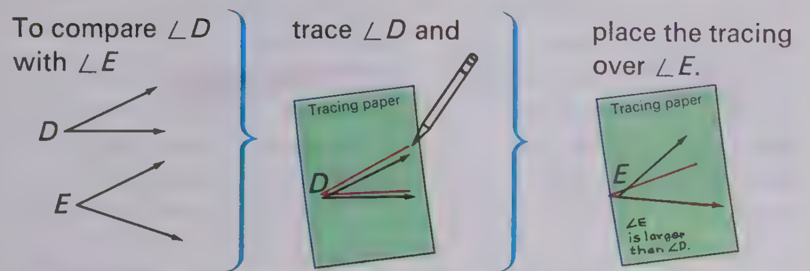
When dealing with segments, the children's intuition is sufficient to help them determine whether or not a given segment is longer than another. However, their intuition may not serve them quite so well in determining whether or not one angle is larger than another.

It is important for children to realize that the length of the rays shown in pictures of the given angle does not determine the size of the angle. Remind them that they are to imagine the rays going on and on endlessly. In exercise 3, we first show an angle with the ends of the rays covered, and then show that the rays might be of one length or another, but the angle would be the same. Emphasize that all three illustrations represent the same angle, and that the length of the rays shown does not determine the size of the angle.

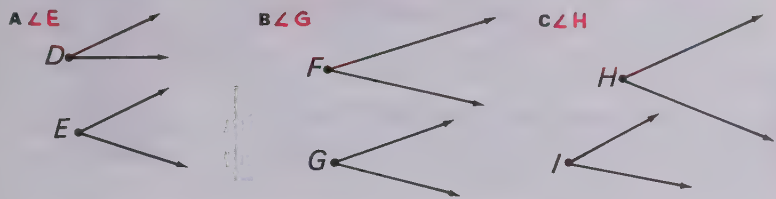
As children explain how we compare angles they might refer to the "opening of an angle" or to the "space between the rays." In the following lesson, they will be introduced to the *measure* of an angle.

Using the Ideas

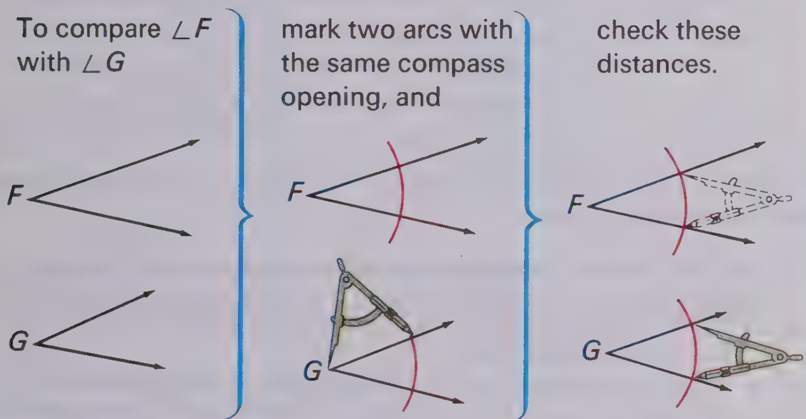
1. Study this method of comparing two angles.



Use the method above to tell which angle is larger.

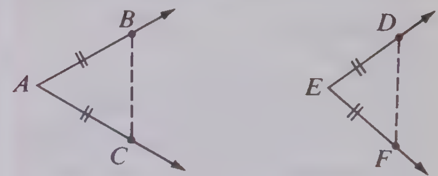


2. Study this method of comparing two angles.



Draw two angles on your paper. Use your compass to decide whether or not the two angles are congruent.
Constructions will vary.

Mathematics
The method of angle comparison suggested in exercise 2 is based upon comparison of segments and is nonmetric in nature.



To compare $\angle A$ and $\angle E$, we choose points B, C, D, and F on the sides of the two angles so that

$$\overline{AB} \cong \overline{AC} \cong \overline{ED} \cong \overline{EF}$$

Then we compare the "openings" of the angles ($\overline{BC}$ and $\overline{DF}$).

If $\overline{BC} \cong \overline{DF}$, then $\angle A \cong \angle E$.
If $\overline{BC} > \overline{DF}$, then $\angle A > \angle E$.
If $\overline{BC} < \overline{DF}$, then $\angle A < \angle E$.

Resources for Active Learning
[Other uses for the compass may be found in the resources listed here.]

Experiments in Mathematics, Stage 1, pp. 32–33, Houghton Mifflin. (Available from Thomas Nelson and Sons)

Franklin Series: *Patterns and Puzzles*, "Line Design," pp. 36-41, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Maths Mini-lab, Card 117; 131–133, Selective Educational Equipment.

Workbook, page 23

Using the Exercises
This exercise set presents two methods by which angles may be compared. The first is one most may have used in the investigation, the use of tracing paper. Stress the importance of tracing the first angle very carefully in order to compare it accurately with the second angle.

In the method suggested in exercise 2, make sure children compare distances between the two rays, not the distances from the vertex to the two points on the rays. They might also use a ruler to measure this distance between the two points of intersection on the arc and the angle.

Assignments (page 77)
Minimum: 1. Average: 1–2.
Maximum: 1–2.

PAGES 78–79

Objective

Given an angle and a unit angle, the child will be able to measure the angle in terms of the unit angle.

Preparation

Materials

paper for tracing and cutting

The nature of this lesson makes it appropriate to begin immediately with the investigation.

Investigation

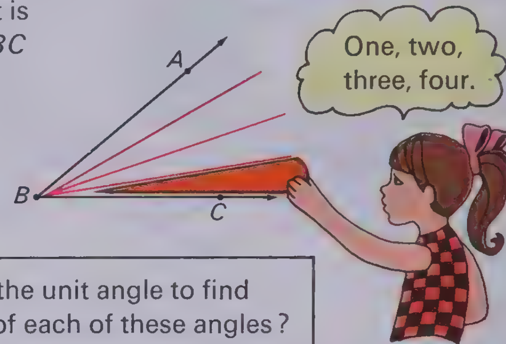
The accuracy of the children's tracing will affect the results of their investigation. Some may choose to trace the angles they are to measure as well as the unit angle. Being able to mark the unit positions with pencil in the interior of each angle would facilitate accurate measuring. You might suggest that those who finish quickly draw an angle which measures 3 of Ann's units, or 5, or some other multiple of it.

● *How are angles measured?*

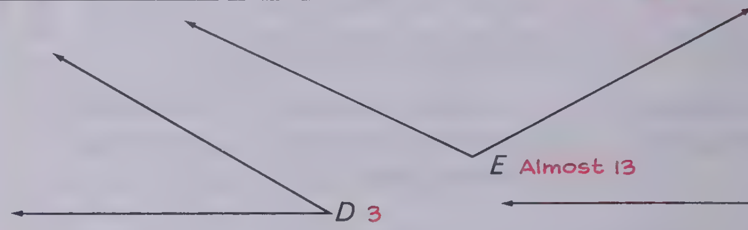
Investigating the Ideas

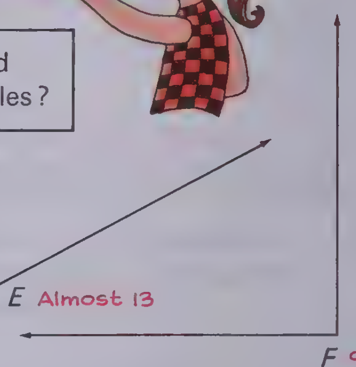
Ann measured $\angle ABC$ using the **unit angle** shown. The red marks show where Ann placed the unit as she measured. What is the measure of $\angle ABC$ using Ann's unit? 4

Trace this unit angle and cut it out.



? Can you use the unit angle to find the measure of each of these angles?





Discussing the Ideas

1. Explain how you measured the angles above. Sample answer: Count the number of times the unit angle can be placed within the angle being measured.
2. A Could you use unit angles of other sizes to measure the angles above? Yes

   B How would the measures of the angles above differ if you used a smaller unit? a larger unit? Measure would be a larger number. Measure would be a smaller number.
3. How is measuring an angle like measuring a segment? To measure each one, you count a chosen unit.

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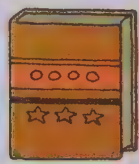
Discussion

As various children explain how they measured the angles in the investigation, point out the fact that all of them *counted the unit* to find the measure. If they were able to place the unit 3 times within an angle, we say the measure of that angle is 3 units. In exercise 2B, be sure to let the children test their answers. For example, they may make an angle unit larger than Ann's and see if the measures of the angles become larger or smaller. As you discuss exercise 3, remind the children that, just as the measure of a segment is a number, the measure of an angle is a number. Also, in both, a unit is chosen and the unit is counted to give the measure.

Using the Ideas

1. Use the unit you cut out on page 78 and draw an angle with measure: A 4 B 7 C 2 D 15 E 5 F 10 G 18



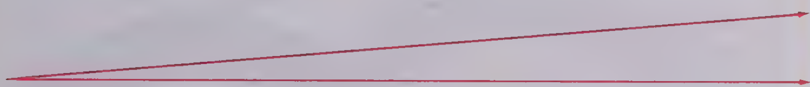
2. The corner of your book suggests a **right angle**. Draw a right angle on your paper and measure the angle. Use the unit you cut out on page 78.
Angle measure will be 9.

3. An **acute angle** is an angle that is less than a right angle. An **obtuse angle** is an angle that is greater than a right angle. Which of the angles you drew for exercise 1 are acute? obtuse?
A,B,C,E D,F,G

4. Here is an angle unit that was chosen by the Sumerians over 4000 years ago. It is one of the units used most often today.

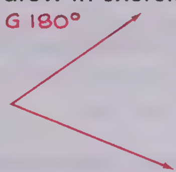
This angle unit is called a degree. The measure of a right angle, using the degree, is 90. We say the measure of the right angle is 90 degrees. We write **90°** for **90 degrees**.

A Trace the angle below. Give its degree measure by placing it over the unit above. 5°



★ B If the degree measure of the unit used in exercise 1 is 10, give the degree measure of each angle you drew in exercise 1.
A 40° B 70° C 20° D 150° E 50° F 100° G 180°

★ **5.** Here is one special angle unit often used. It is called a radian On tracing paper, draw an angle with radian measure 3.



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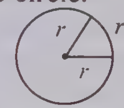
Using the Exercises
Before assigning the exercises on page 79, you might discuss the terms *acute angle, obtuse angle, radian,* and *degree*. However, the topic of degree measure will be treated further in subsequent lessons. The most important point in this lesson is for the children to gain a feeling for the idea of angle measure in general. Note that with the introduction of degree measure, a right angle may be defined as an angle that has degree measure 90. Explain the use of the degree symbol (°), as in 90°.

Assignments (page 79)
Minimum: 1–4A. Average: 1–4.
Maximum: 1–5.

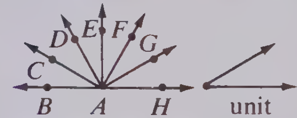
Mathematics
The special unit of angle measure (radian) given in exercise 5 is a useful unit in the study of trigonometry and advanced mathematics. By definition, a radian is an angle which, if its vertex is at the centre of a circle, subtends (cuts off) an arc that is equal in length to the radius of the circle.



Since the circumference of a circle is 2π times the radius of the circle, the length of a semicircle is πr; hence, an angle that measures 180° also has a measure of π radians ($\pi = 3.1416$). Thus, one radian is approximately 57.3°, and one degree is approximately 0.017 radian.

Follow-up
To add to the children's understanding of measuring angles with unit angles, give them worksheets similar to the following:

Each small angle in the figure below is congruent to the unit angle. Find the measure (m) of each angle.



$m \angle FAH = \underline{(2)}$ $m \angle GAC = $ ____
$m \angle DAH = \underline{(4)}$ $m \angle BAH$ ____
$m \angle BAE = $ ____ $m \angle EAF = $ ____

Using only the points labelled in the sketch above, give the symbols for each angle. Answers will vary.

1. An angle with measure 4. (CAG)
2. An angle with measure 2. ____
3. An angle with measure 5. ____
4. Two angles with measure 5. ____
5. Two angles with measure 6. ____
6. Three angles with measure 2. ____

Resources for Active Learning
Applied Mathematics Cards, Group 2/19, 22, Schofield and Sims. (Available from Mafex Associates, Willowdale)
Developmental Math Cards, J²2, Addison-Wesley.
Franklin Series: *Pencil and Paper Geometry,* "Triangles," pp. 81–84, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

PAGES 80–81

Objective

Given an angle unit and suitable materials, the child will be able to make and use a protractor.

Preparation

Materials

scissors; tracing paper

To prepare for the investigation, use the text illustration of the ruler and the unit to review the concept of measurement as the counting of units.

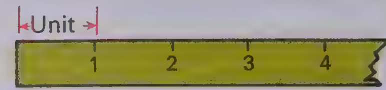
Investigation

Have tracing paper available so that, if necessary, children may trace and cut out the unit angle again. You might find it necessary to help children use the compass in the proper manner.

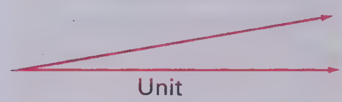
For example, suggest that they draw on tracing paper a circle with a radius of $4\frac{1}{2}$ centimetres. They should then fold and cut out exactly half the circle and trace the angle units side by side. Point out the numerals on the protractor so that children will number theirs *beginning with zero* (0) on the edge. The measures they find for the angles on page 78 should not differ from those found in the investigation for that page. However, children might find that the protractor is easier and more accurate than trying to place and count a single angle unit.

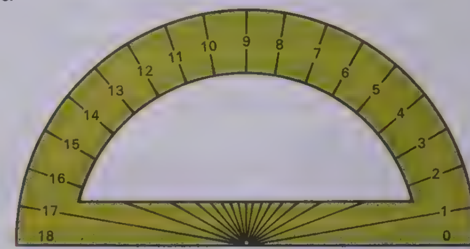
● *Let's make and use a protractor.*

Investigating the Ideas



A **ruler** has several **segment units** placed end-to-end to make it easier for you to find length. A **protractor** has several **angle units** placed side-by-side to make it easier for you to measure angles. The numerals on the ruler and protractor make it easier for you to count units. Use the "unit" from the last lesson and a compass to make a protractor.



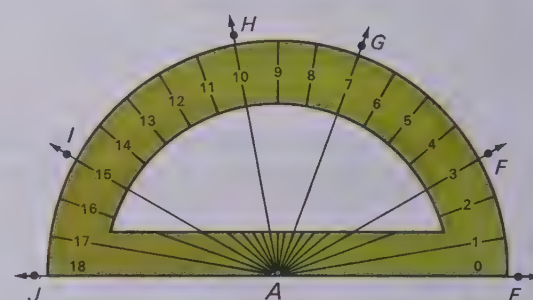


? Can you use your protractor to measure the angles in the Investigation on page 78? See Investigation.

Discussing the Ideas

This picture of a protractor is placed over some rays from point A. Compare this protractor with the one you made.

1. What is the measure of $\angle FAE$? 3
2. Explain how to tell that $\angle GAF$ is 4 units. See Discussion.
3. What is the measure of $\angle IAH$? 5



4. Explain how to find the measure of $\angle IAF$ quickly. See Discussion.

80

Discussion

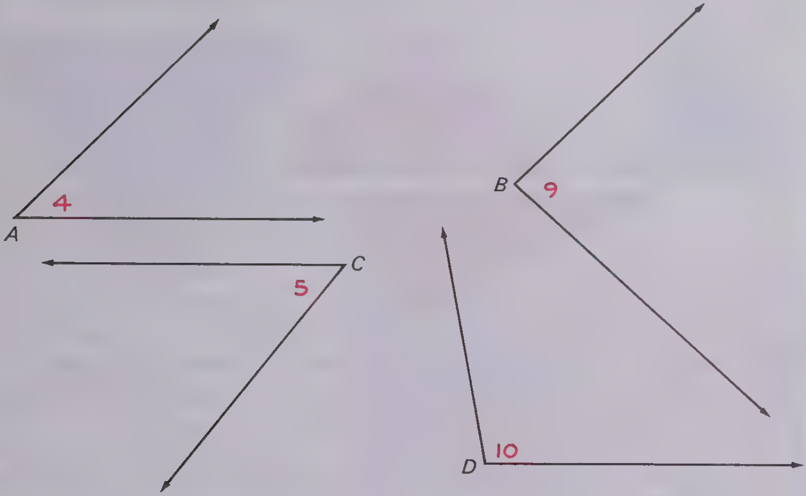
As you discuss and demonstrate the use of a protractor, point out that the vertex of the angle is placed at the centre point and that the "bottom" ray of an angle should run along the edge of the protractor from the ray from the centre point to zero. As you discuss exercises 2, 3, and 4, point out the convenience of subtracting. For example, to prove that the measure of $\angle IAF$ may be found by subtracting 3 (the unit along AF) from 15 (the unit along IA), show the children how to turn the protractor slightly so that the ray AF lines up with its edge, and then measure $\angle IAF$ by counting the number of units from zero. Both measures should be the same—in this case, 12.

It would be helpful to demonstrate measuring angles in degrees with a chalkboard protractor or on the overhead projector. Help children realize that the protractor they made in this lesson is different from an ordinary protractor of degree measure only in the size of the angle unit.

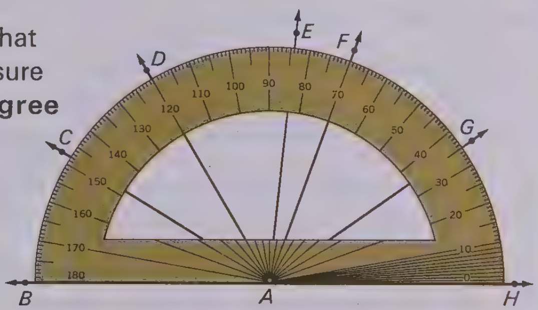
Using the Ideas

1. Use the figure in the Discussion to give the measure of the following angles.
   A $\angle GAE$ 7 B $\angle EAH$ 10 C $\angle EAI$ 15 D $\angle JAI$ 3 E $\angle JAG$ 11 F $\angle FAJ$ 15 G $\angle GAH$ 3 H $\angle IAG$ 8

2. Use the protractor you made to measure these angles to the nearest unit.



3. This is a protractor that can be used to measure angles when the **degree** is the unit. It is placed over some rays from point A. Give the degree measure of these angles.



   A HAF 70° B DAH 120° C HAG 35° D HAE 84° E HAC 148° F BAC 32°

4. Use a degree protractor to measure the angles in exercise 2.
   A 44° or 45° B 85° or 86° C 51° or 52° D 100°

Using the Exercises

Note that exercise 1 on page 81 requires that children refer to the figure at the bottom of page 80.

One of the important goals of this lesson is for children to be able to use an ordinary protractor calibrated in degree measure. Thus, you may choose to discuss and check exercises 3 and 4 rather carefully. If children have commercial protractors which number the units from left to right as well as from right to left, you might explain that they need only concern themselves with the scale which is comparable to the protractor illustrated on page 81. Children who are sufficiently interested might do personal research to learn how to use the left-to-right numerals.

Assignments (page 81)
Minimum: 1–3. Average: 1–3.
Maximum: 1–4.

Follow-up

As a summary, direct the children to measure the unit they used to make their protractors. Then have them compare their protractors with a degree protractor. They should discover that each unit on their protractor has the measure 10° and that their unit angle also has the measure 10°.

More capable and interested children might again measure the angles in exercise 2 with a protractor. They can see how well they measured the first time by comparing the degree measures with their unit measures multiplied by ten.

Resources for Active Learning

Developmental Math Cards, K–3, Addison-Wesley.

Mathex: Geometry No. 9, "Introducing the Use of Instruments," pp. 29–30, Encyclopaedia Britannica Publications Ltd.

Measure and Find Out, Book 2, "Measuring Circles and Angles," Activity 1/4, Scott Foresman. (Available from Gage Educational)

Duplicator Masters, page 13
Workbook, page 24

PAGES 82–83

Objective

Given a triangle, the child will be able to identify it as isosceles, equilateral, scalene, or right.

Preparation

Materials

tracing paper; scissors

To prepare for this lesson, review the concept of symmetry. You might simply use the explanation at the top of page 82 for this purpose. It would also be helpful to exhibit a few symmetrical figures, other than triangles, and show these figures folded along a line of symmetry.

Investigation

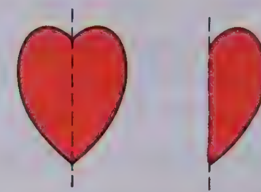
Stress with the children the importance of tracing and cutting the triangles accurately. Also, remind them to label their triangles according to the labels in the book and to record their conclusions. Since triangle D is an equilateral triangle, it has 3 different lines of symmetry.

You might suggest that those who finish quickly try to draw a triangle which has only one line of symmetry, or 3 lines of symmetry, or no lines of symmetry.

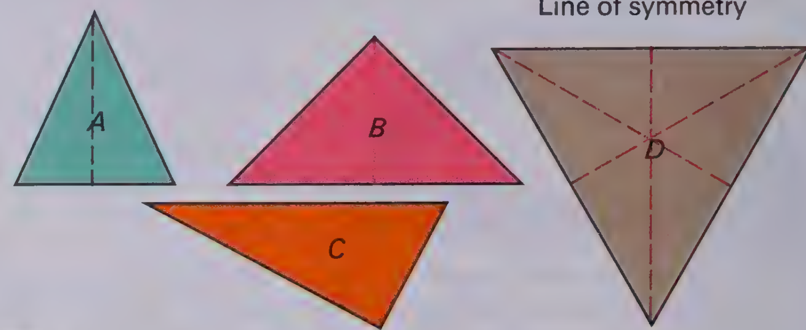
Let's explore triangles and symmetry.

Investigating the Ideas

A figure has a **line of symmetry** if a fold on that line makes two halves of the figure that match exactly.



Trace and cut out these triangular shapes.



? Which of the shapes has 3 different lines of symmetry? only one line of symmetry? no lines of symmetry?	Show the lines by folding the figures.

See above.

Discussing the Ideas

1. An **isosceles triangle** has 1 line of symmetry. Choose an isosceles triangle above and show, by folding, that it has 2 congruent sides and 2 congruent angles. See triangles A and B above. (Triangle D is also a possibility, since equilateral triangles are also isosceles triangles.)
2. An **equilateral triangle** has 3 lines of symmetry. Show, by folding, that it has 3 congruent sides and 3 congruent angles. See triangle D above.
3. A **scalene triangle** has no lines of symmetry and no congruent sides. Which triangle above is scalene? C
4. Which two triangles above do you think are **right triangles**? Why? B and C; each has a right angle.

82

Discussion

The chief purpose of this lesson is to enable children to identify different kinds of triangles. Help them with the pronunciation and meaning of the terms.

Isosceles triangle—Two sides are congruent; two angles are congruent.

Scalene triangle—No sides are congruent; no angles are congruent.

Equilateral triangle—All three sides are congruent; all three angles are congruent.

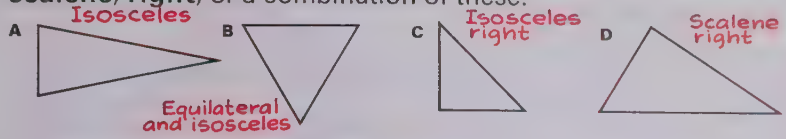
Right triangle—One angle of the triangle is a right angle, that is, its measure is 90°.

Isosceles right triangle—A right triangle which has two congruent sides

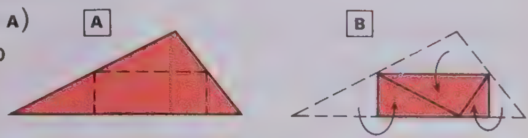
To relate the concept of symmetry to the different kinds of triangles, point out that, when a triangle has a line of symmetry, two sides on either side of the line of symmetry must be congruent. Thus, because the equilateral triangle has 3 lines of symmetry, it has 3 pairs of congruent sides. A triangle which has only one line of symmetry must have only one pair of congruent sides; hence, it is an isosceles triangle.

Using the Ideas

1. Describe each triangle by writing **isosceles**, **equilateral**, **scalene**, **right**, or a combination of these.

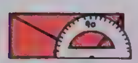


2. A A student drew fold lines on a triangle (picture A) and then folded to form a rectangle (picture B).



The "angle flaps" fit together exactly with no overlap. Can each of the four triangles you cut out in the Investigation be folded to exactly form a rectangle? Yes

B Another student placed a protractor like this:



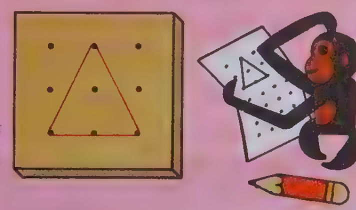
What does this suggest about the sum of the angles of a triangle?
The sum is 180°.

★ 3. Make an equilateral triangle by using
   A a paper clip, 2 pencils, and a ruler. 
   B a compass and a ruler.
   C paper (for folding) and pencil.

See Solution, T.E. page 83.

think

Suppose you have a 3-nail by 3-nail geoboard.



Can you show on dot paper 4 differently shaped isosceles triangles that can be formed with rubber bands on the board?

See Solution, T.E. page 83.

Using the Exercises

Assign the exercises on page 83 as independent or small-group work. Encourage children to treat exercises 2 and 3 as investigative exercises. When they have finished, check their work and allow time for discussion.

In exercise 1, do not expect children to give every possible identification of each triangle. As you discuss exercise 2A, have the children note not only that the isosceles triangle can be folded into a rectangle but that this rectangle is a square. In exercise 2B, make sure the children realize that the protractor is being placed on top of the three angles of a triangle that has been folded into a rectangle, as in exercise 2A. Stress the conclusion that the sum of the angles of a triangle is 180°.

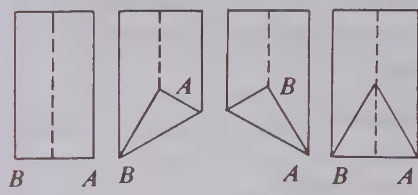
For starred exercise 3A, the children might use the paper clip as a radius and find a point on the arc that is the length of the paper clip away from both points.

Assignments (page 83)

Minimum: 1–2. Average: 1–2.
Maximum: 1–3.

Solution, exercise 3C, page 83

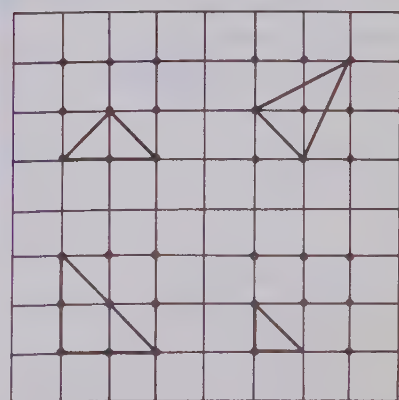
The figure below suggests how an equilateral triangle may be formed by folding paper.



Fold a sheet of paper in half. Segment BA is the base of the triangle. Fold corner A to the middle fold and draw segment AB. Fold corner B to the middle fold and draw segment BA. Since the base, $\overline{BA}$, and the two sides that were drawn all have the same length, the triangle is equilateral.

Solution, Think, page 83

Besides the triangle illustrated in the problem itself, the other isosceles triangles of different shapes that can be formed are as shown below. Note, however, that, although the lower two differ in size, they are the same shape; hence, only one of the two should be included among the child's drawings of "4 differently shaped isosceles triangles."



Resources for Active Learning

Experiments in Mathematics, Stage 1, "Triangles and Quadrilaterals," pp. 20–21, Houghton Mifflin. (Available from Thomas Nelson and Sons)

Math Activity Cards, D16, Macmillan.

Mathex: Geometry No. 9, "Sides of Triangles," pp. 27–28 (pupil pages 35–37), Encyclopaedia Britannica Publications Ltd.

PAGES 84–85

Objective

Given two triangles, the child will be able to determine whether or not they are congruent by tracing one and comparing it to the other.

Preparation

Materials

tracing paper

To prepare for this lesson, review symbols for line segments, ($\overline{AB}$), and for angles ($\angle CAB$). Make sure the children realize that the label of the vertex of an angle is written between the labels of points on the rays. Thus, $\angle CAB$, has its vertex at point A, whereas $\angle ABC$ has its vertex at point B.

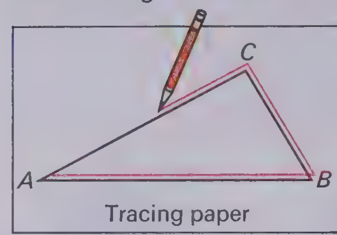
Investigation

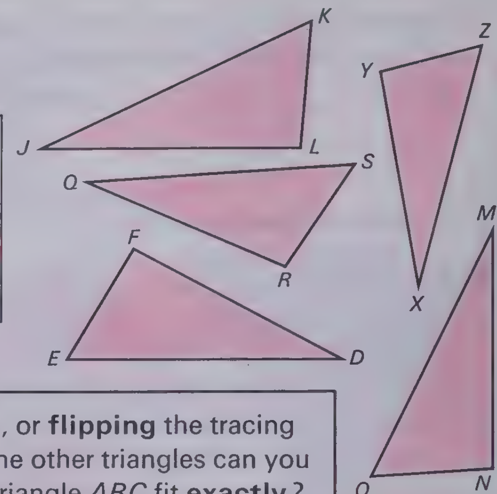
Notice that there is no need for children to cut out triangle ABC in order to conduct this investigation. They need only use the traced figure, turning it over as well as around. However, remind them to label their triangle as in the text. This will facilitate the discussion of the parts of a triangle. Of course, children should record their results as usual. You might demonstrate how to abbreviate an expression such as "triangle ABC" by writing $\triangle ABC$ on the chalkboard.

● *When are two triangles congruent?*

Investigating the Ideas

Trace triangle *ABC*.





? By **sliding**, **turning**, or **flipping** the tracing paper, on which of the other triangles can you make the tracing of triangle *ABC* fit **exactly**?

DEF, MNO

Discussing the Ideas

In the Investigation you found triangles that are **congruent** to triangle *ABC* ($\triangle ABC$). Explain this definition. Then use triangles *ABC* and *DEF* above and give the missing segment or angle for each ▥.

Two triangles are congruent if the parts (angles and segments) of one can be matched with the parts of the other so that the pairs of angles and segments are congruent.

This part of $\triangle ABC$ ↓	is congruent to ↓	this part of $\triangle DEF$. ↓	This part of $\triangle ABC$ ↓	is congruent to ↓	this part of $\triangle DEF$. ↓
$\overline{AB}$	$\cong$	$\overline{DE}$ ▥	$\angle CAB$	$\cong$	$\angle FDE$ ▥
$\overline{AC}$	$\cong$	$\overline{DF}$ ▥	$\angle CBA$	$\cong$	$\angle FED$ ▥
$\overline{BC}$	$\cong$	$\overline{EF}$ ▥	$\angle ACB$	$\cong$	$\angle EFD$ ▥

84

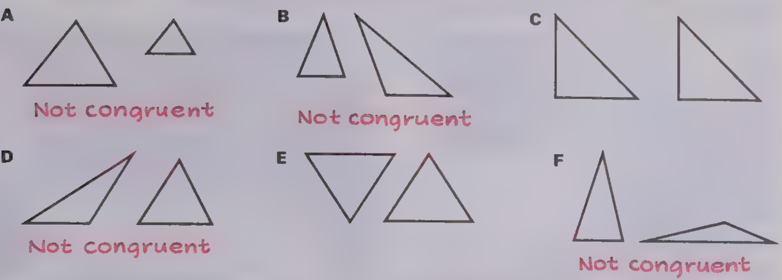
Discussion

To begin this discussion, point out the various parts of $\triangle ABC$. For example, you might say: "One of the angles of this triangle is $\angle CAB$. Name the other two." ($\angle CBA$ or $\angle ABC$, and $\angle ACB$ or $\angle BCA$) Also, you might say: "One of the segments of this triangle is $\overline{AB}$. Name the other two." ($\overline{AC}$ and $\overline{CB}$) You might repeat these questions and have children also identify parts of another triangle, such as $\triangle DEF$.

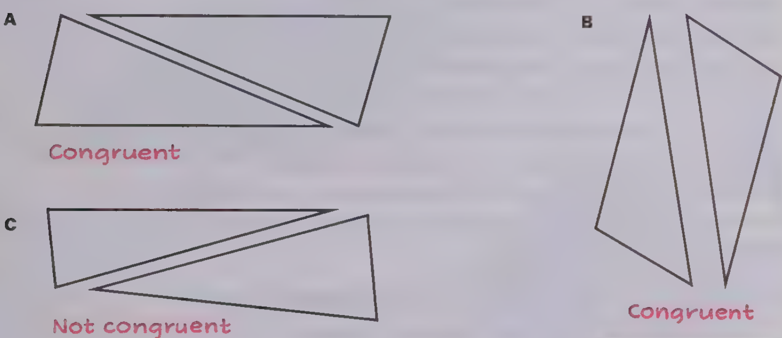
In order to understand and explain the definition of congruent triangles, children must understand the idea of congruence of angles and segments. A development of the table in the text should help clarify the definition. Have children place their traced $\triangle ABC$ over $\triangle DEF$ so that they can see for themselves that each segment and angle of $\triangle ABC$ is congruent to the corresponding part in $\triangle DEF$. You may want to make a table like the one at the bottom of the page that shows the corresponding parts for $\triangle ABC$ and $\triangle MON$, since $\triangle ABC \cong \triangle MON$.

Using the Ideas

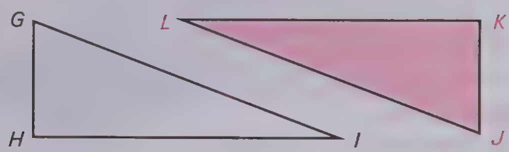
1. For each exercise, tell (just by looking) which pairs of triangles are not congruent.



2. For each exercise, trace one of the triangles on a thin sheet of paper. Use this tracing to tell whether or not the two triangles are congruent.



3. The pair of right triangles (each has one right angle) below is congruent. Give the missing angles or segments in the table.



$\triangle GHI \cong \triangle JKL$
$\overline{GH} \cong \overline{JK}$
$\overline{HI} \cong$ ▮ $\overline{LK}$
$\overline{GI} \cong$ ▮ $\overline{LJ}$
$\angle GHI \cong$ ▮ $\angle LKJ$
$\angle HIG \cong$ ▮ $\angle KLJ$
$\angle IGH \cong$ ▮ $\angle LJK$

85

Mathematics

Congruence of triangles is defined in terms of congruence of segments and congruence of angles. Briefly, two triangles are congruent if there is a one-to-one correspondence of the vertices of the two triangles such that, under this correspondence, the *corresponding parts* of the two triangles (the segments and angles) are congruent. When we write

$$\triangle XYZ \cong \triangle MNP,$$

the letters in the order named in each triangle indicate the correspondence and tell which pairs of sides and angles are congruent. Thus,

$$\angle X \cong \angle M,\ \angle Y \cong \angle N, \text{ and } \angle Z \cong \angle P.$$

Similarly,

$$\overline{XY} \cong \overline{MN},\ \overline{YZ} \cong \overline{NP}, \text{ and } \overline{XZ} \cong \overline{MP}.$$

Follow-up

Children might draw pairs of triangles – some congruent, some not. Then they might exchange papers and try to determine which triangles are congruent. At this level, encourage children to use the tracing method for constructing triangles and checking for congruence.

Resources for Active Learning

Inquiry in Mathematics via the Geoboard, "Congruent Triangles," Geo-Card 22, Walker. (Available from Fitzhenry and Whiteside)

Using the Exercises

On page 85, encourage the children to do exercise 1 simply by observation and without tracing or using any mechanical device. They should note immediately that the only pairs of triangles that could possibly be congruent are those in parts C and E. Perhaps they cannot tell positively that these are pairs of congruent triangles, but they can see that, if any pairs of the triangles are congruent, the congruent pairs would have to be C and E. You may wish to give some children a pair of labelled congruent triangles to manipulate so that they can reinforce their understanding of the concept of congruence.

When the children have completed the exercises, allow time for discussion and checking papers. Note that in these exercises, they apply their understanding of the definition of congruent triangles, but they should not be required to memorize it.

Assignments (page 85)
Minimum: 1–2. Average: 1–3.
Maximum: 1–3.

Workbook, page 25

PAGES 86–87

Objective

Given tangram pieces, the child will be able to use them to form polygonal shapes.

Preparation

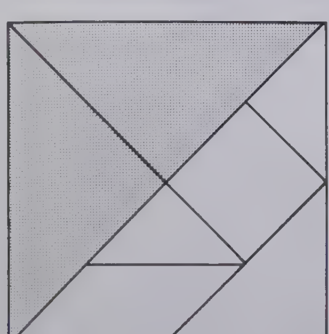
Materials

tracing paper; scissors; envelopes for storing tangram pieces

The nature of this lesson makes it appropriate to begin immediately with the investigation.

Investigation

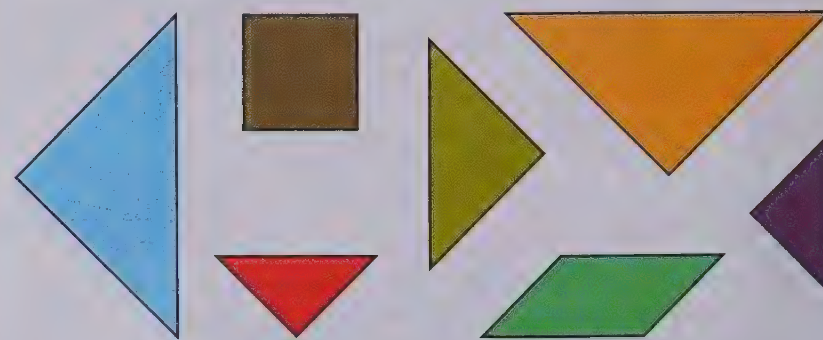
Although this investigation may be done with the tracing paper suggested, you might prefer that the children use stiffer paper. For this purpose, it would be helpful to prepare a duplicating master of the tangram pieces (*Duplicator Masters,* page 67), run them off on stiff paper, and distribute them for the children to cut out. The suggested envelopes should enable children to keep their tangram pieces together for future use. Since all of the triangles are isosceles right triangles, the first two investigation challenges require simply that the hypotenuses of each of the two triangles be placed together so that they form the diagonal of a square. The last activity is quite challenging. Children may need extended time to make a square using all seven pieces. If some children become frustrated, you might ask them if they want a hint; if so, show them the correct position of the shaded figures in the following diagram of the solution.



Can you use tangram pieces to form polygon shapes?

Investigating the Ideas

Here are the seven pieces of the tangram puzzle.

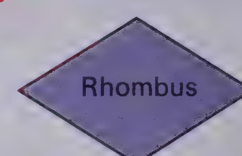


Trace these seven shapes and cut them out.
Use the two large triangles to form a square.
Use the two small triangles to form a square.

? Can you place the seven pieces together to form a square? See Investigation.

Discussing the Ideas

1. The green tangram piece is a **parallelogram**. Which two pieces will exactly cover it?
   The two small triangles
2. Can you find three pieces that will exactly cover one of the large triangles?
   The square and the two small triangles
3. A **rhombus** is a parallelogram which has 4 congruent sides. Do you think you can place some of the tangram pieces together to form a rhombus?
   See Discussion.



86

Discussion

As you discuss the investigation, review with the children the names of the various shapes of the tangram pieces. For example, point out how the two large right triangles can be placed together to form one larger right triangle whose hypotenuse is the diagonal of the whole square. Similarly, point out the trapezoid which is formed by the small square and the two small triangles on opposite sides as they are placed in the square.

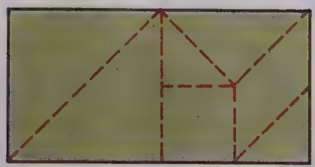
Discussion exercise 1 reviews not only the parallelogram, but also how the diagonal of a parallelogram will separate it into two congruent triangles. In exercise 2, point out how the fitting of the small square into the top vertex of the large triangle shows that that angle is a right angle and therefore that the triangle is a right triangle. In exercise 3, every rhombus formed by the tangram pieces will also be a square.

Continue this review of geometric terms and shapes as you proceed to page 87.

Using the Ideas

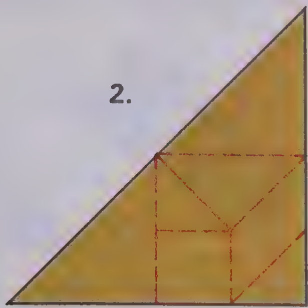
Use all 7 pieces of the tangram puzzle to make each of these shapes. Your shapes will be larger than the ones shown.

1.



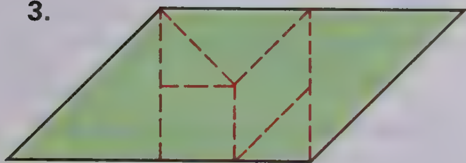
Rectangle

2.



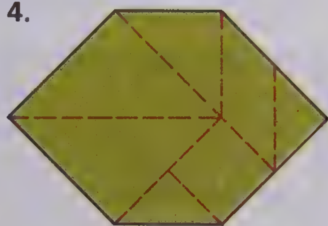
Triangle

3.



Parallelogram

4.



Hexagon

5.

Trapezoid

Mathematics
In this lesson, the names of several polygons are reviewed or introduced by means of work with tangrams.

A *polygon* is a closed plane figure that has segments as its sides.

A *quadrilateral* is a polygon with four sides.

Some of the special kinds of quadrilaterals are described below.

A *trapezoid* is a quadrilateral that has one pair of parallel sides.

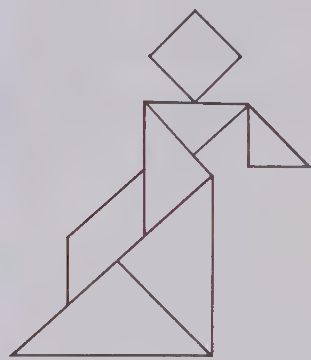
A *parallelogram* is a quadrilateral that has two pairs of parallel sides.

A *rectangle* is a parallelogram that has right angles.

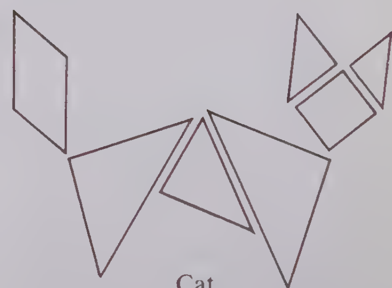
A *square* is a rectangle that has four congruent sides.

A *rhombus* is an equilateral parallelogram.

Follow-up
Encourage children to use the tangram pieces to make a variety of shapes.



Lady



Cat

Resources for Active Learning
Independent Exploration, "Chinese Tangram Puzzle Kit," Concept Co.

Using the Exercises
Before assigning the exercises on page 87, use the illustrations in the text to review polygons and the terms used to name the more common polygons. Remind the children that a *polygon* is a many-sided figure. You might use questions such as the following to review various characteristics of the figures.

"What can you say about opposite sides of a parallelogram?" (They are parallel.)

"Would you say that the opposite sides of a rectangle are parallel? Then a rectangle is also a __?__." (parallelogram)

"How many sides has a hexagon?" (6) "An octagon?" (8)

"What can you say about one pair of opposite sides in a trapezoid?" (One pair of sides are parallel; the other pair are not.)

You should not expect the children to solve all of the puzzles, but encourage them to try to put together at least one or two of the figures.

Assignments (page 87)
Minimum: 1–2. Average: 1–3.
Maximum: 1–5.

PAGES 88–89

Objective

Given a polygon, the child will be able to find its perimeter.

Preparation

Materials

centimetre rulers

You might prepare for this lesson by reviewing polygons studied in the previous lesson. Exhibit a variety of polygons and ask children to identify them with the proper term. You might also review the centimetre ruler, pointing out the tenths between each centimetre mark.

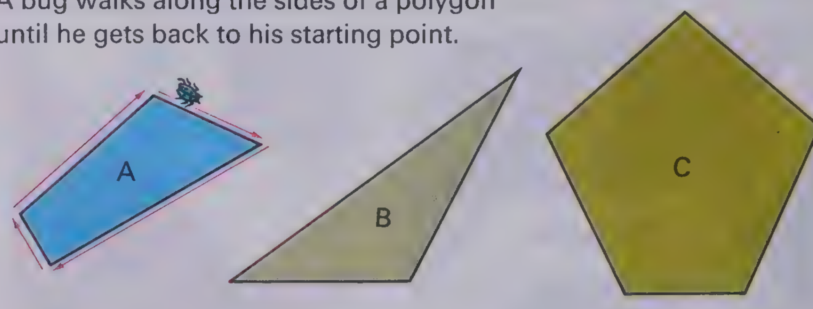
Investigation

In this investigation, children review a concept which, if they have studied from previous books in this series, should not be difficult for them. Encourage independent work and remind children to record their results.

● *How do you find the perimeter of a polygon?*

Investigating the Ideas

A bug walks along the sides of a polygon until he gets back to his starting point.

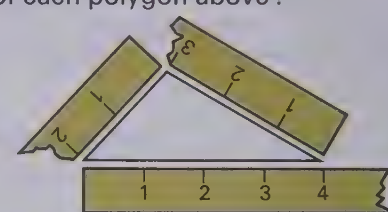


? Can you use your centimetre ruler to find how far the bug has to travel for each polygon above?

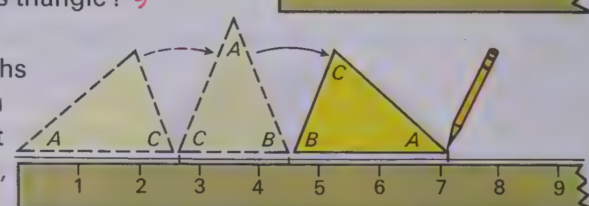
A 10 cm
B 13 cm
C 14 cm

Discussing the Ideas

1. The **perimeter** of a polygon is the sum of the lengths of its sides. What is the perimeter of each polygon above?
   See above.
2. If the lengths of the sides of a polygon are whole numbers, it is easy to find the perimeter with a ruler. What is the perimeter of this triangle? 9



3. When the lengths of the sides of a polygon are not whole numbers, you can find the perimeter by "rolling" the polygon along your ruler. Explain how to find the perimeter of triangle *ABC*.
   See Discussion.



Discussion

Have the children explain the method they used to find the perimeter of each figure in the investigation. Ask children to write on the chalkboard an equation which will illustrate how to find the perimeter. For example, the four-sided polygon might be matched with this equation:

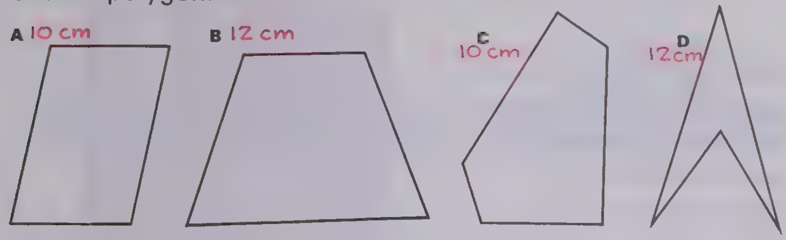
$5 + 2 + 4 + 3 = 14$ (centimetres)

As you work through these exercises, stress the word *perimeter* and have the children give the definition in their own words.

In exercise 3, children must be careful to "roll" the polygon accurately, so that the measurement of each segment begins where the preceding one ended. After they complete the rolling process, they could use their centimetre ruler to measure the total segment representing the perimeter.

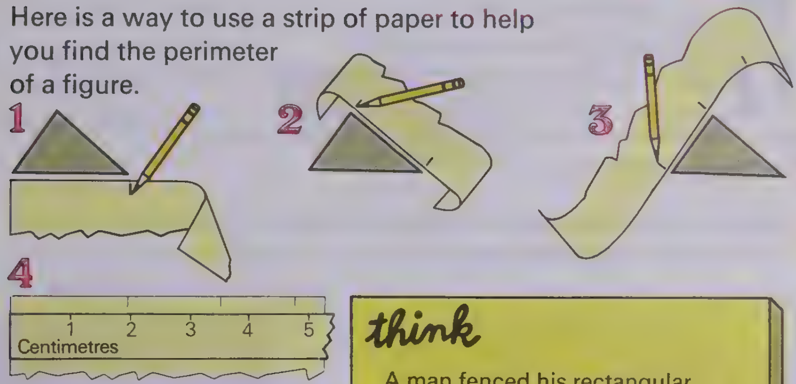
Using the Ideas

1. Use your centimetre ruler to find the perimeter of each polygon.

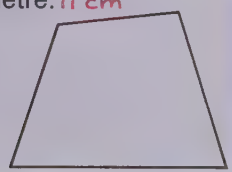


2. Draw a triangle, a quadrilateral, a pentagon, and a hexagon, and find the perimeter of each. Constructions and perimeters will vary.

3. Here is a way to use a strip of paper to help you find the perimeter of a figure.



Use this method to find the perimeter of this polygon to the nearest centimetre. 11 cm



think

A man fenced his rectangular shaped garden and used 40 posts. He had 14 posts on each long side of the garden. How many did he have on each short side? 8

BE CAREFUL! ! !

More practice, page A-6, Set 11

Using the Exercises

Assign the exercises on page 89 as independent work. The method suggested in exercise 3 is somewhat similar to the "rolling process" described on page 88. When children have finished, check their work and encourage them to share opinions as to which method of finding perimeters they think is easiest.

The trick to the *Think* problem is that, although only twelve posts are left for the short sides (thus, six posts to be placed on each short side), two corner posts count both ways, so each short side will actually have eight posts.

Assignments (page 89)
Minimum: 1. Average 1–2.
Maximum: 1–3.

Follow-up

Suggest that some children draw a variety of polygons on newsprint, measure the sides to the nearest centimetre, and then find the perimeters. Or, give three groups of children a duplicated sheet showing 6 or 7 polygons. Ask one group to measure to the nearest whole unit and find the sum of the sides for the perimeter of each polygon. Direct another group to cut out each polygon and use the "rolling" technique demonstrated on page 88 to find the perimeters. Direct the third group to use the strip-of-paper method suggested in exercise 3 on page 89. These groups can then compare results to gain some insight into accurate measurement.

Resources for Active Learning

Developmental Math Cards, "Conservation of Perimeter," I²8, I²17, Addison-Wesley.

Franklin Series: *Pencil and Paper Geometry*, "Perimeter," pp. 71–74, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Duplicator Masters, page 14
Workbook, page 26

PAGES 90–91

Objective

Given a parallelogram, or another polygon of rectangular shape, the child will be able to find the area.

Preparation

Materials

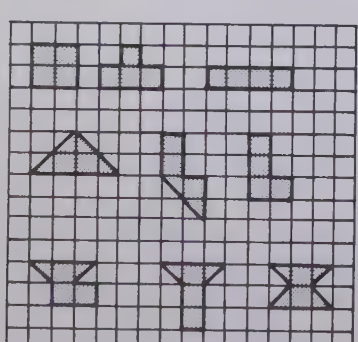
geoboard (1 per child) or dot paper; graph paper (*Duplicator Masters*, page 61)

To prepare for this lesson, briefly review with the children the concept of area. For example, ask them about the amount of surface covered by a rug in the classroom, or by the chalkboard, or by a picture, and remind them of the term *area*. If you are sure children recall the term, begin immediately with the investigation.

Investigation

Although it is possible to work with this investigation on dot paper, the geoboard is a much more suitable device. Notice that in the sample region, figure A, the children must consider half units. Children should be encouraged to try to use half units in their figures as well. Note that both convex and concave polygons (see "Mathematics," T.E. page 91) are acceptable.

Remind children to record on graph paper each figure they find in this investigation. Some regions with area of 4 square units are the following:

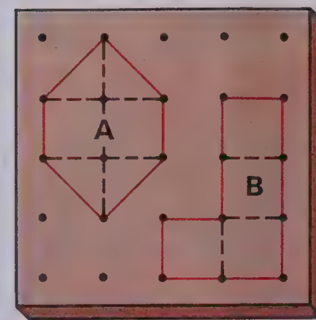


You might also have children repeat the investigation looking for shapes that have an area of 6 square units.

Let's find the area of some polygons.

Investigating the Ideas

If each small square on the geoboard is **1 unit of area**, the **area** of each of the polygons **A** and **B** is 4 square units.



See Investigation.

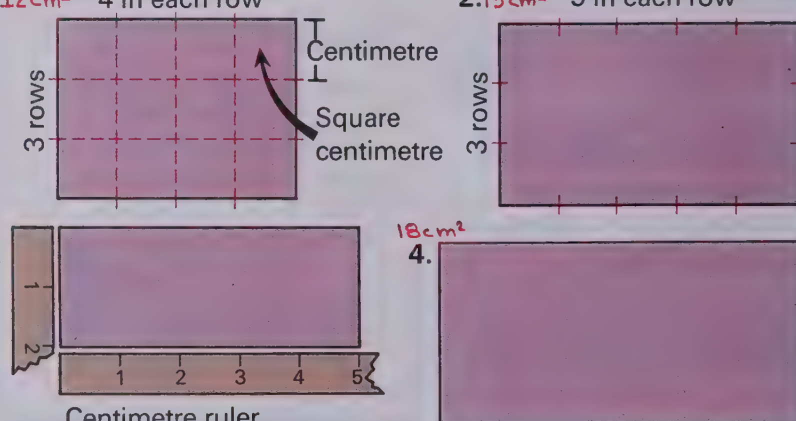
? Can you find at least 8 more differently shaped regions that each have an area of 4 square units?	Record your regions on graph paper.

Discussing the Ideas

Explain how you would find the area of each rectangle. The unit is a square centimetre.

1. 12 cm² 4 in each row

2. 15 cm² 5 in each row



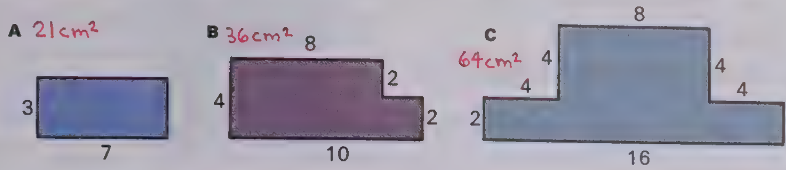
90

Discussion

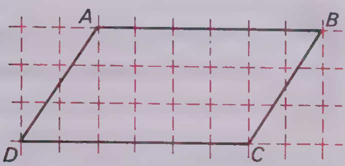
After you discuss the various shapes children found on the geoboard, use the discussion exercises to develop the method of finding area of a rectangle by multiplying the length and the width. Also refer to any shapes in the investigation which might be used to illustrate that the number of rows times the number in each row will give the area of a rectangle.

Using the Ideas

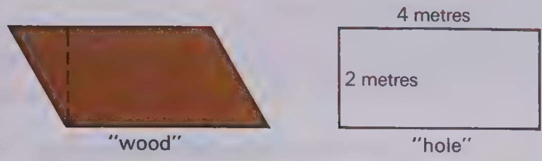
1. Find the areas of each polygon. Pretend the unit of area is the square centimetre and the lengths given are in centimetres.



2. Estimate the area of parallelogram *ABCD*. 18



3. Trace the rectangular "hole" and the parallelogram of "wood."

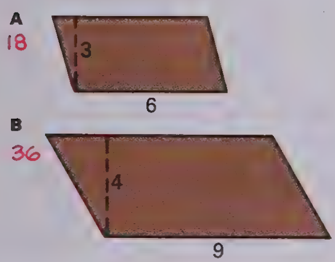


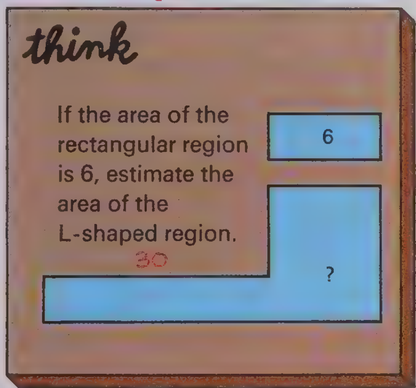
Cut out the piece of "wood", and then cut along the dotted line. Paste the pieces so they completely "fill" the hole.

A What is the area of the rectangle? 8 m²

B What is the area of the parallelogram? 8 m²

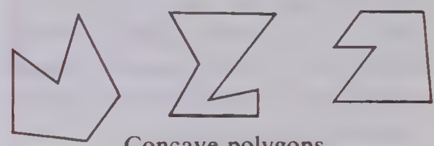
4. Think about forming a rectangle as in exercise 3. Then give the area.



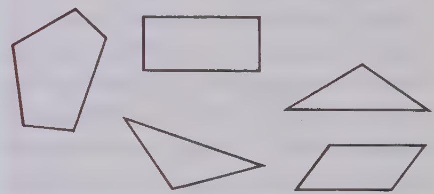


Mathematics

A polygon is a closed geometric figure made up of line segments. Polygons may be either concave or convex. A segment joining two points of a convex polygon will have all its points on the polygon or in the interior of the polygon. Observe that this is not the case for concave polygons.



Concave polygons



Convex polygons

Follow-up

Encourage children to continue to use the geoboard to make various shapes, and to find the area of the shapes.

It would also be instructive for children to find areas of various surfaces in the classroom. For example, one group of children might use an actual cardboard square-metre unit and measure the floor or a part of the floor. Another group of children might measure the length and width of this same region, and find the area arithmetically. Then they should compare the accuracy of the measurements and decide which method they prefer.

Resources for Active Learning

Applied Mathematics Cards, Group 2/16, Schofield and Sims. (Available from Mafex Associates, Willowdale)

Math Activity Cards, D19, Macmillan.

Maths Mini-lab, Card 125, Selective Educational Equipment.

Using the Exercises

Assign the exercises on page 91 as independent work. After children have worked on exercises 2, 3, and 4, use them as a basis for a discussion of the area of a parallelogram. Help the children realize that they may think of cutting off a right triangle at one end of the parallelogram and then fitting this onto the other side. They actually do this in exercise 3. The point to emphasize here is that the segment along the cut is the height of the rectangle, or the height of the parallelogram, and either of the horizontal segments can be thought of as a base of the parallelogram.

Thus, the general formula for finding area, base times height, is applicable for a parallelogram as well as for a rectangle.

Accept any good estimate for the area of the L-shaped region shown in the *Think* problem. For children who are interested, secure some precise measuring devices and have them find the areas as accurately as possible.

Assignments (page 91)

Minimum: 1–2. Average: 1–3. Maximum: 1–4.

PAGES 92–93

Objective

Given a triangle, the child will be able to find the area.

Preparation

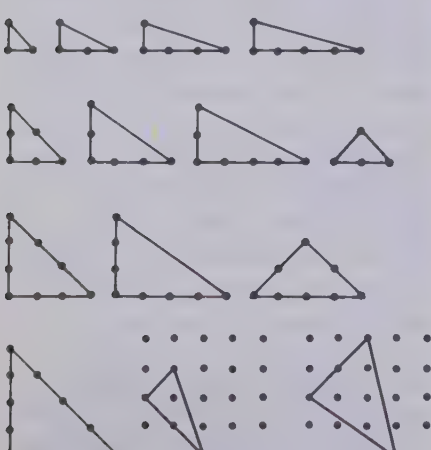
Materials

geoboard or dot paper (*Duplicator Masters*, page 64)

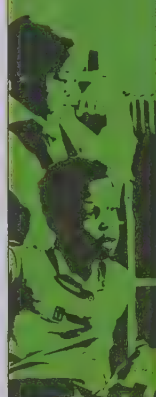
To prepare for this lesson, review right triangle and the terms *legs* and *hypotenuse*. You might do this by exhibiting a rectangle with one of the right triangles formed by the diagonal shaded in color.

Investigation

Once children construct one right triangle on a corner, they might easily change the shape simply by lengthening a leg of the triangle. Right angles in other positions on the board will yield other right angles, so at least 16 possible right triangles may be formed. If some children think they have found all the right triangles, suggest that they check to see if they have studied all sizes of squares and rectangles that can be found on the geoboard. Below are some of the possible right triangles.

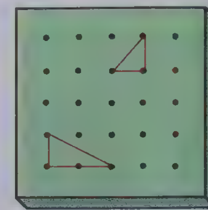


● *Can you find the area of any triangle?*



Investigating the Ideas

Two **right triangles** of different shapes are shown on the geoboard.

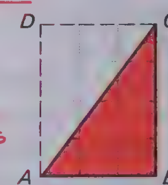


? How many right triangles of different sizes or shapes can you find on a geoboard?	Show your triangles on dot paper.

See Investigation.

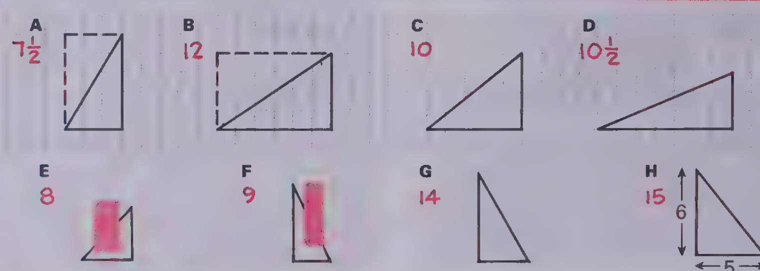
Discussing the Ideas

1. Can you give the area of any of the triangular regions you found in the Investigation? See Discussion.



2. Every right triangular region is one half of some rectangular or square region.
   - A What is the area of the region *ABCD*? 12 sq units
   - B The area of region *ABC* is what part of the area of the rectangular region? $\frac{1}{2}$
   - C What is the area of triangular region *ABC*? 6 sq units

3. Give the area of each triangular region. Can you find a rule for finding the area of any right triangle? See Discussion.



92

Discussion

The development of this lesson progresses from finding the area of a right triangle to finding the area of any triangle. It is important that children discover how to find the area of a right triangle before they proceed to page 93.

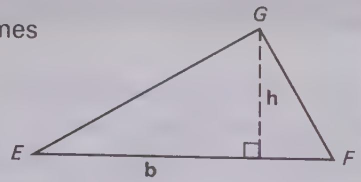
In exercise 1, have children discuss and demonstrate ways of finding the area of some of the right triangles they constructed in the investigation.

Exercise 2 is important because it provides the key to finding area of right triangles. Each right triangle has an area that is one half the area of some rectangle.

If children realize the rule for finding the area of a rectangle, they will be better prepared for exercise 3. Thus, you might remind them that we can find the area of a rectangle by finding the product of the length of two adjacent sides. As you work through exercise 3, it would be helpful to have the children find the area of the rectangle as well as the area of the triangle related to it. This should help them realize that the area of a right triangle is one half its base times its height.

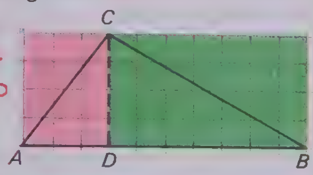
Using the Ideas

The length of segment EF is sometimes called the base (**b**).
The length indicated by the dotted line is called the height (**h**).

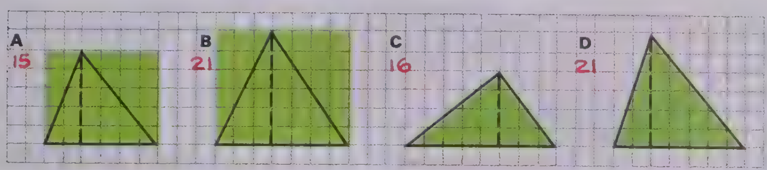


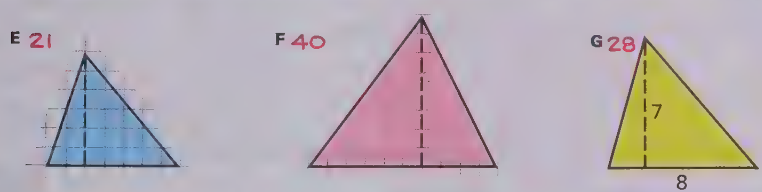
These exercises will help you find the area of a triangular region when you know the base and the height from that base.

1. A What is the area of the region shaded pink? 12
B What is the area of the region shaded green? 28
C What is the area of the two regions together? 40
D What is the area of triangle ADC? 6
E What is the area of triangle BDC? 14
F What is the area of triangle ABC? 20
G The area of triangle ABC is what part of the entire shaded region? $\frac{1}{2}$



2. Find the area of each large triangular region.





3. Find the areas of the following triangular regions. Only the base and height are given.

A $b = 10$, $h = 4$ — 20
B $b = 18$, $h = 4$ — 36
C $b = 12$, $h = 10$ — 60
D $b = 24$, $h = 36$ — 432
E $b = 48$, $h = 35$ — 840

More practice, page A-6, Set 12

Using the Exercises

You might choose to use the top of page 93 as a basis for discussion, although many children would find the content in the text clear enough to study on their own. The main idea on this page is that each triangle shown may be thought of as two right triangles, whose areas they already know how to find.

When the children have finished the exercises, emphasize the fact that, in exercise 1, $\triangle ADC$ is half of the rectangle which is to the left of segment CD, and that the area of $\triangle CDB$ is half of the rectangle to the right of CD. The fact that each of these triangles is half of one of the two rectangles indicates that the whole $\triangle ABC$ is half of the entire rectangle with base AB. Thus, for any triangle, area is equal to one half of its base times its height.

Assignments (page 93)
Minimum: 1, oral; 2A–D.
Average: 1, oral; 2. Maximum: 1–3.

Mathematics

You will notice that on page 93 base and height are defined as numbers. That is, the base is the length of a particular segment of the triangle, and the height is the length of a segment from one vertex perpendicular to the side opposite that vertex. Determining which side of a particular triangle to consider to be its base is purely an arbitrary matter. Once we select the base for a given area problem, however, we must use as the height the length of the perpendicular line segment to that base from the opposite vertex. This should cause the children little trouble since all of the triangles given in this lesson are drawn in such a way that the base is the length of the horizontal side of the triangle and the height is a vertical length from the vertex at the top of the triangle.

Follow-up

Many children would find it helpful to draw various triangles on graph paper and construct the related rectangles. Then they may cut out the triangle and the remaining parts of the rectangles, and compare the areas by placing appropriate sections over the triangle. For optimum benefit from this activity, suggest that the children also work out the area arithmetically.

Resources for Active Learning

Inquiry in Mathematics via the Geoboard, "Area of Triangles," Geo-Cards 16-18, Walker. (Available from Fitzhenry and Whiteside)

Math Activity Cards, C26, Macmillan.

Maths Mini-lab, Card 126, Selective Educational Equipment.

Duplicator Masters, page 15
Workbook, page 27

PAGES 94–95

Objectives

The child will demonstrate his ability to work with the concepts presented in this chapter.

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

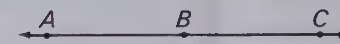
Materials

protractors

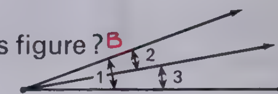
You might have the children begin immediately with the exercises, or you might review some topic with which they had particular difficulty. You might also review one of the topics treated on page 95, such as reading large numbers.

Reviewing the Ideas

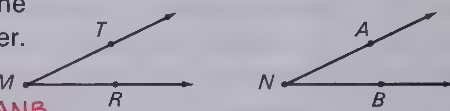
1. Only one of the following could be true for this figure. Which one? C
   A $\overline{AB} \cong \overline{AC}$ B $\overline{AC} \cong \overline{BC}$ C $\overline{AB} \cong \overline{BC}$



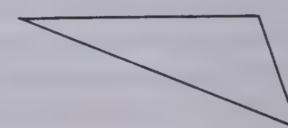
2. Which one of the following could be true for this figure? B
   A $\angle 2 \cong \angle 1$ B $\angle 2 \cong \angle 3$ C $\angle 1 \cong \angle 3$



3. The four segments shown on the rays are congruent to each other. Tell whether or not $\angle TMR$ is congruent to $\angle ANB$. $\angle TMR \cong \angle ANB$

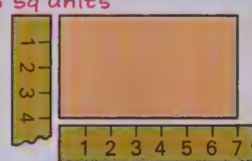


4. Use your centimetre ruler to find the perimeter of this triangle. 11 cm



5. Find the area of each region by using the unit indicated.

A 28 sq units

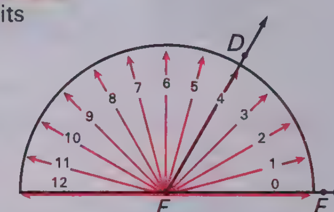


B 15 sq units

5 units

6 units

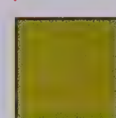
6. A Use the unit shown on the protractor to find the measure of $\angle DEF$. 4
   B Find the degree measure of $\angle DEF$. 60°



7. How many lines of symmetry does each figure have?

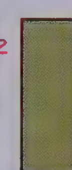
A 3

B 4



C 1

D 2



94

Discussion

Review page 94 places some emphasis on mastery of factual material. Certain terminology is expected in a few exercises, and concepts of area and degree measure must be clear in order for children to complete this page successfully. (Note that exercise 6B requires the use of a protractor.)

Depending on the ability of the children and your method of teaching, you might use this page as a review and work through it with the children, or you might use it as an evaluation of the children's grasp of these concepts. In either case, clarify concepts with a discussion and allow sufficient time for children's questions.

Keeping in Touch with

Addition | Subtraction | Multiplication | Division | Place value | Inequalities

1. Find the sums.
   - A 4000 + 300 + 60 + 9 4369
   - B 50 000 + 8000 + 700 + 20 + 1 58 721
   - C 70 000 + 3000 + 10 + 8 73 018
   - D 600 000 + 90 000 + 4000 694 000
2. Write each number as in the example. See Answers, T.E. page 95.
   Example: 3254 = 3000 + 200 + 50 + 4
   - A 4695
   - B 8329
   - C 17 264
   - D 29 435
   - E 843 672
3. Give the sign $<$ or $>$ for each ▒.
   - A 4280 ▒ 4279 $>$
   - B 32 496 ▒ 32 500 $<$
   - C 416 837 ▒ 417 213 $<$
4. Solve the equations.
   - A $5 \times 6 = n$ 30
   - B $8 + 7 = r$ 15
   - C $15 - 9 = s$ 6
   - D $48 \div 6 = a$ 8
   - E $4 \times 9 = c$ 36
   - F $8 + 9 = t$ 17
   - G $13 - 8 = b$ 5
   - H $9 \times 8 = m$ 72
   - I $5 + 9 = d$ 14
   - J $7 \times 6 = n$ 42
   - K $32 \div 8 = b$ 4
   - L $49 \div 7 = s$ 7
   - M $5 \times 8 = a$ 40
   - N $56 \div 7 = t$ 8
   - O $35 \div 5 = r$ 7
   - P $8 \times 8 = f$ 64
5. Find the products.
   - A 26×10 260
   - B 49×100 4900
   - C 60×40 2400
   - D 30×200 6000
6. Find the quotients.
   - A $240 \div 6$ 40
   - B $320 \div 4$ 80
   - C $240 \div 30$ 8
   - D $1800 \div 3$ 600
   - E $1800 \div 300$ 6
   - F $1800 \div 30$ 60
7. Estimate the products.
   - A 99×46 4600
   - B 19×31 600
   - C 48×52 2500
   - D 199×31 6000

think

The dots below should help you see why 3, 6, and 10 are sometimes called triangular numbers.

3 6 10

Give the next 5 triangular numbers. 15, 21, 28, 36, 45

You are invited to explore

ACTIVITY CARD 4 Page 335

95

Using the Exercises

For page 95, have the children do the exercises and, when they have finished, allow time for discussion and checking papers.

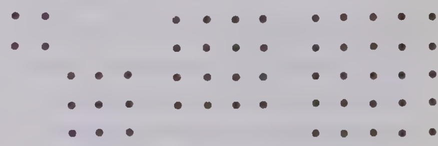
The pattern for solving the *Think* problem should be easy for most of the class to see. Encourage the children to experiment with drawing models of groups of dots to work out the next five numbers, if they find it necessary. Some children may note that, since the number in the last row increases by one each time, they can find the new numbers by adding the next consecutive integer. Most of the children in the class will benefit from a discussion of the different approaches to a solution.

Answers, exercise 2, page 95

A 4695 = 4000 + 600 + 90 + 5
B 8329 = 8000 + 300 + 20 + 9
C 17 264 = 10 000 + 7000 + 200 + 60 + 4
D 29 435 = 20.000 + 9000 + 400 + 30 + 5
E 843 672 = 800 000 + 40 000 + 3000 + 600 + 70 + 2

Follow-up

Encourage children to expand the suggestion in the *Think* problem to other shapes. For example, they might try to find groups of dots which they can draw in the shape of squares.



You might suggest that several groups of children each take a different object in the classroom and study it in terms of as many concepts in this chapter as possible. To help them get started, present questions such as the following.

1. Are there any angles in your object? What is the degree measure of each angle?
2. What shape is the top surface of your object? Can you find its area? What shape is one of the side surfaces of your object? Can you find its area?
3. Are any two sides of your object parallel if you think of them as being in the same plane?
4. Can you divide any of the surfaces of your object into congruent shapes?

Objects which have rectangular surfaces would be most applicable for this activity, for example, windows, tops of tables or desks, chalkboards, doors, etc.

Workbook, page 28 (Use with text page 94.)

CHAPTER 5 | Estimation

Pages 96–113

General Objectives

To develop skill in using multiples of 10 and 100 to estimate products and quotients

To develop skill in finding products that have factors which are multiples of 10 and 100

To develop skill in finding quotients when the divisor and the dividend are multiples of 10

To provide a variety of experiences in estimation

The beginning pages of this chapter provide general experiences in estimation, involving all operations and the idea of "rounding" numbers in order to make good estimates. Work with the number line is provided to help the children visualize the idea of "rounding" numbers for estimation.

Following this, children develop guidelines for finding products of multiples of 10 and 100. The inverse relation of multiplication and division is the foundation for the study of quotients involving multiples of 10 and 100.

Next, children are given a variety of opportunities to practice estimating skills in operations and to try estimation projects. These later pages of estimation, such as the lesson titled "Estimation for Fun," are designed to stimulate the children's interest and enjoyment.

Mathematics

Estimation is seldom thought of as a topic that lies entirely within the realm of pure mathematics. However, nearly everyone would agree that estimation plays a vital role in practical applications of mathematics. Estimation is a valuable tool, for example, when the mathematician uses 1.414 as an approximation of the square root of 2, or 3.14159 as an approximation of π. These numbers are, of course, merely rational estimations for the irrational numbers $\sqrt{2}$ and π.

Because a precise definition of estimation would not be practical, we prefer to define it loosely and in terms readily comprehended by the children. Estimation is explained by giving several statements that the children themselves might use if asked to tell what they think estimation means. The main objective is to instill the children with some feeling for the general ideas involved in finding reasonably close approximations.

Since much of the skill in estimating numerical situations involves the use of multiples of 10, 100, and 1000, it is helpful for children to develop skills with these numbers.

The following example illustrates how, by using the basic principles, a product such as 60×80 can be reduced to multiplications which employ nothing more than basic multiplication facts and products involving a factor of 10 or of 100.

$$\begin{aligned} 60 \times 80 &= (6 \times 10) \times (8 \times 10) \\ &= (6 \times 8) \times (10 \times 10) \\ &= 48 \times 100 \\ &= 4800 \end{aligned}$$

The commutative and associative principles allow us to rearrange factors in the convenient manner shown. Certainly, several steps would be involved if we were to show all the regrouping and reordering of the factors that is actually involved in going from step 2 to step 3. Although children are asked to find a rule for multiplying such products, they, of course, are not expected to relate their steps directly to the appropriate basic principles.

Teaching the Chapter

Materials

Containers of various sizes (mixing bowl, 2-litre jar, litre jar, coffee cup)
Marbles
Peas or beans
Ruler
Stopwatch

Vocabulary

estimate	nearest multiple
factor	products
flow chart	quotients
multiple	

The materials mentioned above are most appropriate for use with the word-problem lessons at the end of the chapter, but you may find it quite stimulating for the children to use these materials during the preparation and study of each lesson. For example, when first explaining estimation to the children, you can have them guess how many marbles are in the jar. You might hold a contest, giving the children a chance, over a period of two or three days, to make a "guesstimate" of the number of marbles in the jar. You can also use some large and small containers to give the children an opportunity to guess, or estimate, how many times the contents of a small container must be put in a large container to fill it. Of course, you should determine the best estimate by having children actually count the items. Although these "guessing activities" are not at the heart of the topics taught in this chapter, they do involve estimation and serve to stimulate children's interest. The children will often guess in numbers that are multiples of 10 or 100, and this is one of the principal goals of the chapter: to accustom the children to "rounding" to multiples of 10, 100, or 1000.

Most of the vocabulary terms will not be new to the children. But, when they are introduced in this text, use them carefully, making sure children understand them.

Lesson Schedule

Plan to spend about a week and a half or two weeks on this chapter. Some of the estimation activities are not absolutely essential to children's work in the text, but you may discover that it enhances their interest in the study of arithmetic, and you may want to spend extra time on the chapter for this purpose. However, skill in finding products such as 60×30 or 60×300 is vital to the children's experiences for subsequent work. Therefore, you should give the children extra time, if needed, for this type of exercise.

Evaluation of Progress

Since one important objective of this chapter is to help children become skillful at finding special products and quotients, be sure to include a large portion of work on special products involving multiples of 10 and 100.

Evaluation is not a crucial part of the treatment of estimation, and you should not place a great premium on achievement of these skills. We do not expect the children to memorize rules or gain great skill at estimation. We do, however, expect them to develop an intuitive feeling for the ideas of estimation – to attain some skill in substituting multiples of 10, 100, and 1000 for other numbers and then arrive at "common-sense" answers. We realize that evaluating whether or not children are able to do this is difficult. It is not easy to tell from a child's written work whether he has actually computed the answer or whether he has estimated it. We therefore recommend that you employ a light touch in teaching these lessons, attempting as much as possible to see that the children enjoy the chapter, and encouraging participation in the activities suggested for the chapter.

Resources for Active Learning

General Activities

A Cloudburst, Vol. 2, Nos. 8313–8393, Midwest Publications

Elementary Science Study: *Peas and Particles,* McGraw-Hill Ryerson

Franklin Series: *Making and Using Graphs and Nomographs,* pp. 71-86, Lyons and Carnahan (Available from McGraw-Hill Ryerson)

Modern Math Games . . ., pp. 47-48, Fearon (Place-value puzzles and games) (Available from Clarke, Irwin)

Nuffield Project: *Computation and Structure* 4, "Large Numbers and Indices," pp. 50–63, Wiley

Objective

Given a 2- or 3-digit numeral, the child will be able to name the nearest multiple of 10 or 100.

Preparation

To provide motivation for this chapter, you might hold up a jar filled with marbles, peas, paper clips, or other small objects and ask the children to guess how many are in it. You might keep the jar on display and encourage children to write down a guess each day for a week. Activities of this type will do much to motivate the children to study the material in this chapter.

Investigation

This investigation may be worked on independently by the children, but it is also appropriate for small-group work. Children should benefit from trying to explain to each other the rule which they think applies for each table. Notice that the ordinary standards of rounding numbers are applied here: When the digit 1, 2, 3, or 4 is in the key place, we round *down* to the nearest multiple, and with the digits 5, 6, 7, 8, or 9, we round *up* to the nearest multiple. However, it is not essential that children verbalize this rule on their own; it is more important that they be able to identify the nearer multiple of 10 or 100 through their understanding of the order and value of numbers.

Discussion

Children must understand the rounding process to give the outputs in discussion exercises 1 and 2. However, as mentioned above, they need not verbalize it. You might point out to them that a number which has 5 in the key position is rounded up to the nearest multiple of 10 or 100 that is greater than the number given. For example, 654 (exercise 1I) would be rounded down to 650 when rule A is applied, that is, when it is rounded to the nearest multiple of 10. But in exercise 2G, when 651 is rounded to the nearest multiple of 100, 651 should be rounded up to 700.

5 Estimation

What is the nearest multiple of 10 or 100?

Investigating the Ideas

What are the last four numbers in each function table?

A

??? RULE ???

n	32	33	34	35	36	37	38	39	40	41	76	92	134	275
output	30	30	30	40	40	40	40	40	40	40				

80 90 130 280

B

??? RULE ???

n	689	695	721	749	750	758	1243	1250	1276	3284	4516
output	700	700	700	700	800	800	1200				

1300 1300 3300 4500

? Can you write or explain a function rule for each table?
See Investigation and Discussion exercise 3 below.

Discussing the Ideas

1. Use the function rule for Function Machine A to give the output for each of these inputs.
   A 52 50 C 84 80 E 64 60 G 138 140 I 654 650 K 1273 1270
   B 57 60 D 76 80 F 65 70 H 132 130 J 655 660 L 1276 1280

2. Use the function rule for Function Machine B to give the output for each of these inputs.
   A 278 300 C 316 300 E 650 700 G 651 700 I 2346 2300 K 7239 7200
   B 244 200 D 361 400 F 649 600 H 987 1000 J 2356 2400 L 7293 7300

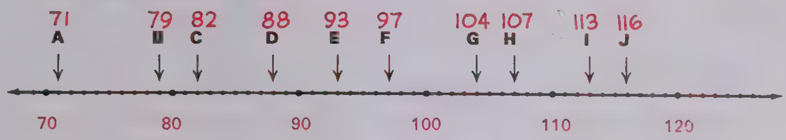
3. State each rule in your own words. Sample answers: A: If ones' digit is less than 5, round down to nearest multiple of 10; If 5 or greater, round up. B: If tens' digit is less than 5 round down to nearest multiple of 100; If 5 or greater, round up.

96

Using the Ideas

The number-line pictures and exercises will help you choose multiples of 10 and 100 that you will use in estimation.

1. Give the numbers (A through J) that go with the points on the number line.

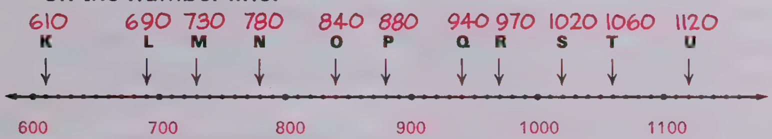


2. Give the multiples of 10 that are closest to the numbers for A through J. If the number is halfway between, give the larger multiple of 10.
A 70; B 80; C 80; D 90; E 90; F 100; G 100; H 110; I 110; J 120

3. Give the multiple of 10 that is closest to each number.

A 27 30	C 45 50	E 89 90	G 127 130	I 136 140	K 298 300
B 44 40	D 99 100	F 36 40	H 133 130	J 289 290	L 1256 1260

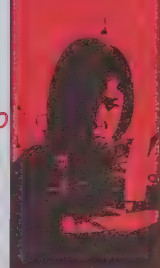
4. Give the numbers (K through U) that go with the points on the number line.



5. Give the multiples of 100 that are closest to the numbers for K through U. If a number is halfway between, give the larger multiple of 100.
K 600; L 700; M 700; N 800; O 800; P 900; Q 900; R 1000; S 1000; T 1100; U 1100

6. Give the multiple of 100 that is closest to each number.

A 207 200	C 84 100	E 573 600	G 649 600	I 2651 2700	K 4059 4100
B 489 500	D 326 300	F 456 500	H 1286 1300	J 3438 3400	L 37 0



Using the Exercises

The number line used in the exercises on page 97, is very useful for helping children understand the rounding process. Suggest that children list the answers to exercises 1 and 4 vertically so that they may write the answers to exercises 2 and 5 next to them, respectively. When children have finished the exercises, allow ample time for discussion. Continue to stress basic understanding of order of numbers, but also be sure children develop the skill of rounding down for numbers ending in 1, 2, 3, or 4 and rounding up for numbers ending in 5, 6, 7, 8, or 9.

Assignments (page 97)
Minimum: 1–5, oral.
Average: 1–3, oral; 3–5.
Maximum: 1–6.

Workbook, page 29

PAGES 98–99

Objective

The child will be able to round numbers in a given problem to the nearest multiple of 10 or 100 and then estimate the answer to the problem.

Preparation

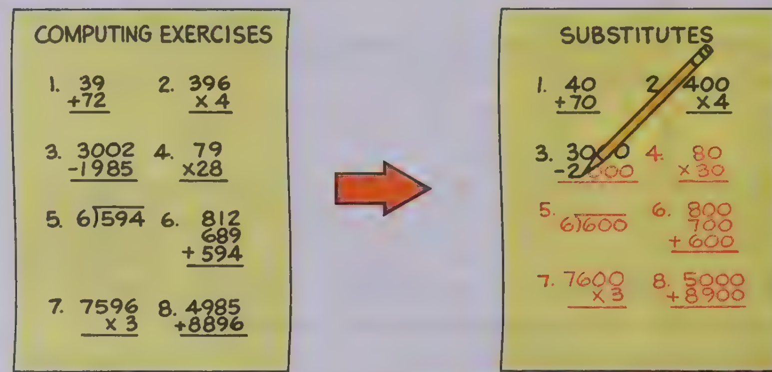
To prepare for this lesson, briefly discuss the terms *about, approximation, guessing,* and *estimating.* Relate these words to examples from the previous lesson and to the "guesstimating" problem you used to introduce that lesson. Stress that, in exercises of this kind, the principal concern is not to arrive at exact answers but to make estimates which are near to the correct answer, based on correct estimating procedures.

Investigation

In this investigation, children are to replace each of the "computing exercises" with a related exercise involving multiples of 10 or 100. Allow children to work with a classmate if their skills in performing basic operations are weak. Others may benefit from small-group discussion of how to round the numbers for the substitute exercises. Encourage free sharing of ideas in investigations of this type.

How are multiples of 10 and 100 used in estimation?

Investigating the Ideas



See above and Investigation.

Can you copy and complete the page of "substitute" problems? Use multiples of 10 and 100 so that the answers to the two sets will be "very close" to each other.

Discussing the Ideas

1. Your substitute problems will help you find estimates—often without pencil and paper. How many of your substitute problems above can you solve without pencil and paper?
   1. 110; 2. 1600; 3. 1000; 4. 2400; 5. 100; 6. 2100; 7. 22 800; 8. 13 900
2. The closer you can get to the correct answer, the better your estimate is. Which problem—1, 2, or 3—will give the best estimate for the one in red?

	red	1	2	3
A (2)	38 +84	40 +90	40 +80	30 +80
B (2)	79 ×8	70 ×8	80 ×8	80 ×10
C (1)	604 −289	600 −300	700 −300	600 −200
D (3)	68 ×71	60 ×70	60 ×60	70 ×70

Discussion

One of the main purposes of this lesson is to provide motivation for working with the special products and quotients that will be studied subsequently. Use discussion exercise 1 as a basis for working through each exercise to see how the "computing exercises" may best be replaced by rounded numbers.

As you go on to exercise 2, stress that a better estimate will be achieved if numbers are rounded carefully. Encourage children to discuss these examples and the reasoning underlying their various estimates. For example a child who is estimating 53–38 might well explain that to arrive at this estimate he subtracted 40 from 50. Some children may be sufficiently skilled in mental computation to arrive at the exact answer, 15. However, you should point out that for such an exercise they need only think of an estimate that is a multiple of 10, and that in this case you are not interested in the exact answer.

Using the Ideas

1. Give the multiple of 10 that you think should go in each ▥.

- **A** To estimate 42 + 59, we can find the sum 40 + ▥. 60
- **B** To estimate 25 + 72, we can find the sum ▥ + 70. 30
- **C** To estimate 81 − 48, we can find the difference 80 − ▥. 50
- **D** To estimate 148 − 29, we can find the difference 150 − ▥. 30
- **E** To estimate 173 − 36, we can find the difference ▥ − 40. 170
- **F** To estimate 245 + 328, we can find the sum ▥ + 330. 250
- **G** To estimate 29 × 82, we can find the product ▥ × 80. 30
- **H** To estimate 54 × 26, we can find the product ▥ × 30. 50
- **I** To estimate 323 ÷ 83, we can find the quotient ▥ ÷ 80. 320
- **J** To estimate 207 ÷ 65, we can find the quotient 210 ÷ ▥. 70

2. Give the multiple of 100 that you think should go in each ▥.

- **A** To estimate 213 + 487, we can find the sum ▥ + 500. 200
- **B** To estimate 528 + 1176, we can find the sum 500 + ▥. 1200
- **C** To estimate 969 + 850, we can find the sum 1000 + ▥. 900
- **D** To estimate 806 − 479, we can find the difference 800 − ▥. 500
- **E** To estimate 1146 − 357, we can find the difference ▥ − 400. 1100
- **F** To estimate 950 − 373, we can find the difference ▥ − 400. 1000
- **G** To estimate 93 × 123, we can find the product 100 × ▥. 100
- **H** To estimate 549 × 451, we can find the product ▥ × 500. 500
- **I** To estimate 3214 ÷ 789, we can find the quotient ▥ ÷ 800. 3200
- **J** To estimate 4461 ÷ 850, we can find the quotient ▥ ÷ 900. 4500

Kevin is three times as old as his sister Karen. In four years, he will only be twice as old as Karen. How old are both children now?

More practice, page A-7, Set 13

Using the Exercises

If necessary, work through a few parts of each exercise on page 99 before assigning them as independent work. Continue to ask children for their reasons for choosing a particular estimate. Allow ample time for discussion when they finish.

Encourage those who try the *Think* problem to explain their solution to the class. Some children may need a chart similar to the one at the right to understand it.

Now		4 Years Later	
Karen	Kevin	Karen	Kevin
1	3	5	7
2	6	6	10
3	9	7	13
4	12	8	16

Assignments (page 99)
Minimum: 1. Average: 1–2.
Maximum: 1–2.

Duplicator Masters, page 16
Workbook, page 30
Skill Masters, page 16

PAGES 100–101

Objective

Given multiplication problems with factors which are multiples of 10 and 100, the child will be able to find the products by applying a simple rule.

Preparation

Since this lesson depends on the children's knowledge of basic facts, you might find it helpful to use a game as a review. For example, use the "What's My Rule" game described on page 46. However, depending on your class organization, you might choose to begin immediately with the investigation.

Investigation

For this investigation, encourage the children to study the flow chart independently and then try to make one for each of the given exercises as directed. The chart should simply be drawn on paper, as illustrated in the text. Note that for the examples given, children should first find 5×3 (or 3×5) and then multiply by the appropriate power of 10. You might give the children other examples to work, such as, 2×40, 20×400; 3×60, 30×60; and so on.

Let's explore some special products.

Investigating the Ideas

Study the **flow chart** to help you review finding products like 4×60.

Input box — Instruction boxes — Output box

Problem 4×60 → First find 4×6. → Then multiply by 10. → The product is 240.

$4 \times 60 = 24 \times 10 = 240$

Can you make your own flow chart for one of these products? See Investigation.

1. 5×30 150   2. 3×500 1500   3. 30×50 1500   4. 5×3000 15 000

Discussing the Ideas

1. Find these products.

A 7×10 70	D 46×10 460	G 37×100 3700	J 32×1000 32000
B 10×35 350	E 8×100 800	H 48×100 4800	K 1000×14 14 000
C 276×10 2760	F 100×24 2400	I 7×1000 7000	L 87×1000 87000

2. Give a simple rule for multiplying by 10. By 100. By 1000.
See Discussion.

3. Finding the product for ***a*** will help you understand a simple rule for finding ***b***.

A $3 \times 8 \times 100 = a$ 2400
$3 \times 800 = b$ 2400

B $4 \times 10 \times 7 \times 10 = a$ 2800
$40 \times 70 = b$ 2800

C $3 \times 10 \times 6 \times 100 = a$ 18 000
$30 \times 600 = b$ 18 000

D $4 \times 100 \times 7 \times 10 = a$ 28 000
$400 \times 70 = b$ 28 000

4. Give a rule for finding each type of product in exercise 3.
See Discussion.

Discussion

Notice that what the children studied in the investigation is studied in two different steps in the discussion exercises. In exercises 1 and 2, children multiply by 10, 100, and 1000, and will probably respond by explaining the shortcut of counting zeros or thinking of 48×100 as 48 hundreds, and of 1000×14 as 14 thousands. In exercises 3 and 4, they use multiples of 10, 100, and 1000. Their rule will probably relate to the example in the investigation in which the factor that is a multiple of 10, 100, or 1000 is broken apart so that basic facts can be used. Encourage children to explain how they found the numbers *a* and *b*, so that they can verbalize their rule.

Using the Ideas

1. Find the products.

A 32×10 320 C 42×100 4200 E 1000×42 42 000 G 342×100 34 200

B 63×10 630 D 30×100 3000 F 20×100 2000 H 10×675 6750

2. Give the number for a. Then give the number for b.

A $4 \times 2 \times 10 = a \rightarrow 4 \times 20 = b$ 80 80 D $8 \times 6 \times 100 = a \rightarrow 8 \times 600 = b$ 4800 4800

B $9 \times 7 \times 10 = a \rightarrow 9 \times 70 = b$ 630 630 E $6 \times 9 \times 100 = a \rightarrow 6 \times 900 = b$ 5400 5400

C $7 \times 6 \times 10 = a \rightarrow 7 \times 60 = b$ 420 420 F $8 \times 5 \times 100 = a \rightarrow 8 \times 500 = b$ 4000 4000

3. Give the number for a. Then give the number for b.

A $3 \times 10 \times 2 \times 10 = a \longrightarrow 30 \times 20 = b$ 600 600

B $4 \times 10 \times 3 \times 10 = a \longrightarrow 40 \times 30 = b$ 1200 1200

C $5 \times 10 \times 7 \times 10 = a \longrightarrow 50 \times 70 = b$ 3500 3500

D $6 \times 10 \times 2 \times 10 = a \longrightarrow 60 \times 20 = b$ 1200 1200

E $4 \times 10 \times 9 \times 10 = a \longrightarrow 40 \times 90 = b$ 3600 3600

F $5 \times 10 \times 5 \times 10 = a \longrightarrow 50 \times 50 = b$ 2500 2500

4. Find the products.

A 20×30 600 E 70×20 1400 I 70×60 4200 M 30×90 2700

B 50×30 1500 F 60×60 3600 J 70×70 4900 N 80×90 7200

C 40×50 2000 G 80×40 3200 K 70×80 5600 O 90×90 8100

D 60×50 3000 H 40×70 2800 L 80×80 6400 P 70×90 6300

5. Find the products.

1200 A 40×30 J 20×600

12 000 B 40×300 K 60×200

1800 C 30×60 L 70×300

18 000 D 30×600 M 700×30

180 000 E 300×600 N 3×7000

3500 F 50×70 O 300×700

35 000 G 50×700 P 60×800

350 000 H 500×700 Q 600×800

350 000 I 50×7000 R 60×8000

J 12 000; K 12 000; L 21 000; M 21 000 N 21 000; O 210 000; P 48 000; Q 480 000; R 480 000

think

A special number pair are we.
100 is our sum, you see.
Our product lets you
know some more.
It's our sum times
twenty-four.

More practice, page A-7, Set 14

101

Using the Exercises

Have the children do the exercises on page 101 and again give them an opportunity to explain how they found their answers. Following this discussion, you might give the children other practice problems similar to exercises 4 and 5.

The more able children will be stimulated by the *Think* problem, and, certainly, nearly all the children will be able to understand the correct answer once it is given. The children will easily see that $40 \times 60 = 2400$ and that $40 + 60 = 100$.

Assignments (page 101)
Minimum: 1–3. Average: 1–5I. Maximum: 1–5.

Mathematics

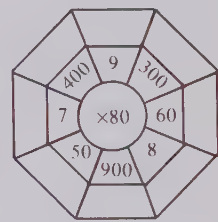
Finding products such as 70×80 and 40×300 involves repeated use of the commutative and associative principles for multiplication. The shortcuts children discover for finding such products can be easily justified by using the basic principles. Thus,

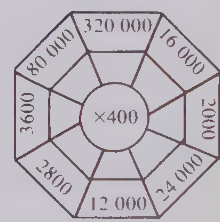
$$\begin{aligned} 40 \times 300 &= (4 \times 10) \times (3 \times 100) \\ &= (4 \times 3) \times (10 \times 100) \\ &= 12 \times 1000 \\ &= 12\ 000 \end{aligned}$$

Follow-up

Practagons may provide children with another way to review multiplication using multiples of 10, 100, or 1000. Be sure to leave some of the practagons blank and ask the children to make some practice practagons for themselves.

Find the products. Find the missing factors.





Now make some of your own.

Duplicator Masters, page 17
Skill Masters, page 17

PAGES 102–103

Objective

Given a division problem, the child will be able to find its quotient by finding a missing factor of the related multiplication problem.

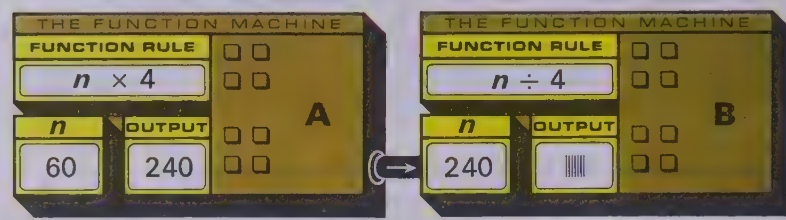
Preparation

Depending on the needs of the children and the organization of your class, spend a few minutes orally reviewing basic multiplication and division facts. You might use a variation of the "What's My Rule" game (page 46), or expressions such as: "I'm thinking of the product 70×20 (or 24×10, etc.). What's my number?"

How are special products and quotients related?

Discussing the Ideas

1. The two function machines help you see how special quotients are related to special products. What is the output number for the second machine? 60



2. You can find special quotients by thinking of missing factors. Solve the two equations.

You have found this quotient $630 \div 10 = n$ 63 when you find this factor. 63 $n \times 10 = 630$

To find $630 \div 10$, think
"What number times 10 gives 630?"

3. Solve the equations.

A $34 \times 10 = a$ 340 → $340 \div 10 = b$ 34; $340 \div 34 = c$ 10

B $100 \times 69 = a$ 6900 → $6900 \div 69 = b$ 100; $6900 \div 100 = c$ 69

C $10 \times 587 = a$ 5870 → $5870 \div 587 = b$ 10; $5800 \div 10 = c$ 587

D $7 \times 90 = a$ 630 → $630 \div 90 = b$ 7; $630 \div 7 = c$ 90

E $6 \times 80 = a$ 480 → $480 \div 80 = b$ 6; $480 \div 6 = c$ 80

F $80 \times 40 = a$ 3200 → $3200 \div 40 = b$ 80; $3200 \div 80 = c$ 40

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Discussion

The main point in this lesson is to help children use the inverse relation of multiplication and division in order to solve division equations by thinking of missing factors. As you work through discussion exercise 1, study the two function machines with the children. Notice with them that we start with 60, multiply by 4, then feed this product into the second function machine, and divide by 4. This gives us the number with which we started. After you discuss exercise 2, give the children other examples, showing the two related division equations for each multiplication equation you discuss, as in exercise 3. After working through exercise 3, you might exhibit a division equation such as $350 \div 70 = n$ and ask children to give a related multiplication equation and explain how finding a missing factor will help them solve the division equation.

Using the Ideas

1. Solve the equations.

56 **A** $a \times 10 = 560 \rightarrow 560 \div 10 = b$ 56
10 **B** $a \times 48 = 480 \rightarrow 480 \div 48 = b$ 10
72 **C** $a \times 10 = 720 \rightarrow 720 \div 10 = b$ 72
56 **D** $a \times 100 = 5600 \rightarrow 5600 \div 100 = b$ 56

think

Find 40 times me.
Divide the product by 4.
It's 700 you'll see.
I will tell no more.

2. Give the number for a.
Then give the number for b.

6 **A** $a \times 40 = 240 \rightarrow 240 \div 40 = b$ 6
40 **B** $a \times 9 = 360 \rightarrow 360 \div 9 = b$ 40
7 **C** $a \times 70 = 490 \rightarrow 490 \div 70 = b$ 7
9 **D** $a \times 300 = 2700 \rightarrow 2700 \div 300 = b$ 9
800 **E** $a \times 6 = 4800 \rightarrow 4800 \div 6 = b$ 800
800 **F** $a \times 7 = 5600 \rightarrow 5600 \div 7 = b$ 800

3. Give the quotients.

A $28 \div 7$ 4	**E** $280 \div 4$ 70	**I** $5400 \div 900$ 6	**M** $42\,000 \div 7000$ 6
B $280 \div 7$ 40	**F** $560 \div 70$ 8	**J** $36\,000 \div 6$ 6000	**N** $4800 \div 6$ 800
C $2800 \div 7$ 400	**G** $3600 \div 400$ 9	**K** $1000 \div 10$ 100	**O** $400 \div 8$ 50
D $280 \div 70$ 4	**H** $6300 \div 7$ 900	**L** $810 \div 90$ 9	**P** $5600 \div 800$ 7

4. Solve the equations.

50 **A** $a \times 30 = 1500 \rightarrow 1500 \div 30 = b$ 50
70 **B** $a \times 50 = 3500 \rightarrow 3500 \div 50 = b$ 70
70 **C** $a \times 60 = 4200 \rightarrow 4200 \div 60 = b$ 70
70 **D** $a \times 40 = 2800 \rightarrow 2800 \div 40 = b$ 70
30 **E** $a \times 70 = 2100 \rightarrow 2100 \div 70 = b$ 30
60 **F** $a \times 90 = 5400 \rightarrow 5400 \div 90 = b$ 60
40 **G** $a \times 80 = 3200 \rightarrow 3200 \div 80 = b$ 40

5. Give the quotients.

A $2400 \div 60$ 40	**H** $4800 \div 60$ 80
B $1600 \div 40$ 40	**I** $3600 \div 90$ 40
C $2700 \div 30$ 90	**J** $1800 \div 20$ 90
D $1400 \div 70$ 20	**K** $6300 \div 70$ 90
E $1000 \div 50$ 20	**L** $7200 \div 80$ 90
F $2400 \div 80$ 30	**M** $8100 \div 90$ 90
G $1200 \div 30$ 40	**N** $3200 \div 40$ 80

More practice, page A-8, Set 15

Using the Exercises
Have the children do the exercises on page 103 independently. When they finish and the papers have been checked, discuss any points which gave the children difficulty. If time permits, you might provide the children with oral practice in finding some of the special products and quotients of this lesson.

Resources for Active Learning
Mathematics in Modules, S5, Addison-Wesley.

Assignments (page 103)
Minimum: 1, oral; 2–3.
Average: 1–4. Maximum: 1–5.

Duplicator Masters, page 18
Workbook, page 31
Skill Masters, page 18

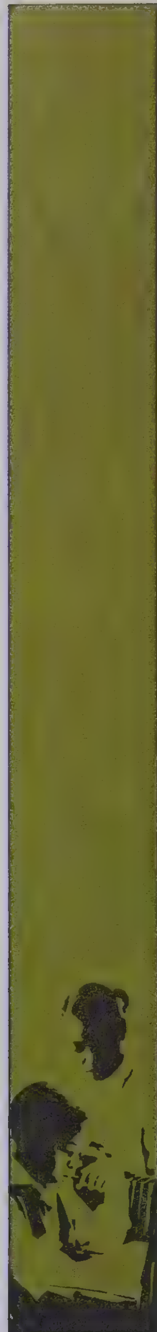
PAGES 104–105

Objective

The child will be able to use multiples of 10 or 100 to estimate the answers for given problems and equations.

Preparation

You might refer to the marbles or peas in the jar suggested in the first lesson of this chapter and ask children to use multiples of ten to tell how many objects the jar contains. Some of them might simply round the previous guesses they made. Others might build an estimate by counting areas in groups of tens or hundreds. During this preparation, use words such as *about, estimate,* and *exact,* when appropriate.



Let's explore estimation.

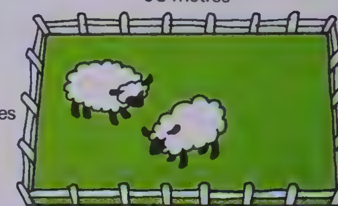
Discussing the Ideas

Each problem below requires an **estimate**. In order to estimate the answers to problems such as these, it is helpful to add, subtract, multiply, or divide, using multiples of 10 or 100 that are "close" to the numbers in the problems.

1. Suppose you purchase items costing the amounts shown. Without finding the exact total amount, how would you decide quickly whether or not you could pay the bill with $20? See Discussion.

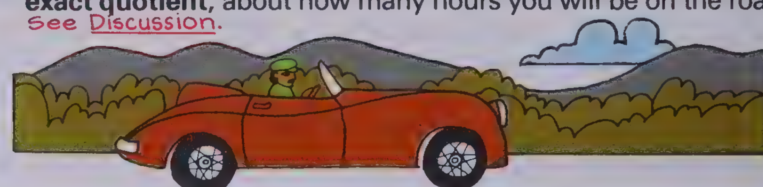
$1.98
3.49
6.52
4.98
1.06

2. Suppose you were going to plant clover on a plot of ground that is 98 metres long and 56 metres wide. To decide how much grass seed to buy, you must find the area of the plot of ground. How could you quickly find **about** how many square metres of ground you have? See Discussion.



3. Suppose that there are 9026 people in a city and that 2987 of them are men. How could you quickly find, without finding the exact difference, **about** how many women and children live in this city? See Discussion.

4. Suppose you are driving 327 kilometres. If you travel 81 kilometres each hour, how could you tell, **without finding the exact quotient,** about how many hours you will be on the road? See Discussion.



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Discussion

Encourage all the children to participate in arriving at the solutions to the discussion exercises. You might point out to the children that one of the easiest ways to make estimates is to think about numbers that are multiples of 10 or 100 that are closest to the actual numbers and to do the computing with these estimates. Expect them to use this kind of thinking in answering exercise 1: "The first amount is about $2; the next two amounts total about $10; and these add up to $12. The next amount is about $5, which gives us $17; and the last amount is about $1, or about $18. Therefore, we spent less than $20, and we could pay the bill with $20."

For exercise 2, you should draw from the children the fact that they can estimate this area by multiplying 100 × 56. Hence, a good estimate would be 5600 square metres.

The children should see that they can give a quick estimate for exercise 3 by subtracting 3000 from 9000, which would mean that there are approximately 6000 women and children in the city. For exercise 4, the children should estimate the number of hours by dividing 80 into 320 to get an answer of about four hours.

Using the Ideas

Write an equation that shows how you estimate the answer for each exercise. Use multiples of 10 for exercise 1 and multiples of 100 for exercise 2.

1. A $19 + 18 + 42$
Answer:
$20 + 20 + 40 = 80$
B 45×79
Answer:
$50 \times 80 = 4000$
C $48 + 51 + 47$
See Answers, T.E. page 105.
D $29 + 68 + 97$
E $323 + 639$
F $163 - 88$
G $361 \div 39$
H 19×48
I 78×57
J $265 \div 91$
K $104 - 48$
L 45×65
M 36×34
N $548 \div 9$
O 403×69
P 98×11
Q $526 - 109$

2. A $387 + 416 + 721$
Answer:
$400 + 400 + 700 = 1500$
B $5617 \div 811$
Answer:
$5600 \div 800 = 7$
C $519 + 787$
D $706 - 289$
E 95×116
F $2431 \div 590$
G $5489 - 2496$
H $409 + 688 + 716$
See Answers, T.E. page 105.
I $1607 - 569$
J $2050 \div 271$
K 97×683
L 667×123
M $2348 + 653$
N $543 + 553$
O $4029 \div 817$
P $5359 \div 94$
Q 350×738
R $5327 - 1965$
S $3276 + 3341$
T 651×749

3. **Estimate** the missing factor. Then **estimate** the quotient.
A $n \times 28 = 308$ 10
$308 \div 28 = n$ 10
B $n \times 31 = 1519$ 50
$1519 \div 31 = n$ 50
C $n \times 97 = 5917$ 60
$5917 \div 97 = n$ 60

think

I'm ninety and a little more.
My partner is a little less.
You miss our product by just 4
When 90 nineties is your guess.

WHO ARE WE?
92, 88

105

Answers, exercises 1 and 2, page 105

1.C $50 + 50 + 50 = 150$
D $30 + 70 + 100 = 200$
E $320 + 640 = 960$
F $160 - 90 = 70$
G $360 \div 40 = 9$
H $20 \times 50 = 1000$
I $80 \times 60 = 4800$
J $270 \div 90 = 3$
K $100 - 50 = 50$
L $50 \times 70 = 3500$
M $40 \times 30 = 1200$
N $550 \div 10 = 55$
O $400 \times 70 = 28\ 000$
P $100 \times 10 = 1000$
Q $530 - 110 = 420$
2.C $500 + 800 = 1300$
D $700 - 300 = 400$
E $100 \times 100 = 10\ 000$
F $2400 \div 600 = 4$
G $5500 - 2500 = 3000$
H $400 + 700 + 700 = 1800$
I $1600 - 600 = 1000$
J $2100 \div 300 = 7$
K $100 \times 700 = 70\ 000$
L $700 \times 100 = 70\ 000$
M $2300 + 700 = 3000$
N $500 + 600 = 1100$
O $4000 \div 800 = 5$
P $5400 \div 90 = 60$
Q $400 \times 700 = 280\ 000$
R $5300 - 2000 = 3300$
S $3300 + 3300 = 6600$
T $700 \times 700 = 490\ 000$

Using the Exercises

The exercises on page 105 give the children further practice in using multiples of 10 and 100 to estimate answers for equations which employ the basic operations. Have the children work them independently using the sample solutions as guidelines, if necessary.

The *Think* problem should challenge your most able children. You may need to provide hints by telling the children that they should try multiplying two numbers, one of which is a little more than 90 and the other a little less than 90. They should discover soon that 88×92 is a number that is 4 less than 8100.

Assignments (page 105)
Minimum: 1A–J, 2A–L.
Average: 1–2. Maximum: 1–3.

PAGES 106–107

Objective

Given a variety of estimation projects, the child will demonstrate his ability and understanding of estimation by completing these projects or problems.

Preparation

To prepare for this lesson, you might simply review the main points treated so far in this chapter. Point out the meaning and usefulness of making estimates. Review the skill of rounding numbers to the nearest multiples of 10 or 100 and the convenience of using these multiples in figuring out problems and equations. Explain to the children that in this investigation they will have a chance to use their understanding and skills by working on an estimation project.

Investigation

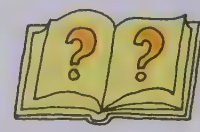
For this investigation, it would be best for children to work in small groups. You might choose to give each group a project, so that all projects will receive some treatment and results may be shared later. However, each group that works on a project may have different ways of making the estimates, so you may want the children to choose any project of their liking. Remind the children to apply their estimating skills wherever possible. For example, in exercise 1, they might first estimate the number of problems on a page and then the number of pages in the book. Throughout each step, multiples of 10 and 100 would be applicable. For exercise 2, children may need a scale, and for exercise 3, they may need a clock or watch.

Let children use their own ingenuity in figuring out where to find the information for these investigations, but have it available for them when they need it.

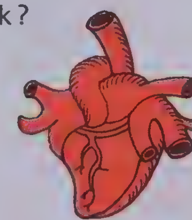
Let's use our estimating skills.

Investigating the Ideas

Choose one of these estimation projects.

1. 
How many problems are in your mathematics book?

2. What is the total weight of all the children in your classroom?

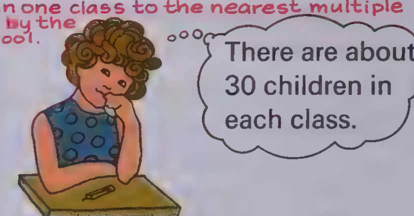
3. How many times does your heart beat in a year?

Can you give an estimate and write an explanation of how you made your estimate? See Investigation.

Discussing the Ideas

1. A Estimate the number of children in your school. Estimates will vary.
   B Explain how you made your estimate. Sample answer: Round the number of children in one class to the nearest multiple of 10, and multiply that number by the number of classes in the school.

2. In Della's school there are 4 fifth-grade classes. What should be Della's estimate for the number of children in the fifth grade?
$4 \times 30 = 120$



3. If there are 1760 bricks in 1 pile, which is the best estimate of the number of bricks in 5 similar piles? B
   A 5000 bricks B 9000 bricks C 12 000 bricks

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Discussion

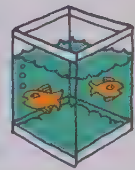
In the discussion section, it is intended that children share their ideas of ways of estimating the answers. If you wish, you may simply use the discussion exercises as further investigations for the children. Throughout the discussion, continue to stress the use of multiples of 10 or 100.

For those who finish a project early or for more variety, you might have children try other projects similar to the following.

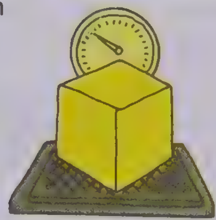
1. How many pieces of tablet or binder paper would be needed to reach a stacked height of 1 metre? 10 metres? 100 metres?
2. How many jars like the one exhibited in the classroom would be needed to hold 10 000 marbles (or peas)? 1 000 000 marbles (or peas)?

Using the Ideas

Choose the best estimate from the three estimates given for each problem.

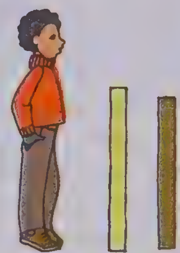


1. One tank of water can hold about 12 fish. Choose the best estimate for the number of fish 88 tanks would hold: C
   A 800 fish B 100 fish C 900 fish



2. A piece of gold weighs about 19 times as much as a piece of ice the same size. If an ice cube weighs 62 g, choose the best estimate for the weight of a similar block of gold. C
   A 80 g B 1000 g C 1200 g

3. A piece of aluminum weighs about 169 grams. A lead pipe weighs 850 grams. Estimate the difference in the weights: B
   A 600 grams B 700 grams C 800 grams



4. The distance between the posts is about 39 centimetres. Estimate the distance in centimetres between 72 such posts: A
   A 2800 cm B 2100 cm C 280 cm

5. One truck holds 36 barrels. Estimate the number of trucks needed to hold 278 barrels: A
   A 7 B 9 C 80

6. A car travelling 96 kilometres per hour is travelling about 27 metres per second. Estimate the distance travelled in 55 seconds: C
   A 4000 metres B 1 kilometre C 1400 metres

7. Sound travels 322 metres per second in air. Estimate how many seconds it takes sound to travel one kilometre in air: B
   A 2 B 3 C 4

More practice, page A-8, Set 16

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Follow-up
Gather together children who still need help in rounding to the nearest multiple of 10 or 100. Furnish them with a demonstration number line showing points for tens, but labels for fifties and hundreds only. Give each child a worksheet of problems similar to those on text page 97. Encourage the children to locate the numbers on the number lines on their papers, or to use the larger demonstration number line if they are unable to decide how to round the numbers.

Ultimately, the children should be able to verbalize, at least roughly, the generalization that they will round *down* to the nearest multiple of 10 if the ones' digit is 4 or less, and round *up* to the nearest multiple of 10 if the ones' digit is 6 or more (and they should see that a similar generalization holds for multiples of 100). The children should also see that the agreement on handling numbers exactly halfway between multiples is arbitrary, but they should decide on some explicit way of handling them.

Resources for Active Learning
Applied Mathematics Cards, "Estimating Length . . .," Group 1/8-10, Schofield and Sims. (Available from Mafex Associates, Willowdale)

Duplicator Masters, page 19

Using the Exercises
Assign the exercises on page 107 as independent work. When the children have finished, allow them time to share the ways in which they chose their estimates.

Assignments (page 107)
Minimum: 1–2, oral; 3–5.
Average: 1–2, oral; 3–7.
Maximum: 1–7.

PAGES 108–109

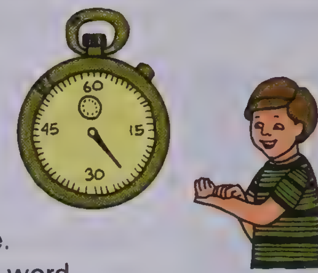
Objective

Given a variety of problems involving estimation, the child will demonstrate and practice his estimating skills.

Preparation

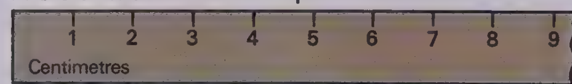
Since this lesson should be treated with a light touch, you might want to use the preparation time for a review of skills with multiples of 10 or 100. You might provide the children with some oral work in rounding numbers ("274 is nearest what multiple of 10?") or in finding products of multiples of 10 or 100.

Estimation for Fun



1. Count the number of times your heart beats in one minute. Estimate the number of beats in one hour.
Estimates will vary.
2. One of the longest words in the Oxford Dictionary is shown below. Count the numbers of letters in a 1-centimetre space. Then estimate the number of letters in the word.

floccinaucinihilipilification

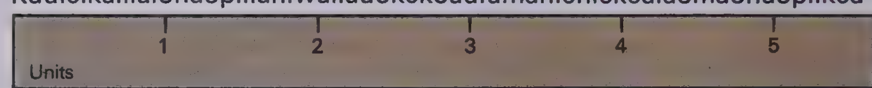


(The Oxford Dictionary defines this word as "the action of estimating as worthless.")

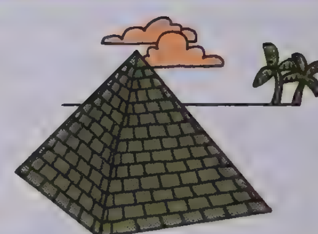
30 is a good estimate.

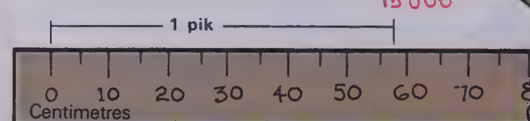
3. One of the longest names known is that of a Hawaiian girl. Estimate the number of letters in her name. 60 is a good estimate.

Kuuleikailialohaopiilaniwailauokekoaulumahiehiekealaomaonaopiikea

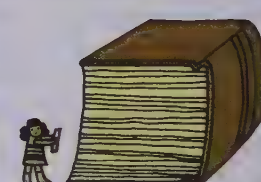


4. The picture shows the length of a unit, called a **pik**, which is used in Egypt. The pyramid of Cheops, near Cairo, was originally about 253 piks high. Estimate the height of the pyramid in centimetres.
15 000





★ 5. Use your mathematics book and your ruler to estimate the number of pages in a book 3 metres thick.
See Discussion.



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Discussion

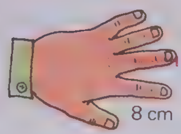
Since these problems are intended to be done independently, you might save any discussion until the children finish them. Keep in mind that an important feature of this type of lesson is the interest that can be generated by allowing the children to digress from purely mathematical discussion. For example, some children might know other facts about pyramids that they would like to relate during the discussion of exercise 4.

Note that exercise 4 introduces a new unit of measure called a *pik*. The exact length of a pik is 58 centimetres. You can expect a wide variety of answers for exercise 5, since the children's estimates of the thickness of the book will vary. Children might use one centimetre or one and a half centimetres as the estimate for the thickness of the mathematics book (excluding the cover) and 300 or 400 as the estimate for the number of pages. Thus, answers may vary from as many as 120 000 pages to 60 000 pages.

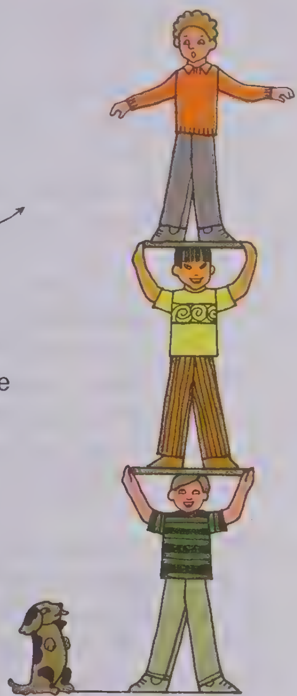
6. It takes about 48 marbles (of approximately 1 cm diameter) to cover the bottom of a litre milk carton. Estimate the number of marbles the carton holds when it is filled to the red arrow. 1000

7. The height of a full-grown human is about 21 times the length of his middle finger.
   - A Estimate the height of a person whose middle finger is the length shown. About 160 cm
   - ★ B Estimate the heights of a tall adult and a short adult by measuring the lengths of their middle fingers. Check your estimates by finding their correct heights.

   Estimates will vary.



8. Measure the height (in centimetres) of three of your classmates.
   - A Estimate the height of a tower formed by the three classmates as shown.
   - B If all the children in your class formed a tall tower, about how many metres high would it be? Estimates will vary.



★ 9. Make the following estimates and then compare your answers with those of your classmates.
   - A Estimate the number of litres of milk you drink in a week, a month, and a year.
   - B Estimate in kilograms the total weight of all the children in your class.
   - C Estimate in metres and in kilometres the distance you walk in one week.
   - D Estimate in square metres the area of your classroom floor.

   Estimates will vary.

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Assignments (pages 108–109)
Minimum: 1–4. Average: 1–7.
Maximum: 1–9.

Follow-up
Encourage children to extend any ideas the estimation problems might inspire. Some may make up task cards with estimation problems they found in other books or invented themselves. Others may want to study lengths of various words or other interesting facts about pyramids, such as dimensions, shape, and time required for construction.

Resources for Active Learning
Math Activity Cards, "Leaves," C38, Macmillan.
Maths Mini-lab, Cards 139, 141 (a game), Selective Educational Equipment.

Workbook, page 32

PAGES 110–111

Objective

The child will demonstrate his ability to work with the concepts presented in this chapter.

Preparation

It would be helpful to review rounding numbers and using them to find estimates for exercises in the basic operations. However, keep the review brief. All children should not be expected to master the estimating skills developed in this chapter, so treat the estimating ideas on page 111 with a light touch and provide the children with a variety of experiences in estimation.

Reviewing the Ideas

1. Find the products.

A 10 × 10 100 B 10 × 100 1000 C 100 × 100 10 000 D 10 × 1000 10 000

2. Find the products.

A 10 × 6 60 C 10 × 28 280 E 6 × 100 600 G 18 × 100 1800 I 100 × 28 2800

B 17 × 10 170 D 10 × 35 350 F 100 × 9 900 H 27 × 100 2700 J 54 × 100 5400

3. Give the number for ***a***.
Then give the number for ***b***.

A 40 × 3 × 10 = ***a*** 1200 → 40 × 30 = ***b*** 1200

B 50 × 7 × 10 = ***a*** 3500 → 50 × 70 = ***b*** 3500

C 90 × 6 × 10 = ***a*** 5400 → 90 × 60 = ***b*** 5400

D 70 × 9 × 10 = ***a*** 6300 → 70 × 90 = ***b*** 6300

4. Find the products.

A 30 × 30 900 F 80 × 50 4000

B 40 × 20 800 G 90 × 6 540

C 60 × 10 600 H 10 × 90 900

D 4 × 70 280 I 90 × 90 8100

E 70 × 40 2800 J 7 × 30 210

5. Find the quotients.

A 900 ÷ 30 30 F 4000 ÷ 50 80

B 800 ÷ 40 20 G 540 ÷ 6 90

C 600 ÷ 10 60 H 900 ÷ 90 10

D 280 ÷ 7 40 I 8100 ÷ 90 90

E 2800 ÷ 40 70 J 210 ÷ 30 7

think

Copy the last six equations and give the numbers covered by the red screens.

1, 2, 3 1 × 3 = (2 × 2) − 1
2, 3, 4 2 × 4 = (3 × 3) − 1
3, 4, 5 3 × 5 = (4 × 4) − 1
4, 5, 6 4 × 6 = (5 × 5) − 1
6 5 × 7 = (▮ × ▮) − 1
8 7 × 9 = (▮ × ▮) − 1
10 9 × 11 = (▮ × ▮) − 1 = ▮ 99
20 19 × 21 = (▮ × ▮) − 1 = ▮ 399
50 49 × 51 = (▮ × ▮) − 1 = ▮ 2499
99 × 101 = (▮ × ▮) − 1 = ▮ 9999
100

6. Find the products and quotients.

A 50 × 300 15 000 E 800 × 50 40 000 I 60 × 90 5400 M 5400 ÷ 90 60

B 70 × 400 28 000 F 90 × 400 36 000 J 3000 ÷ 6 500 N 40 000 ÷ 80 500

C 600 × 40 24 000 G 600 × 70 42 000 K 28 000 ÷ 70 400 O 42 000 ÷ 70 600

D 50 × 600 30 000 H 80 × 900 72 000 L 15 000 ÷ 300 50 P 81 000 ÷ 90 900

Discussion

Assign exercises from both pages for independent work. Page 110 stresses the products and quotients of multiples of 10 and 100. Page 111 stresses the estimating skills. When children have finished, allow ample time for checking their work and discussing their answers. Encourage children to give reasons for their answers, particularly for the exercises on page 111.

The pattern in the *Think* problem on page 110 may challenge the children, but once they recognize it they should be able to fill in the screens.

The *Think* problem on page 111 involves estimation, so ask those who solve it to explain how they estimated their answers.

7. Give the multiple of 10 that is closest to each number.
A 39 40 C 67 70 E 866 870
B 41 40 D 134 130 F 1068 1070

8. Give the multiple of 100 that is closest to each number.
A 509 500 C 86 100 E 6945 6900
B 197 200 D 1367 1400 F 5999 6000

think

How many 1-digit numbers are there?
How many 2-digit numbers are there?
Estimate the number of 3-digit numbers.

9. Write an equation that shows how to estimate the answer for each exercise. Use multiples of 10.
A $39 + 78$ $40+80=120$ D 27×52 $30 \times 50 = 1500$ G 14×98 $10 \times 100 = 1000$ J $801 - 98$ $800-100=700$
B $397 + 88$ $400+90=490$ E $423 \div 58$ $420 \div 60 = 7$ H 65×35 $70 \times 40 = 2800$ K 48×54 $50 \times 50 = 2500$
C $503 - 59$ $500-60=440$ F 85×46 $90 \times 50 = 4500$ I $719 \div 83$ $720 \div 80 = 9$ L $561 \div 72$ $560 \div 70 = 8$

10. Write an equation that shows how to estimate the answer for each exercise. Use multiples of 100.
A $513 + 978$ $500+1000=1500$ C 92×112 $100 \times 100 = 10\ 000$ E $6729 - 3687$ $6700-3700=3000$
B $1503 - 498$ $1500-500=1000$ D $3543 \div 726$ $3500 \div 700 = 5$ F $4864 \div 697$ $4900 \div 700 = 7$

11. Ironwood is one of the heaviest types of wood. It weighs about 1450 grams per cubic decimetre. Balsa is one of the lightest types of wood. It weighs about 125 grams per cubic decimetre.
A Estimate the weight of a cubic metre of ironwood. 1 450 000 g or 1450 kg
B Estimate the weight of a cubic metre of balsa wood. 125 000 g or 125 kg

12. Some large cars are nearly 6 metres long. One of the largest buses ever built was $17\frac{1}{2}$ metres long. Estimate the number of cars that could fit into a space long enough for the bus. 3

★ 13. One of the longest bicycles ever built was a 10-seater built in 1898. It was 763 centimetres long. Estimate the difference between the length of this bicycle and the total length of 10 ordinary bicycles, each 175 centimetres long. 1200 cm is a good estimate (2000 - 800)

111

Follow-up

Children might enjoy working with a function-machine game such as the following.

Use this notation: $\overline{n}$ means round up to the given multiple; $\underline{n}$ means round down to the given multiple; $\boxed{n}$ means round to the nearest given multiple.

Sample Tables

Function Rule: $\overline{n}$ to multiple of 10

n	Output
18	20
37	40
92	100
49	
51	
68	
23	

Function Rule: $\underline{n}$ to multiple of 100

n	Output
150	100
534	500
372	300
689	
426	
708	
789	

Function Rule: $\boxed{n}$ to multiple of 100

n	Output
160	200
543	500
475	500
869	
624	
718	
987	

Function Rule:

n	Output
73	70
89	90
401	400
379	380
123	
681	
239	

Workbook, page 33

PAGES 112–113

Objective

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

If you have been conducting any contests involving estimation, such as guessing the number of items in a container, you may want to use this preparation period to determine and discuss the best answers.

Keeping in Touch with Addition, Subtraction, Multiplication, Division, Place value

1. Solve the equations.

A $9 \times 6 = n$ 54 D $63 \div 9 = n$ 7 G $72 = n \times 8$ 9 J $8 = 56 \div n$ 7
B $8 + 7 = n$ 15 E $50 = n \times 10$ 5 H $n = 54 \div 9$ 6 K $49 \div 7 = n$ 7
C $16 - 7 = n$ 9 F $67 = n + 47$ 20 I $n = 9 \times 7$ 63 L $n \times 10 = 90$ 9

2. Find the products.

A 7×4 28 D 5×8 40 G 7×5 35 J 8×8 64 M 5×9 45
B 4×9 36 E 9×8 72 H 7×9 63 K 6×7 42 N 7×7 49
C 8×6 48 F 3×7 21 I 8×4 32 L 9×6 54 O 7×8 56

3. Solve the equations.

A $837 = 800 + n + 7$ 30 D $864 = 800 + 50 + n$ 14
B $973 = n + 70 + 3$ 900 E $358 = 200 + n + 8$ 150
C $796 = n + 90 + 700$ 6 F $592 = n + 190 + 2$ 400

4. Solve the equations.

A $843 = (n \times 100) + 43$ 8 H $803 = (80 \times 10) + n$ 3
B $843 = (n \times 10) + 3$ 84 I $803 = (79 \times 10) + n$ 13
C $369 = (n \times 100) + 69$ 3 J $700 = (70 \times 10) + n$ 0
D $369 = (n \times 10) + 9$ 36 K $700 = (69 \times 10) + n$ 10
E $4582 = (4 \times n) + 582$ 1000 L $3005 = (300 \times 10) + n$ 5
F $4582 = (45 \times n) + 82$ 100 M $3005 = (299 \times 10) + n$ 15
G $4582 = (458 \times n) + 2$ 10 N $3023 = (29 \times 100) + n$ 123

5. Solve the equations.

A $3 \times 57 = (3 \times 50) + (3 \times n)$ 7 C $5 \times 84 = (5 \times n) + (5 \times 4)$ 80
B $4 \times 36 = (n \times 30) + (n \times 6)$ 4 D $3 \times 24 = (3 \times n) + (3 \times 4)$ 20
E $3 \times 124 = (3 \times n) + (3 \times 20) + (3 \times 4)$ 100
F $3 \times 5124 = (3 \times n) + (3 \times 100) + (3 \times 20) + (3 \times 4)$ 5000

You are invited to explore

ACTIVITY CARD 5
Page 335

112

Discussion

Assign the exercises on page 112 as independent work. Review any topic which may have caused difficulty previously and discuss any questions the children may have. In review pages such as these, it is important to check the papers carefully and correct any misunderstandings the children may have.

Sequoia Trees

The giant sequoia trees have been said to be the "oldest and largest living things on earth." Some are from 3000 to 4000 years old. One of the largest sequoias, called the "General Sherman," is 83 metres tall. Another type of sequoia, called a redwood tree, grows taller but not as large. A tree called the "New Tree" is one of the tallest of the redwoods. It is 112 metres tall.

1. The Toronto Dominion Centre is 219 metres tall. Find the difference between the height of the building and the height of the "New Tree."
107 m

2. How much taller is the "New Tree" than the "General Sherman"?
29 m

3. Is the Toronto Dominion Centre 2 times as tall as the "New Tree"?
Almost

4. It takes 17 men with outstretched arms to encircle the "General Sherman." Suppose a man with outstretched arms can reach 183 cm. Give the circumference of the "General Sherman."
3111 cm

5. The "General Sherman" contains enough wood to build 35 small five-room houses. Find the number of houses that could be built with the lumber from 24 such trees. 840

6. The oldest dated giant sequoia was 3212 years old when it was cut down in 1892. Find the date when this tree started to grow.
1320 B.C.

113

Using the Exercises

On page 113, read and discuss the first paragraph with the children. Allow the children an opportunity to discuss sequoia trees briefly and then have them do the exercises.

During the discussion of these exercises, keep in mind that one of the most important aspects of word problems is the interest they stimulate within the children. If the children show interest in this particular lesson, you might encourage them to do further research about sequoia and redwood trees and then report on them to the class.

Follow-up

Children may be motivated to prepare reports on other natural phenomena such as heights of waterfalls or depths of oceans. World almanacs or the *Book of Facts* would be useful resources for this type of research.

Pages 114–151

General Objectives

To develop skill in working with the addition and subtraction algorithms

To review the multiplication algorithm and to develop skill in its use

To review and extend ideas and skills involving division with 1-digit divisors

To prepare the child for work in division using 2-digit divisors

To provide a variety of word problem experiences

The initial pages of this chapter are devoted to reviewing and extending work with the addition and subtraction algorithms. Accompanying these pages of practice and extension of the algorithms are several sets of appropriate word problems. Following this, the multiplication algorithm is reviewed and extended. This, too, is accompanied by appropriate word-problem sets. This review and extension of the addition, subtraction, and multiplication algorithms constitutes about one half of the chapter. The rest of the chapter is devoted to work with division with single-digit divisors and some preparatory work with larger divisors. However, the complete development of the division algorithm using 2-digit divisors is deferred until Chapter 7.

The degree of emphasis you will need to place on the early portions of the chapter will depend, of course, upon the background and ability of your children. It is quite likely that you will want to place the greater emphasis on the work with division, so that your children will have the skills needed to work efficiently with the division algorithm in the following chapter.

Mathematics

Most of the work with the addition algorithms involves no new mathematical concepts. Rather, this development relies primarily on mathematical concepts which were developed earlier in the program. For example, in the addition algorithm the chief mathematical concepts are those concerning use of the associative and commutative principles. Of course, it is these principles, and the generalization from these principles (that of rearranging addends), which allow us complete freedom when adding in columns.

The most important principle in the multiplication algorithm is that of distributivity. In the multiplication algorithm, we are often confronted with the need to find partial products and then add these products to get the final product. This procedure is very much dependent upon the use of the distributive principle.

Our basic definition for division (if $a \times b = c$, and $b \neq 0$, then $a = c \div b$) must be extended to allow for remainders in division. Thus, we must define $c \div b$ for the case in which there is no whole number a such that $a = c \div b$. There are two alternatives: we can reconsider the definition above without the restriction that a, b, and c are whole numbers, or we can redefine division to allow for nonzero remainders. Consider the following definition.

> If a, b, c, and d are whole numbers such that $(a \times b) + d = c$, and $b \neq 0$ and $d < b$, then for $c \div b$, a is the quotient and d is the remainder.

Notice in this definition that, if $d = 0$, we have the original definition. In other words, $a \times b = c$ simply implies $c \div b = a$. Of course, if we wanted to make a statement of equality with regard to $c \div b$ when $d \neq 0$, fractional numbers would be involved, and we would have

$$c \div b = a + \frac{d}{b}.$$

An example of this would be $14 \div 3$.

$$\begin{array}{r} 4 \\ 3\overline{)\,14} \\ \underline{12} \\ 2 \end{array} \qquad 14 \div 3 = 4 + \tfrac{2}{3}, \text{ or } 4\tfrac{2}{3}$$

When we develop fractional numbers in future work, we can return to this idea and establish that nonzero remainders give a quotient which is not a whole number.

You will note that we are considering two different ways of thinking about quotients: (1) as whole numbers with a number called the remainder related to the quotient and (2) as fractional numbers such that the quotient times the divisor equals the dividend in each case. The latter case will be explored in Book 6.

Teaching the Chapter

Materials

Atlases, almanacs, and other reference books
Catalogues and advertisements
Colored strips
Crayons
Graph paper (1-cm grid)
Maps (highway or airline maps, if possible)
Watch with second hand

Vocabulary

average	rate
distance	regrouping
expanded notation	time

The short materials and vocabulary lists for such an extensive chapter as this reflect the fact that it includes little new mathematical content. This is a chapter devoted primarily to the development of skills and the utilization of mathematical concepts which have been developed previously.

Lesson Schedule

In a chapter as long as Chapter 6, it is easy to let the time schedule become too flexible. Because a

large amount of material is covered and skills are emphasized, it might be tempting to devote too much time to this work. Some two-page lessons are obviously a fairly easy day's work, while others clearly will require more than a single day. There may even be areas in the chapter where you will choose to cover more than two pages in a single day. However, as a general rule, you will find that the work is sufficiently intense that you will prefer to cover no more than two pages per day in order that you may devote considerable time to explanatory development. In some instances, you may choose to spend two, or even three, days on a single lesson. For average classes, five to six weeks should be sufficient time for this chapter.

Evaluation of Progress

Assessing children's progress and achievement in a chapter which emphasizes skills is one of the easier tasks of evaluation in the program. It is relatively easy to determine whether or not the children are able to work with a given algorithm. To test whether or not they understand the algorithm is another matter. Naturally, we feel that it is extremely important that the children understand the algorithms, and, certainly, such understanding facilitates mastery of any algorithm. We therefore suggest that you evaluate understanding of the algorithms on a day-to-day basis, and conduct written tests to evaluate whether or not the children are actually able to perform the necessary computations. Children's ability to work with word problems should be part of your considerations in evaluation.

The chapter review on pages 148–149 can be used either as a review or reserved for use as an instrument of evaluation. Pages 150–151 provide a cumulative review to help you check the children's retention of basic skills and concepts.

Resources for Active Learning

GENERAL REFERENCES

A Cloudburst, Vol. 2, No. 1353, Midwest Publications

A Cloudburst, Vol. 2, Nos. 5914–5964, Midwest Publications [Pressure/density problems in division]

Discovery, Section II, Units 11/4,5; 12/4,5; 18/5, Encyclopaedia Britannica Educational Corp.

Nuffield Project: *Problems*—Red Set, No. 6, Wiley

MANIPULATIVE DEVICES

Papy Minicomputer (Macmillan)

SEE Calculator (Selective Educational Equipment)

Ten-Inch Slide Rule (Selective Educational Equipment)

COMMERCIAL GAMES

Equations (Creative Publications; Wff 'N Proof)

Heads Up (Creative Publications; Hammett; Math Media)

Imout (Imout)

Krypto (Creative Publications; Edmund Scientific)

Numble (Hammett)

Orbiting the Earth (Scott Foresman)

Playing Card Number Games—Whole Numbers (Heath)

Quinto (Hammett; Selective Educational Equipment)

Real Numbers Game (Wff 'N Proof)

Sum Times (Hammett; Selective Educational Equipment)

TUF (Creative Publications; Cuisenaire Co.; TUF)

Twin Choice (Holt, Rinehart and Winston)

The Winning Touch (CCM School Materials; school supplier)

PAGES 114–115

Objective

Given addition problems with addends of 2, 3, 4, or 5 digits, the child will test his ability to find the sums by using the addition algorithm.

Preparation

To prepare for this lesson, you might provide a short oral review of basic addition facts. Give the children three addends less than 9 and ask them for the sum. For example, say: "7 + 8, add 5. What's your answer?" Or: "3 + 8, add 6 more. What's your answer?" If the children are able, have them do these exercises without paper and pencil. They should try to remember partial sums in their heads as they go along.

Investigation

An investigation of this kind is most effective if the children work on it independently. This investigation is actually a little self-test which enables children to see how much they recall about adding when use of the addition algorithm is necessary. Stress the fact that the investigation's purpose is to have the children find out for themselves what they need to learn; it is not meant to enable anyone else to judge their work. Make sure they understand how to use the code correctly so that their corrections are valid.

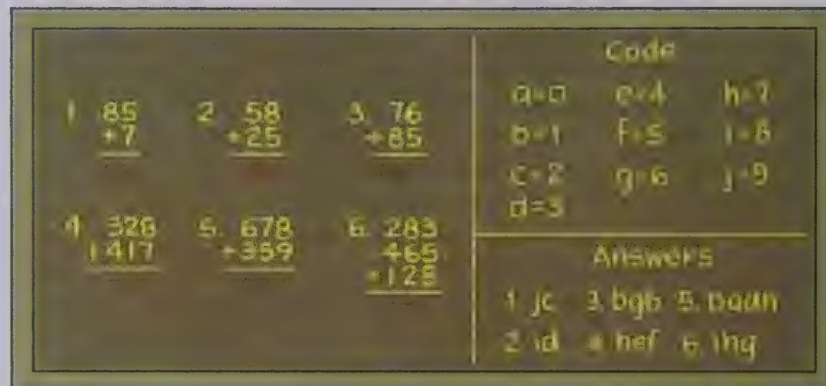
6 Computing

How are your adding skills?

Investigating the Ideas

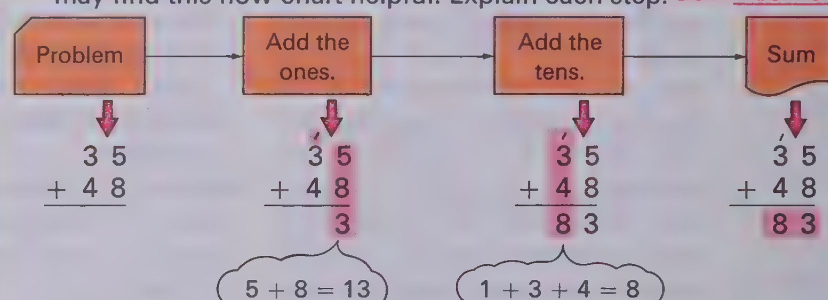
See Investigation.

? Can you find the sums without missing more than one?	Use the code to grade your own paper.



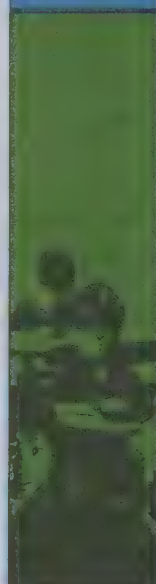
Discussing the Ideas

1. If you missed more than one of the addition problems, you may find this flow chart helpful. Explain each step. See Discussion.



2. Can you correct each mistake you made in the Investigation?

114



Discussion

The children should be able to work through the discussion section without much guidance. However, you will want to give special attention to those children who missed more than one of the problems in the investigation. Help them master the skill of the addition algorithm, which is depicted here in flow-chart form with an accompanying example. It would also be helpful to exhibit a few column addition problems and demonstrate how partial sums are noted and how the algorithm applies. However, since this lesson is largely review, avoid spending excess time on it.

Using the Ideas

1. Find the sums.

	A	B	C	D	E	F
	87	8	9	18	35	24
	+6	+54	+36	+7	+8	+7
	93	62	45	25	43	31

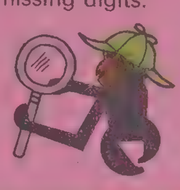
2. Find the sums.

	A	B	C	D	E	F
	64	48	67	58	95	64
	+21	+36	+39	+67	+87	+46
	85	84	106	125	182	110

think

Find the missing digits.

2 9 ▥7
5 35 ▥
7 624
+ ▥ 446
23 ▥95



3. Do not copy the exercises below. Just find each sum and write it on your paper.

A 56 + 8 64   D 5 + 9 + 8 22   G 6 + 7 + 8 + 7 28
B 47 + 9 56   E 6 + 7 + 5 18   H 8 + 9 + 7 + 8 32
C 89 + 8 97   F 6 + 7 + 8 21   I 6 + 9 + 5 + 8 28

4. Find the sums.

	A	B	C	D	E
	348	784	743	984	856
	692	65	806	376	972
	+843	+892	+59	+977	+800
	1883	1741	1608	2337	2628

	F	G	H	I	J
	9283	9037	8651	982	7836
	7651	8066	784	7655	965
	8420	579	97	8	1749
	+9165	+8432	+8465	+93	+88
	34 519	26 114	17 997	8738	10 638

5. Find the sums.

A 6784+932+89+7864+35 15 704   C 76 528+9328+657+9827 96 340
B 6748+297+3608+94 10 747   D 6427+15 348+19+384 22 178

★ **6.** In these exercises, some of the digits are covered. Give all possible digits for the ▥.

	A	B	C
	5▥	4▥	▥9▥
	+2▥	+8▥	+▥5▥
	▥▥	▥▥▥	▥▥▥
	7,8	2,3	4,5

More practice, page A-9, Set 17

Using the Exercises

Assign the exercises on page 115 as independent work. Have the children check their papers carefully when they are finished, and then provide further review work for any children who might need it.

Let the children who finish their work quickly attempt the *Think* problem. Probably all the children will be able to understand this problem once the correct answer is given; it is merely a standard addition problem. The entire class might benefit from hearing some of the children explain how they were able to arrive at the missing digits.

Assignments (page 115)

Minimum: 1–4E. Average: 1–5.
Maximum: 1–6.

Follow-up

The use of chain games is a suitable method of providing additional practice of addition facts. Duplicate chains such as the following on a worksheet or exhibit them on the chalkboard or with the overhead projector.

Fill in the missing addends ☐ or sums ▥.
Use the digits 0 through 9.

Start — End

8		15		▥	+9	▥		36		44

Start — End

▥		▥		17		▥		▥		27

Start — End

▥		▥	+8	▥		▥		32		▥

Since there are many possible solutions, you might have the children trade papers and check each other, thus doubling the review possibilities.

Resources for Active Learning

Mathematics in Modules, WN19, Addison-Wesley.

Duplicator Masters, page 20
Workbook, page 34
Skill Masters, page 20

Objective

Given subtraction problems involving 2-, 3- and 4-digit numbers, the child will test his ability to find the differences by using the subtraction algorithm.

Preparation

To prepare for this lesson, you might provide the children with a short oral practice of the basic subtraction facts. This might be done by giving the children subtraction equations or asking them to find missing addends in an addition equation. For example, ask them, "If I start with 17 and I take away 9, how many do I have left?" Or, "If I have 8 and want 15, how many more do I need?" You might also give the children some oral chain games such as the following:

"Start with 8. . . Add 5. . . Subtract 7. . . Add 9. . . What's your answer?" (15)

"Start with 16. . . Subtract 7. . . Add 8. . . Subtract 4. . . What's your answer?" (13)

Remember that the desirability of using preparation like this depends on the need of the children and the organization of your classroom.

Investigation

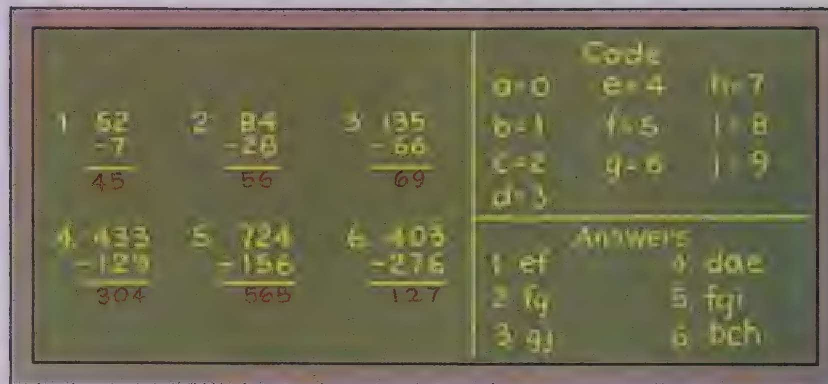
As in the previous lesson, children should work independently on this investigation. Again they are given an opportunity to test themselves on how well they recall a basic algorithm. Do not help the children beyond making sure that they understand what they are being asked to do. If they have forgotten how to do a problem and ask for assistance, explain that you want them to find out for themselves what they need to learn or review.

How are your subtracting skills?

Investigating the Ideas

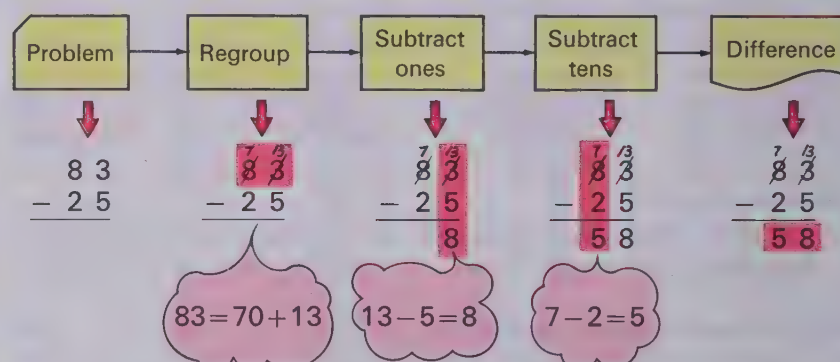
See Investigation.

? Can you find the differences without missing more than one? | Use the code to grade your own paper.



Discussing the Ideas

1. If you missed more than one of the subtraction problems, you may find this flow chart helpful. Explain each step. See Discussion



2. Can you correct each mistake you made in the Investigation?

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Discussion

Again, this discussion section may be handled by the children without much teacher guidance. However, it would be helpful to exhibit and work through a few examples such as parts 5 and 6 of the investigation or exercise 2 on page 117. Allow children who have mastered the algorithm skill to continue to page 117 or to work on supplementary activities. Provide any necessary guidance for those who still need it.

Using the Ideas

1. Find the differences.

A 64 − 28 = 36	B 72 − 34 = 38	C 83 − 29 = 54	D 27 − 18 = 9	E 95 − 75 = 20	F 61 − 54 = 7
G 158 − 69 = 89	H 137 − 58 = 79	I 62 − 21 = 41	J 157 − 78 = 79	K 93 − 65 = 28	L 80 − 56 = 24
M 182 − 91 = 91	N 156 − 87 = 69	O 327 − 164 = 163	P 643 − 156 = 487	Q 827 − 287 = 540	R 650 − 193 = 457
S 723 − 327 = 396	T 8427 − 6543 = 1884	U 6513 − 2465 = 4048	V 4327 − 1651 = 2676	W 7214 − 3888 = 3326	

2. The two examples show a convenient way to regroup when there are zeros involved. Complete each subtraction.

Think 39 tens and 12.

A 402 − 136 → $\overset{3}{\cancel{4}}\,\overset{9}{\cancel{0}}\,\overset{12}{\cancel{2}}$ − 1 3 6 = 2 6 6

Think 699 tens and 14.

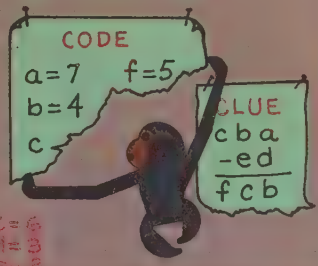
B 7004 − 4265 → $\overset{6}{\cancel{7}}\,\overset{9}{\cancel{0}}\,\overset{9}{\cancel{0}}\,\overset{14}{\cancel{4}}$ − 4 2 6 5 = 2 7 3 9

3. Find the differences.

A 701 − 137 = 564	B 600 − 133 = 467	C 507 − 489 = 18
D 5726 − 4314 = 1412	E 6592 − 1399 = 5193	F 8462 − 3597 = 4865
G 7302 − 1765 = 5537	H 8000 − 1672 = 6328	I 5001 − 3679 = 1322
J 600 − 257 = 343	K 800 − 69 = 731	L 6034 − 1655 = 4379
M 8006 − 1739 = 6267	N 6205 − 1984 = 4221	O 7026 − 1550 = 5476

think

Here is part of a code and a clue. See if you can find the rest of the code.



c=6
d=3
e=8

More practice, page A-9, Set 18

117

Follow-up

You might prepare a worksheet of reconstruction problems such as those that follow. (Point out that each screen does not necessarily represent the same number in a particular example.)

3▮ − 8 = ▮8	7▮0 − 3 2▮ = ▮1 6	1▮7▮ − 9▮4 = 3 5 6	▮4▮ − 9 1 = 2▮7
▮5 4▮ + 1 7▮1 = 8▮8 7	▮2 7▮ + 8 4▮5 = 13,▮3 5	4 8▮ + ▮5 6 = 6▮3	

Duplicator Masters, page 21
Workbook, page 35
Skill Masters, page 21

Using the Exercises

Assign the exercises on page 117 as independent work. If you did not do so during the discussion period, point out in exercise 2 the convenient way to regroup in situations involving zero.

The children who finish the page early can work on the *Think* problem. This problem may prove particularly difficult for some children, and they may need some suggestions, such as: "Write the subtraction problem and put in 7 for *a*, 4 for *b*, and 5 for *f*. Then try to find the other missing letters." Most of the children should understand the problem once the correct answer is given. Take care, though, to see that all the children have an opportunity (perhaps overnight) to work on this *Think* problem before the correct answer is given.

Assignments (page 117)
Minimum: 1A–L, 2–3F.
Average: 1A–R, 2–3I.
Maximum: 1–3.

PAGES 118–119

Objective

Given addition and subtraction problems involving amounts of money, the child will be able to solve the problems using the addition and subtraction algorithms and the appropriate money notation.

Preparation

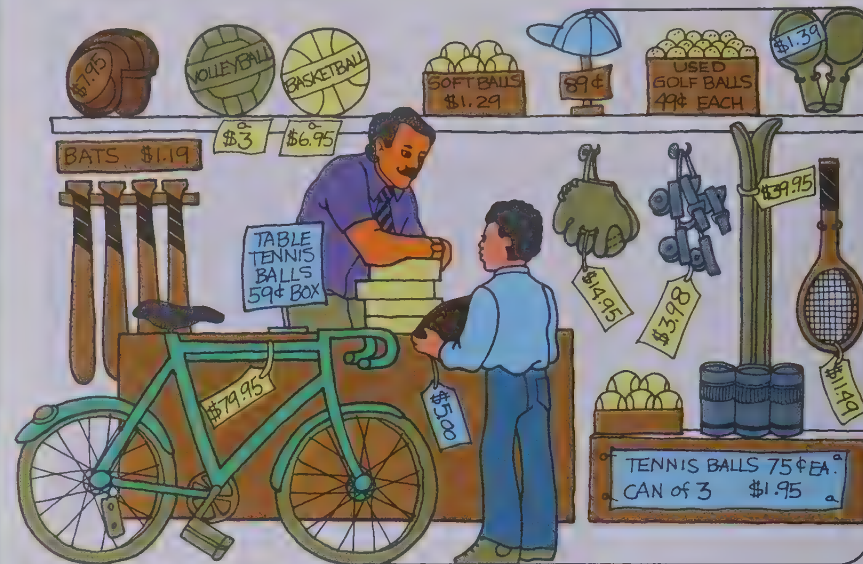
It would be suitable to begin this lesson without specific preparation. However, if you choose, you might stimulate the children's interest by asking them about things they would like to buy if they had $20.00 to spend.

Investigation

You might have children work independently or in small groups for this investigation. In order to answer the question, the children will have to add a column of prices. As you move around the room, notice which children recall how to use the money notation, but wait until the discussion period to point out its use. You might want to ask some groups to suggest why the investigation question specifies that they should save "about a dollar for tax." Encourage children to explain what they know about such sales or other taxes on purchases.

Let's add and subtract amounts of money.

Investigating the Ideas



? You have $20.00 to spend. Can you figure out a way to spend nearly all of it, except about a dollar for tax?
See Investigation and Discussion.

Discussing the Ideas

1. When you add and subtract amounts of money, what must you be sure to do when you write your problem on paper? Why?
Line up the decimal points correctly, so the answer will show the proper number of dollars and cents.

2. A What two items are more than $20.00? Bicycle and skis
   B What items are between 10 and 20 dollars?
   Baseball glove and tennis racket

3. Explain why the total is not correct. How would you write the problem?

$ .89
3.00
$3.89

ball cap	89¢
volleyball	$3
total	92¢

118

Discussion

Encourage children to compare the combinations which they chose in response to the investigation question. You might have several children write their lists of amounts of money on the chalkboard and use these as examples to point out the importance of using the correct notation for dollars and cents. Also, use examples to review with the children the idea of converting dollar notation to cents notation and cents notation to dollar notation. Stress the fact that the decimal point merely separates the numerals representing cents from those representing dollars.

Using the Ideas

1. How much more does the football cost than the roller skates? $1.02
2. How much would the tennis racket and a can of 3 tennis balls cost? $13.44
3. The bicycle costs how much more than the skis? $40.00
4. How much would it cost to buy the ball glove, a softball, and a bat? $17.43
5. If you bought the basketball for $6.95 and gave the clerk $20, how much money should you get back? $13.05
6. How much money do you save by getting a can of 3 tennis balls rather than 3 separate balls? $.30
7. How much would it cost to buy 4 table tennis paddles and a box of table tennis balls? $6.15

★ 8. What two things could you buy (without tax) so that you would get 5 cents in change if you gave the clerk 20 dollars?
Baseball glove and football

9. Find the total amounts.

A	B	C	D	E
$3.27	$9.23	$2.39	$7.45	$5.67
4.68	6.58	6.98	1.68	.85
$7.95	$15.81	$9.37	$9.13	$6.52

F $4.26, 3.78, 4.65 — $12.69
G $7.37, .06, 5.04 — $12.47
H $6.73, $5.86, $.95, $10.75, $64.30
I $46.20, $34.95, $68.50, $94.27
H $88.59
I $243.92

10. Find the difference in the amounts.

A	B	C
$6.78	$21.95	$7.98
2.39	16.56	4.06
$4.39	$5.39	$3.92

D $78.60, $21.78 F $6.75, $29.95
E $2.98, $54.50 G $60.00, $29.88
D $56.82 F $23.20
E $51.52 G $30.12

think

Replace each ▮ with the same digit.

```
     3 ▮52
     1 07▮
     5 4▮2
 6 + ▮ 73▮
   1 ▮ 92▮
```


Using the Exercises
Have the children do the problems and exercises on page 119 independently. When they finish and as you check their papers with them, stress the importance of correctly aligning the columns of numerals and the money notation for the amounts in exercises 9 H and I and 10 D–G.

Point out to those who try the *Think* problem that here each ▮ represents the same digit.

Assignments (page 119)
Minimum: 1–4, oral; 9A–C, 10A–C. Average: 1–4, oral; 5–7, 9A–E, 10A–F. Maximum: 1–4, oral; 5–10.

Mathematics
The use of the decimal point in exercises for finding total amounts of money could involve fractional-number concepts and some study of fractional numbers. However, until fractional numbers are developed carefully, tell the children simply that the decimal point is a dot separating the numerals which tell the number of dollars from those which tell the number of cents. Review with the children that the first, or ones', place tells the number of pennies, that the second, or tens', place records the number of dimes, and that ten pennies make a dime. Then note that the third place shows the number of dollars, and that ten dimes make a dollar. Thus, we draw an analogy between this situation and ordinary 3-digit problems. The children should understand that the addition algorithm is used to find the total amounts of money in the standard notation for dollars and cents, and you need not mention mathematical concepts involved in decimal or fractional-number notation. Similarly, money subtraction problems will be handled as though they are whole-number problems.

Follow-up
Perhaps a comparative study of the cost of sports equipment in your area would provide data for other problems requiring subtraction. If you assign each child to a small group and assign each group the task of studying prices in a given store, the children can later pool data to find average prices, differences, determine "best buys," organize data by making graphs, and so on.

Resources for Active Learning
A Cloudburst, Vol. 2, Nos. 1563, 1564, Midwest Publications
Experiences in Mathematical Ideas, Vol. 2, Unit 8, Experience 1, NCTM.
Mathematics in Modules, WN16, Addison-Wesley.

Duplicator Masters, page 22
Workbook, page 36
Skill Masters, page 22

PAGES 120–121

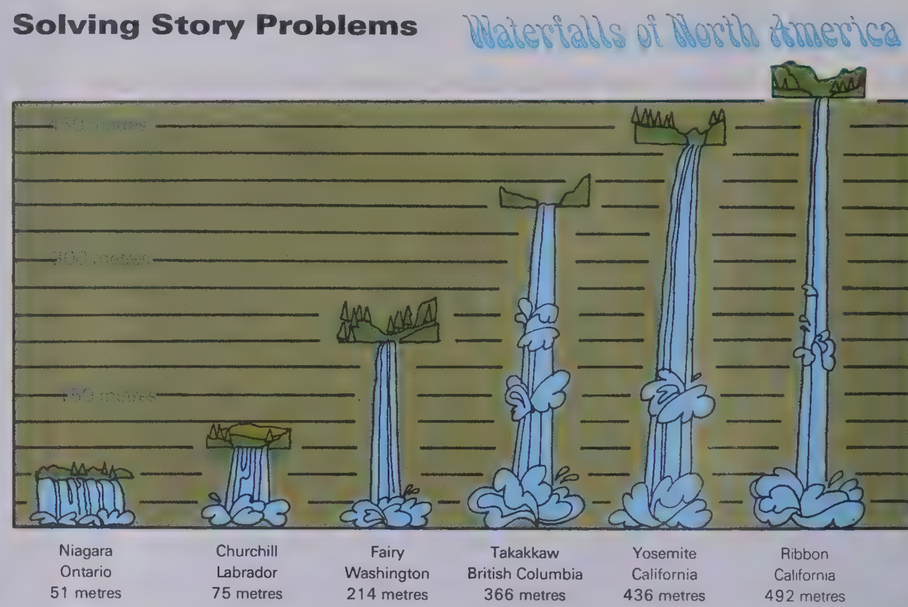
Objective

Given problems with accompanying charts, the child will be able to solve the problems by using the data in the charts and by using addition and subtraction.

Preparation

To prepare for this lesson, you might stimulate interest by asking if any children have ever seen any of the waterfalls or bridges discussed in these two pages. You might also use this time for a brief review of the addition and subtraction algorithm.

Solving Story Problems



1. Tell how much farther the water falls in the first waterfall than in the second.

A Ribbon, Yosemite 56m
B Ribbon, Takakkaw 126m
C Yosemite, Niagara 385m
D Churchill, Niagara 24m
E Fairy, Churchill 139m
F Takakkaw, Churchill 291m
G Takakkaw, Niagara 315m
H Fairy, Niagara 163m
I Ribbon, Niagara 441m

2. At Angel Falls in Venezuela the water falls 488 metres farther than at Ribbon Falls. Give the number of metres for Angel Falls. 980m

3. Yosemite Falls has 3 sections. The upper falls is shown above. The water drops 206 metres in the middle section and 97 metres in the lower falls. Give the total distance that the water drops 739m

4. At Multnomah Falls in Oregon the water drops a distance 176 metres less than at Takakkaw Falls. What is the height of Multnomah Falls ? 190m

★ **5.** Place Ville-Marie in Montreal is 117 metres higher than Churchill Falls. How much higher is Takakkaw than Place Ville-Marie ? 174m

120

Discussion

Discuss with the children the graph showing the heights of the waterfalls. Have the children observe that they must refer to the graph for all of exercise 1 and parts of the other exercises. Also, observe with them that starred exercise 5 is more difficult than the other problems and intended only for those who wish to try it. You will probably want to provide time for children to work through these problems and then compare their answers, before moving on to the discussion and assignment of the problems on page 121.

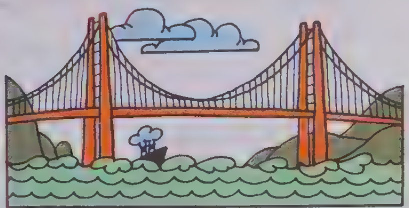
Assignments (page 120)

Minimum: 1–2. Average: 1–4.
Maximum: 1–5.

SUSPENSION BRIDGES

Bridge	Location	Year built	Height above water (Metres)	Length of main span (Metres)
Verrazano-Narrows	New York City	1964	70	1300
Golden Gate	San Francisco Bay	1937	73	1280
Tacoma	Washington	1952	56	854
Pierre Laporte	Quebec	1972	45	668
Lion's Gate	Vancouver	1939	46	473
Narrows Bridge	Halifax	1969	48	421

1. How much longer is the main span of the Verrazano-Narrows Bridge than the Pierre Laporte? 632 m
2. How many years old was the Golden Gate Bridge when the Verrazano-Narrows Bridge was built? 27 years
3. How much longer is the main span of the Golden Gate Bridge than that of Halifax's Narrows Bridge? 859 m
4. The Lion's Gate Bridge was built how many years before the Tacoma Bridge? 13 years



5. The highest bridge listed in the chart is how much higher than the lowest? 28 m
6. What is the total length of the main spans for these three bridges: the Lion's Gate, the Pierre Laporte, and the Narrows? 1562 m

121

Follow-up
Learning to organize and record information in graphs (line, bar, picture, and circle graphs), in charts, and in tables, and learning to interpret this organized data are important skills. Information about your province's crops, minerals, climate, industry, imports and exports, population, occupations, and the like will provide many opportunities to sharpen such skills.

Resources for Active Learning
SMSG: *Probability for Intermediate Grades,* "Using Graphs to Learn About Chance," Lesson 4, Stanford University. [A lesson extension]

Workbook, page 37

Using the Exercises
It would be helpful to discuss the chart on page 121 and read several entries from the table. Then, have the children do the exercises. Again, when they have finished, allow time for further discussion. Keep in mind that, for such word-problem sets as those on these two pages, some of the extraneous accompanying discussion is important in terms of stimulating the children's interest, not only in arithmetic but in the way arithmetic is used in everyday life.

Assignments (page 121)
Minimum: 1–3. Average: 1–5. Maximum: 1–6.

PAGES 122–123

Objective

Given multiplication problems with a single-digit multiplier, the child will be able to find the products by using the distributive principle and the multiplication algorithm.

Preparation

To prepare for this lesson, you might provide a short oral review of basic multiplication facts. Otherwise, begin immediately with the investigation.

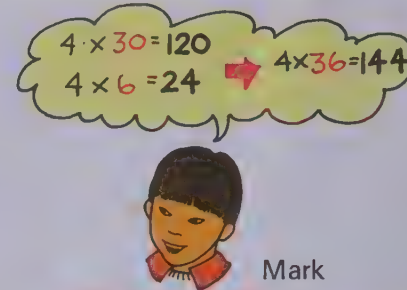
Investigation

In this investigation, children are expected to study and recall the way the distributive principle is used to break apart a 2-digit factor. Then they are given an opportunity to use their understanding and skill in a timed exercise. Note that the children are encouraged to multiply without pencil and paper and without written computation. Use this investigation as a self-test, and provide the children with answers so that they can check their work when they finish. You might devise a code and write the answers in coded form on the chalkboard.

Can you use the distributive principle?

Investigating the Ideas

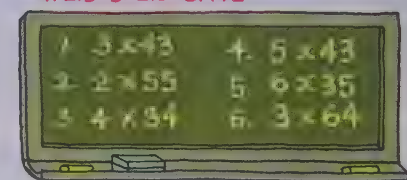
The distributive principle helps you find products like 4 × 36 without doing any writing. Study the example. Now test yourself and grade your own paper.



See Investigation.

? Can you write just the answers for these problems in 2 minutes?

1. 129 2. 110 3. 136
4. 215 5. 210 6. 192



Discussing the Ideas

1. Use this example of the distributive principle to explain how Mark found the product 4 × 36. See Discussion.

$$4 \times 36 = (4 \times 30) + (4 \times 6)$$

2. Give the numbers for ***a*** and ***b***. Then give the product for ***c***.

A $3 \times 40 = a$ 120; $3 \times 2 = b$ 6; $3 \times 42 = c$ 126

B $2 \times 50 = a$ 100; $2 \times 4 = b$ 8; $2 \times 54 = c$ 108

C $4 \times 20 = a$ 80; $4 \times 3 = b$ 12; $4 \times 23 = c$ 92

D $6 \times 40 = a$ 240; $6 \times 5 = b$ 30; $6 \times 45 = c$ 270

E $3 \times 50 = a$ 150; $3 \times 6 = b$ 18; $3 \times 56 = c$ 168

F $2 \times 90 = a$ 180; $2 \times 6 = b$ 12; $2 \times 96 = c$ 192

G $5 \times 70 = a$ 350; $5 \times 3 = b$ 15; $5 \times 73 = c$ 365

H $4 \times 70 = a$ 280; $4 \times 4 = b$ 16; $4 \times 74 = c$ 296

I $3 \times 60 = a$ 180; $3 \times 5 = b$ 15; $3 \times 65 = c$ 195

J $2 \times 60 = a$ 120; $2 \times 7 = b$ 14; $2 \times 67 = c$ 134

122

Discussion

Guide the children in a discussion of the distributive principle. Help them realize that Mark "broke apart" the 36 and multiplied each part and then added his two (partial) products. It would be helpful to work through the other investigation exercises similarly.

Discussion exercise 2 is intended to be used as an oral exercise. Continue to stress how the sums of *a* and *b* equal *c*, and that this type of computation should be done mentally.

You might choose to use exercise 1 on page 123 as part of the discussion section. If so, work through similar examples to show the convenience of the final shortcut method, as shown here.

$$\begin{array}{r} {\scriptstyle 2} \\ 87 \\ \underline{\times 3} \\ 1 \end{array} \rightarrow \begin{array}{r} {\scriptstyle 2} \\ 87 \\ \underline{\times 3} \\ 261 \end{array}$$

Using the Ideas

The flow chart will help you review two ways to write your work for products like 3 × 465.

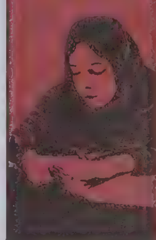
Problem 3 × 465 → 3 × 5 = 15 → 3 × 60 = 180 → 3 × 400 = 1200 → Add → Product 1395

3 × 5 = 15	3 × 60 = 180	3 × 400 = 1200	Add
465 ×3 15	465 ×3 15 180	465 ×3 15 180 1200	465 ×3 15 180 1200 1395
1 465 ×3 5	1 1 465 ×3 95	1 1 465 ×3 1395	You did the adding as you did each step.

Find the products.

1. 34 × 4 = 136	2. 65 × 3 = 195	3. 38 × 6 = 228	4. 67 × 4 = 268	5. 75 × 7 = 525
6. 94 × 8 = 752	7. 185 × 4 = 740	8. 359 × 5 = 1795	9. 287 × 4 = 1148	10. 651 × 9 = 5859
11. 704 × 8 = 5632	12. 839 × 7 = 5873	13. 534 × 9 = 4806	14. 576 × 8 = 4608	15. 680 × 9 = 6120
16. 3276 × 2 = 6552	17. 5843 × 3 = 17,529	18. 8439 × 4 = 33,756	19. 8037 × 4 = 32,148	20. 9206 × 8 = 73,648

More practice, page A-10, Set 19



Using the Exercises

After discussing exercise 1 on page 123, assign the remaining exercises as independent work. You might have several children demonstrate at the chalkboard how they worked the exercises. When they have finished, have them check their papers carefully.

Assignments (page 123)

Minimum: Odd-numbered problems.

Average: 1–15. Maximum: 1–20.

Mathematics

Since the concepts necessary for understanding the multiplication algorithm have been developed previously, this lesson presents no new mathematical concepts. The chief mathematical concept that is involved is the utilization of the distributive principle for breaking apart numbers and adding partial products in the multiplication algorithm. The development of the algorithm requires little more than helping the children learn to write the work in a specific convenient way.

Follow-up

To provide practice in finding the multiples of 3 and 8, provide worksheets presenting a problem similar to that below. Encourage children to work on it during their free time.

> During a sale 5¢ candy bars cost 3¢, and 10¢ candy bars cost 8¢. Use the numbers 0 through 9 to show how many of each kind of candy bar the children purchased during the sale.
>
> Alice
> (__ × 3) + (__ × 8) = __ + __ = 12
> Ben
> (__ × 3) + (__ × 8) = __ + __ = 22
> Cheryl
> (__ × 3) + (__ × 8) = __ + __ = 27
> Dale
> (__ × 3) + (__ × 8) = __ + __ = 32
> Ed
> (__ × 3) + (__ × 8) = __ + __ = 48
> Frank
> (__ × 3) + (__ × 8) = __ + __ = 53
> Gail
> (__ × 3) + (__ × 8) = __ + __ = 67
> Helen
> (__ × 3) + (__ × 8) = __ + __ = 79
> Jon
> (__ × 3) + (__ × 8) = __ + __ = 81
> Kris
> (__ × 3) + (__ × 8) = __ + __ = 83

Resources for Active Learning

Mathematics in Modules, WN14, Addison-Wesley.

Duplicator Masters, page 23
Workbook, page 38
Skill Masters, page 23

PAGES 124–125

Objective

Given multiplication problems with factors of 2 or 3 digits, the child will be able to use the multiplication algorithm to solve the problems.

Preparation

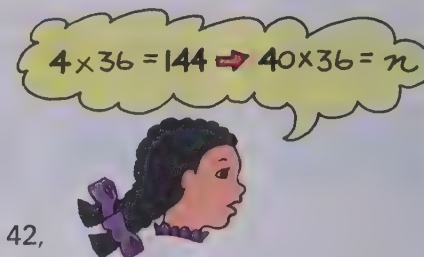
To prepare for this lesson, you might review multiplication with single-digit multipliers. For example, give the children oral exercises such as 3×54; 4×67; 5×72; 2×46; etc.

Encourage children to try to do their work without pencil and paper. However, since accuracy is important, allow them to use pencil and paper if they wish.

Let's explore 2- and 3-digit factors.

Discussing the Ideas

1. If you can find products like 4×36, you can easily find products like 40×36. Can you solve the equation for Becky? 1440



Becky

2. If you know the product 3×42, what must you multiply this product by to find 30×42? 10

3. Describe how you can find the product 40×23.
Think: $4 \times 23 = 92 \rightarrow 40 \times 23 = 92 \times 10 = 920$

4. Solve the equations.

A $3 \times 54 = 162 \longrightarrow 30 \times 54 = m$ 1620
B $4 \times 26 = 104 \longrightarrow 40 \times 26 = p$ 1040
C $7 \times 34 = 238 \longrightarrow 70 \times 34 = r$ 2380
D $2 \times 93 = 186 \longrightarrow 20 \times 93 = s$ 1860
E $8 \times 35 = 280 \longrightarrow 80 \times 35 = t$ 2800
F $5 \times 67 = 335 \longrightarrow 50 \times 67 = v$ 3350
G $6 \times 83 = 498 \longrightarrow 83 \times 60 = x$ 4980
H $9 \times 54 = 486 \longrightarrow 90 \times 54 = w$ 4860
I $7 \times 86 = 602 \longrightarrow 86 \times 70 = y$ 6020
J $8 \times 49 = 392 \longrightarrow 80 \times 49 = z$ 3920

5. Explain each step in the example below. See Discussion.

Step 1	Step 2	Step 3
$\begin{array}{r} 37 \\ \times\ 48 \\ \hline 296 \end{array}$	$\begin{array}{r} 37 \\ \times\ 48 \\ \hline 296 \\ 1480 \end{array}$	$\begin{array}{r} 37 \\ \times\ 48 \\ \hline 296 \\ 1480 \\ \hline 1776 \end{array}$
$8 \times 37 = 296$	$40 \times 37 = 1480$	$296 + 1480 = 1776$

124

Discussion

In this lesson, children progress from using a multiple of 10 as a multiplier to using 2- and 3-digit multipliers. As you discuss exercises 1, 2, and 3, be sure the children understand that, in order to find a product such as 40×65, they begin by writing a zero and then multiply 4×65. This is an important step in the initial work with the 2- and 3-digit algorithm, and it should be stressed as you discuss step 2 of exercise 5. Work through each of the steps in exercise 5 carefully. Help the children realize that this algorithm is a way of working out the equation: $48 \times 37 = (40 \times 37) + (8 \times 37)$. If you feel the children need more practice, present other examples and ask children to explain them.

It would also be helpful to present exercises in which the multiplier has three digits. Exercise 3 on page 125 could serve as the basis for such a presentation.

Using the Ideas

1. Find the products.

A	24×36 = 864	B	36×25 = 900	C	52×41 = 2132
D	14×56 = 784	E	35×24 = 840	F	46×37 = 1702
G	62×26 = 1612	H	75×33 = 2475	I	83×57 = 4731
J	49×68 = 3332	K	91×37 = 3367	L	38×49 = 1862

think

Each letter stands for a digit. Can you find each digit?

$$\begin{array}{r} a\,a\,a\,a \\ \times\ a\,a \\ \hline a\,a\,a\,a \\ a\,a\,a\,a\ \ \\ \hline a\,b\,b\,b\,a \end{array} \qquad \begin{array}{r} c\,d\,d \\ \times\ c\,d\,d \\ \hline c\,d\ \ d\,d\,d \end{array}$$

2. Study the example. Then find the products.

$34 \times 526 = (30 \times 526) + (4 \times 526)$		
Step 1	Step 2	Step 3
$\begin{array}{r} 526 \\ \times\ 34 \\ \hline 2104 \end{array}$	$\begin{array}{r} 526 \\ \times\ 34 \\ \hline 2104 \\ 15780 \end{array}$	$\begin{array}{r} 526 \\ \times\ 34 \\ \hline 2104 \\ 15780 \\ \hline 17884 \end{array}$
$4 \times 526 = 2104$	$30 \times 526 = 15,780$	$2104 + 15,780 = 17,884$

A 256×25 = 6400	B 324×36 = 11664	C 514×23 = 11822	D 137×45 = 6165	E 284×62 = 17608	F 891×34 = 30294

3. Find the products.

A	341×3 = 1023	341×90 = 30690	341×200 = 68 200	341×293 = 99913
B	956×8 = 7648	956×40 = 38 240	956×300 = 286800	956×348 = 332 688

More practice, page A-10, Set 20

125

Follow-up

Children might enjoy studying and completing the following patterns of 9.

1. $7 + 9 \times 9 = 88$
   $6 + 98 \times 9 = 888$
   $5 + 987 \times 9 = 8888$
   $4 + 9876 \times 9 = 88888$
   .
   .
   .

2. $(1 \times 9) + 2 = 11$
   $(12 \times 9) + 3 = 111$
   $(123 \times 9) + 4 = 1111$
   .
   .
   .

Resources for Active Learning

Math Activity Cards, D28, Macmillan.

Mathematics in Modules, WN20, Addison-Wesley.

Duplicator Masters, pages 24–25
Workbook, page 39
Skill Masters, pages 24-25.

Using the Exercises

Have the children do the exercises on page 125 independently. You might want some children to study exercise 2 together and then discuss the computations that were involved. In exercise 3, the children should notice the steps that are used in problems with a 3-digit multiplier. If this was not discussed previously, treat it carefully here. Relate part A to the equation $293 \times 341 = (3 \times 341) + (90 \times 341) + (200 \times 341)$. Point out how 200×341 may be solved by writing two zeros and multiplying by two. You might also present examples like 602×2793 and 600×2793.

Assignments (page 125)

Minimum: 1A–F, 2A–C.
Average: 1A–I, 2. Maximum: 1–3.

PAGES 126–127

Objective

Given addition, subtraction, and multiplication word problems, the child will be able to solve the problems by using the appropriate algorithm.

Preparation

To prepare for these problems, discuss the two maps. Have children read several distances between two cities and have them observe the historical data on the transportation map. You might ask them what year it would have been 50 years ago, or 100 years ago. You may point out to the children that the nine weeks which the stage coach took to cross the country was considered then to be very fast time and probably required travelling day and night.

Solving Story Problems Distances

1. Give the total distance for each trip.
   - A Montreal to Quebec to Halifax 1526 km
   - B Vancouver to Regina to Winnipeg to Sault Sainte Marie 3793 km
   - C Edmonton to Whitehorse to Vancouver to Edmonton 6142 km
2. Use the map to plan these trips for the smallest number of kilometres.
   - A Vancouver to Winnipeg Via Regina
   - B Edmonton to Sault Sainte Marie Via Winnipeg
   - C Halifax to Sault Sainte Marie Via Quebec
   - D Quebec to Whitehorse Via Edmonton
3. Which distance is about 4 times as far as the distance from Montreal to Quebec? Sault Ste. Marie to Montreal (1022 km)

★ 4. Mr. Brown had driven his new car 1247 kilometres before leaving for a trip. When he got to one of the cities on the map, he had driven his car a total of 1817 kilometres. He went from this city to another city on the map, and by then he had driven 2617 kilometres. From what city on the map did Mr. Brown start? Winnipeg; see Discussion

126

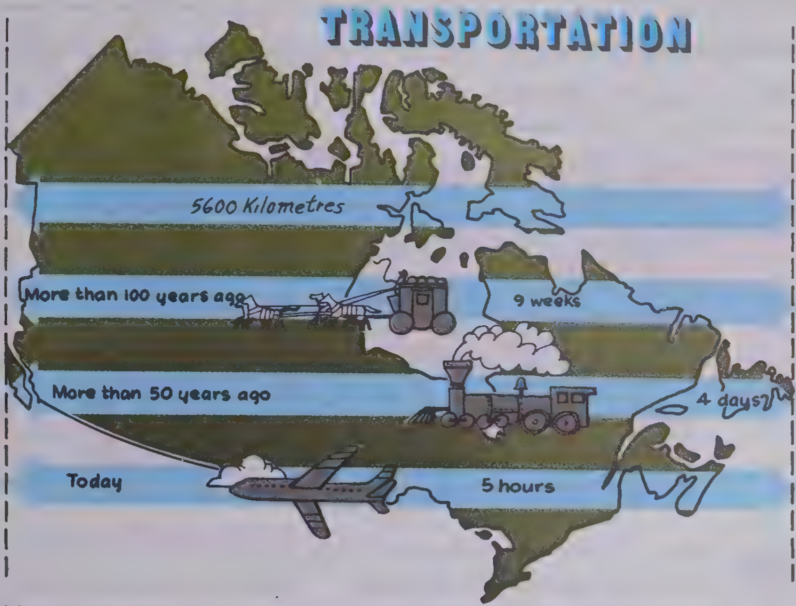
Discussion

After the introductory discussion, have the children work the problems. Remind them to read each problem carefully two or three times, find the required information from one of the maps, and solve the problem by using the suitable operation.

Note that problem 4 is starred because it is intended primarily for the faster children. When the explanation to this problem is given, note that the first leg of the trip is 570 kilometres; hence, Mr. Brown started either at Winnipeg or Regina. In order to find out which city he started from, we must do the second part of the exercise and observe that on the second leg of his trip he travelled 800 kilometres; since the only 800-km trip listed on the map is from Regina to Edmonton, he must have started from Winnipeg.

Assignments (page 126)

Minimum: 1–2. Average: 1–3. Maximum: 1–4.



1. About how many hours did it take the train to cross Canada ? 96

2. About how many minutes does it take the jet plane to cross the country ? 300

3. How many days did it take the stagecoach to cross Canada ? 63
How many hours would this be? How many minutes ?
1512 90 720

4. More than 100 years ago, the first train between Montreal and Vancouver took 5 days.
How many hours was this ? 120

5. The trip from Montreal to Vancouver is about 869 kilometres less than a trip all the way across the country. About how far is the trip ?
4731 km

6. Today a train can cross Canada in about 58 hours. Could a modern train make it across and back while the old train is going across ? No

127

Using the Exercises

The first four problems on page 127 are multiplication problems, while problem 5 is a simple subtraction problem. Note that problem 6, however, requires the children to find, first, the number of hours needed for a modern train to make a round trip across the country and then compare that number with the number of hours it took an older train to make the trip one way. Most children will probably realize that they have already found the latter number in response to problem 1.

When the children have finished the problems on both pages, have them compare their answers and explain how they thought about the problem and what operation they used.

Assignments (page 127)

Minimum: 1–3. Average: 1–5. Maximum: 1–6.

Follow-up

Data such as the following might be used to encourage children to create their own story problems.

The average speed of transoceanic liners is 31 to 35 knots (or approximately 65 kilometres per hour). A knot is one nautical mile per hour. The international unit for a nautical mile is about 1852 metres.

You might wish to give the children copies of the following data and ask them to find the approximate sea distances (nautical miles) between the ports. (The number of hours has been rounded for easier computation.)

Port to Port Sailing Times

Montreal to Cape Town, South Africa — 203 hours
Cape Town to Hong Kong — 200 hours
Hong Kong to Vancouver — 165 hours
Le Havre to Gibraltar – 33 hours
Gibraltar to Port Said, U.A.R. – 55 hours
Port Said to Bombay, India – 88 hours
Bombay to Melbourne, Australia – 159 hours
Melbourne to Honolulu – 142 hours
Honolulu to San Francisco – 60 hours

Some children might like to make up problems based on the following questions.

1. How many ways does the map on page 126 show to get from Vancouver to Halifax? Which route is the longest? the shortest?
2. You are planning a vacation trip through Canada from your home in Whitehorse. Suppose you can make ideal airline connections (at any time you choose), and can spend at least 1½ days visiting each city. Plan at least two trips visiting five different cities in 10 days. How many metres does each trip cover?

Resources for Active Learning

Maths Mini-lab, Card 81, Selective Educational Equipment.

Duplicator Masters, page 26

PAGES 128–129

Objective

Given rectangular or polygonal regions and the lengths of their sides, the child will find the area and/or perimeter by using the appropriate addition or multiplication algorithm.

Preparation

Materials

graph paper, 1-cm grid; crayons

To prepare for this lesson, you might briefly review the concepts of area and perimeter. For example, ask children to describe how they might measure the perimeter of the classroom or the area of the chalkboard. Remind them that it is not always necessary to count units in order to measure something; they may often use operations such as addition and multiplication.

Investigation

Some children may be able to name the possible rectangular regions that have an area of 144 square units without drawing them on graph paper. Others may need to draw all 8 rectangles. Encourage children to list the dimensions of the rectangles systematically so that they can more easily check whether their list is complete.

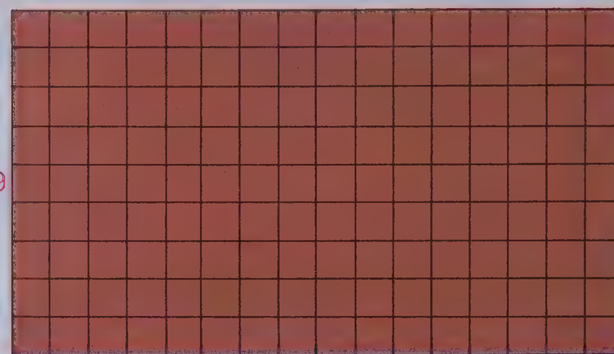
1 × 144	6 × 24
2 × 72	8 × 18
3 × 48	9 × 16
4 × 36	12 × 12

It would also be beneficial for children to group the colored rectangles on a bulletin board, according to height. They would have to tape some strips of graph paper together for the rectangles of 1 × 144; 2 × 72; 3 × 48, unless graph paper of smaller grids is used.

● *Let's compute area and perimeter.*

Investigating the Ideas

Here is a rectangular region that has an area of 144 square units. 9 × 16 = 144

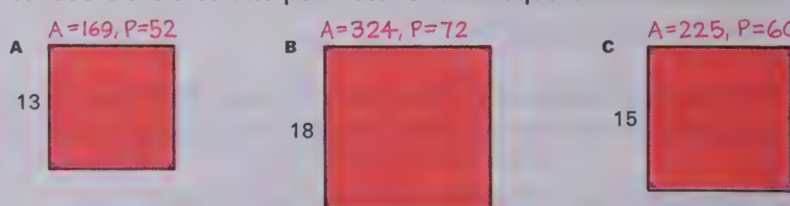


See Investigation.

? How many other **rectangular** regions that have an area of 144 square units can you find?	Draw and color at least one of the regions on graph paper.

Discussing the Ideas

1. What is the perimeter of the region shown in the Investigation? 50
2. Can you give the perimeter of each of the regions you found in the Investigation? 1×144, P=290; 2×72, P=148; 3×48, P=102; 4×36, P=80; 6×24, P=60; 8×18, P=52; 9×16, P=50; 12×12, P=48
3. What are the area and perimeter of each square?



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Discussion

Have the children explain the methods they used in their investigation activities. Make sure they realize that the area can be found as the product of the length times the width. Use the examples from the investigation to emphasize the fact that the same amount of area may be enclosed by sides of different length.

As you discuss exercises 1 and 2, ask children to explain how to find the perimeter of a rectangular region. It would also be helpful to list the perimeter of each region in increasing or decreasing order. Call the children's attention to the fact that the rectangle with the smallest perimeter is the 12-by-12 square.

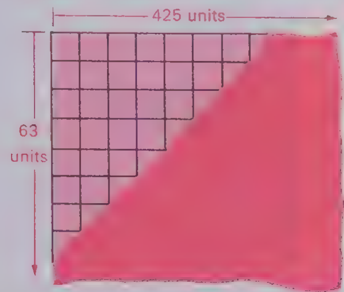
In discussing exercise 3, emphasize the methods of finding the perimeter and the area of a square: to find the perimeter, multiply the length of one side by four; to find the area, multiply the length of a side by itself.

Using the Ideas

1. Find the area of the rectangle indicated in each exercise.

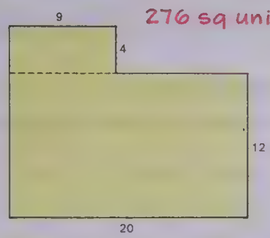
A 9128 sq units

B 26,775 sq units

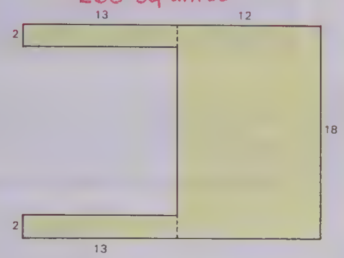


2. Find the area of each figure.

A 276 sq units



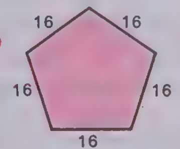
B 268 sq units



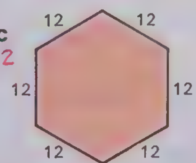
3. Multiply to find the perimeter of each figure. The lengths of the sides are given.

A 48

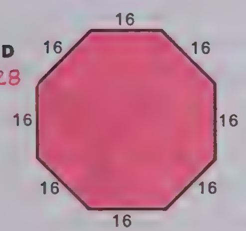
B 80



C 72

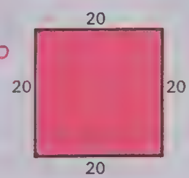


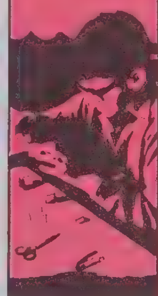
D 128



E 96

F 80





Follow-up

As suggested in the investigation, you might have the children graph or chart their rectangles according to height or according to perimeter. Encourage children to try the same activities for rectangles with other areas, such as 48 or 36 square units. Besides stressing the application and practice of multiplication, this activity should also help children to realize that the same area may be enclosed within a variety of dimensions.

Resources for Active Learning

Developmental Math Cards, J-4, Addison-Wesley.

Experiments in Mathematics, Stage 1, pp. 12–13, Houghton Mifflin. (Available from Thomas Nelson and Sons)

Freedom to Learn, pp. 9-13; 84-85, Addison-Wesley.

Inquiry in Mathematics via the Geo-Board, "Perimeter and Area," Geo-Cards 19; 20/1-4; "Functions . . .," Geo-Cards 30/1-31/3, Walker. (Available from Fitzhenry and Whiteside)

Duplicator Masters, page 27

Using the Exercises

Before assigning the exercises on page 129 as independent work, help children observe that only a small part of each rectangle in exercise 1 is pictured and that multiplication should be used to find the area. In exercise 2, children should find the area of each of the partial regions and then find the total area.

After you discuss the perimeters of the figures in exercise 3, have the children compare the areas of some of the figures. They might enjoy trying to decide whether figure B or figure F has the greater area. It would be difficult for the children to find the exact areas of these figures, so you should not belabor the issue. However, simple sight comparisons of the two figures will provide a stimulating discussion. Some children might wish to do some research to try and find out how they can tell the area of odd-shaped figures such as those in parts B, C, D, and E.

Assignments (page 129)

Minimum: 1–2. Average: 1–3C. Maximum: 1–3.

PAGES 130–131

Objective

Given addition, subtraction, and multiplication word problems, the child will be able to solve the problems by using the appropriate algorithm.

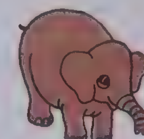
Preparation

To prepare for this lesson, you might review the phrase, "as many times as." For example, give children oral practice with problems such as these:

"John has $4.00. Sue has three times as much. How much money does Sue have?" ($12.00)

Jack travels 2 kilometres to school. Bill travels 7 times as many kilometres to school. How many kilometres does Bill travel to school?" (14 kilometres)

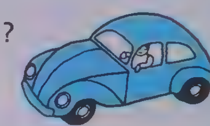
Short Stories Weights



1 Baby elephant: 89 kilograms. Full-grown elephant: 57 times that much. How many kilograms does a full-grown elephant weigh? 5073 kg

2 Baby whale: weighs twice as much as a full-grown elephant. How much does a baby whale weigh? 10 146 kg

3 Full-grown whale: weighs 954 times as much as a baby elephant. How much does a full-grown whale weigh? 84 906 kg



4 Automobile: weighs 21 times as much as the baby elephant. How much does the automobile weigh? 1869 kg

5 Professional football player: 31 kg heavier than the baby elephant. What is the football player's weight? 120 kg

6 Beauty queen: 35 kg lighter than the baby elephant. How much does the beauty queen weigh? 54 kg

7 One tonne: 68 kilograms less than 12 times as much as the baby elephant. How many kilograms? 1000 kg

8 Sigma 7 (a Mercury space capsule): weighed 763 kilograms less than 30 times the weight of the baby elephant. How much did Sigma 7 weigh? 1907 kg

9 Circus fat man: weighs 30 kilograms more than three times as much as the baby elephant. How much does the fat man weigh? 297 kg

10 Strong man: lifts twice as much as 8 kilograms more than the baby elephant weighs. How much does he lift? 194 kg

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Discussion

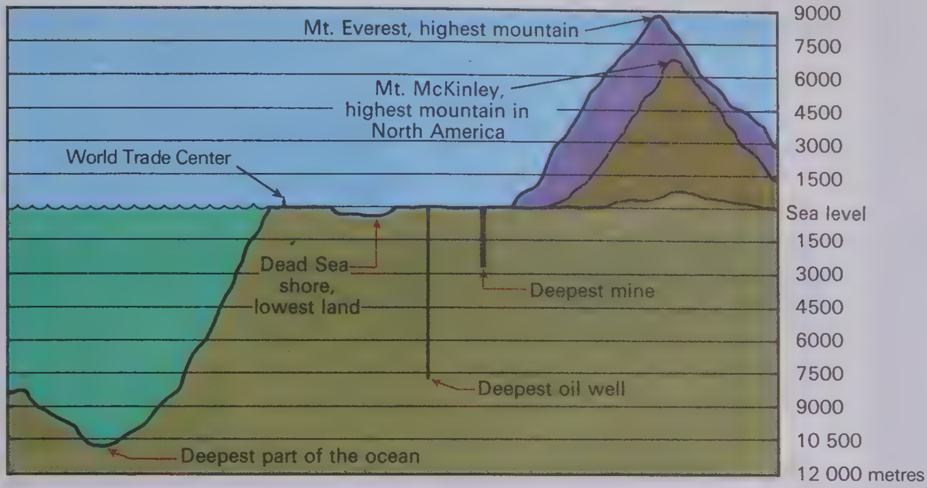
Page 130 may be assigned as independent work. The children might be intrigued by some of the interesting, perhaps strange, information in the problems on page 130. They may even question the validity of some of it. If so, you can assure them that, although some of the numbers given are approximations, the information is essentially factual.

Assignments (page 130)

Minimum: Odd-numbered problems.

Average: 1–8. Maximum: 1–10.

HEIGHT AND DEPTH



1. Use the graph above to estimate the height or depth of each item listed. Write the name of each item and your estimate beside it.
   See figures (to the nearest hundred metres) on the right.
2. The World Trade Center is 412 metres tall. Mount McKinley is 5749 metres higher than the World Trade Center. How high is Mount McKinley? 6161 m
3. Mount Everest is almost 22 times higher than the World Trade Center. Use the height of the building, given in exercise 2, to estimate the height of Mount Everest.
   Over 8800 m (22 x 400)
4. The lowest land is 395 metres below sea level. The deepest mine is 2406 metres deeper. How deep is the deepest mine?
   2801 m below sea level
5. How deep is the deepest oil well if it is 4929 metres deeper than the mine? 7730 m below sea level
6. The deepest part of the ocean is about 305 metres deeper than 27 times the depth of the lowest land. Use this information and the depth of the lowest land, given in exercise 4, to estimate the depth of the deepest part of the ocean.
   About 11 105 m [(27 x 400) + 305] or 11 105 m to the nearest hundred metres) below sea level

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Using the Exercises

Before assigning page 131, it would be helpful to discuss the illustration. You might observe with them that the picture is not intended to imply that the World Trade Center is right beside the Dead Sea or that the deepest oil well is right beside the Dead Sea or that the deepest mine is close to Mt. Everest or that Mt. McKinley is close to Mt. Everest. Point out that this is a diagram which shows the relationship between the various objects.

Assignments (page 131)
Minimum: 1–3. Average: 1–5.
Maximum: 1–6.

Follow-up

Help the children collect and organize data using a bar or line graph. You might suggest that they make a project of classifying information about the class members. For example:

Birthdate by days and months
Range of weights, boys and girls
Range of heights, boys and girls
Color of hair
Color of eyes
Hobbies

This project could also include other kinds of data such as food preferences, pets, and television viewing habits.

Answers exercise 1, page 131

Mt. Everest, 9100 m
Mt. McKinley, 6100 m
World Trade Center, 400 m (500 is a good estimate)
Dead Sea Shore, 400 m
Deepest mine, 2600 m or 2700 m or 2800 m
Deepest oil well, 7700 m or 7800 m
Deepest part of the ocean, 10 700 m or 10 800 m

Resources for Active Learning

Applied Mathematics Cards, Group 2/6–10, Schofield and Sims. [Measurement and computation] (Available from Mafex Associates, Willowdale)

Duplicator Masters, page 28

PAGES 132–133

Objective

Given a problem such as $6\overline{)96}$, *or* $n \times 6 = 96$, *the child will be able to find the quotient or missing factor by estimating the answer.*

Preparation

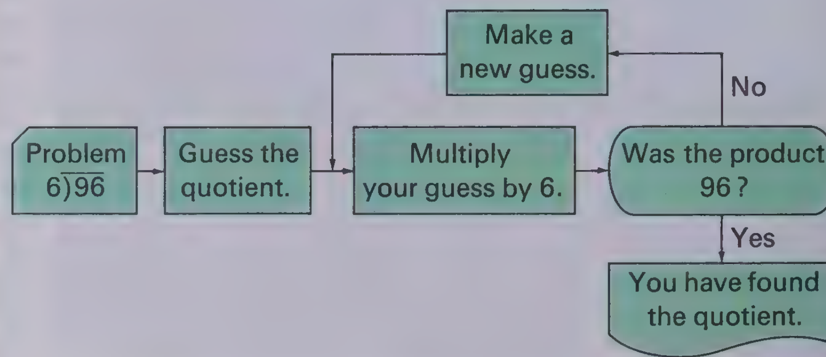
To prepare for this lesson, you might briefly review division as the inverse of multiplication. For example, you might say to the children: "If you know that $25 \times 3 = 75$, then what do you know about $75 \div 3$?" Or: "If you know that $4 \times 12 = 48$, then what do you know about $48 \div 12$?"

Investigation

In order for children to do this investigation with understanding, it is important that they realize the relation between multiplication and division. With such an understanding, they should be able to follow the steps in the flow chart, and continue to make guesses until they arrive at the correct quotient. If some children are not sure how to begin, suggest that they try a multiple of 10 as their first guess. By trial and error, they may soon realize that the quotient must be greater than 10 and less than 20. Some children may approach this problem with the viewpoint of division as repeated subtraction. Such an approach should help them arrive at the correct answer, though perhaps with less actual "guessing." Try to avoid giving the children direct guidance; encourage them to study the problem and flow chart on their own. However, as you move around the room, make sure each child interprets the directions on the chart correctly. One of the purposes of this investigation is to help children realize the need for a method other than trial and error when looking for quotients.

Let's get ready for dividing.

Investigating the Ideas



? Can you follow the flow chart and find the quotient? 16 (See Investigation.)

Discussing the Ideas

1. How many estimates did you make before you found the quotient? Answers will vary.

2. How could subtraction help you make your new guess? See Discussion.

3. The example on the right will help you review the meaning of divisor, dividend, quotient, and remainder.

$$\begin{array}{rrl} & 8 & \text{quotient} \\ \text{divisor } 4 & \overline{)35} & \text{dividend} \\ & \underline{32} & \\ & 3 & \text{remainder} \end{array}$$

   A What number was the divisor in the Investigation? 6

   B What number was the dividend? 96

   C What was the remainder? 0 (See Discussion.)

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Discussion

As you discuss exercise 1, you might show in inequality form some of the estimates the children made. For example:

$$10 \times 6 < 96$$
$$20 \times 6 > 96$$

In exercise 2, help the children explain how subtraction might be used to solve a problem like $6\overline{)96}$.

$$\begin{array}{rl} 96 & \\ \underline{-60} & (10 \times 6) \\ 36 & \\ \underline{-36} & (6 \times 6) \\ 0 & \end{array}$$

Use exercise 3 to review the meaning of the terms *divisor*, *dividend*, *quotient*, and *remainder*. In exercise 3C, encourage the response "remainder 0," rather than "no remainder."

Using the Ideas

Give the number pair for each gray space in exercises 1 to 6.
Then find the quotient and remainder.

Pairs: 1 2 | 2 3 | 3 4 | 4 5 | 5 6 | 6 7 | 7 8 | 8 9 | 9 10

1. A 3 $\square \times 4 < 15$
4 $\square \times 4 > 15$
B $4\overline{)15}$ 3 R 3

2. A 6 $\square \times 6 < 40$
7 $\square \times 6 > 40$
B $6\overline{)40}$ 6 R 4

3. A 9 $\square \times 5 < 47$
10 $\square \times 5 > 47$
B $5\overline{)47}$ 9 R 2

4. A 5 $\square \times 3 < 16$
6 $\square \times 3 > 16$
B $3\overline{)16}$ 5 R 1

5. A 6 $\square \times 7 < 48$
7 $\square \times 7 > 48$
B $7\overline{)48}$ 6 R 6

6. A 5 $\square \times 6 < 34$
6 $\square \times 6 > 34$
B $6\overline{)34}$ 5 R 4

7. For each exercise, first tell how many digits you think the product has; then estimate the product; finally, find the product to see how you did on your estimates.
Example: 4×49 (Answer: Number of digits, 3; estimate, 200; correct product, 196)

A 40×49 1960 D 8×97 776 G 90×75 6750 J 4×197 788
B 6×51 306 E 80×97 7760 H 90×750 67500 K 40×197 7880
C 60×51 3060 F 9×75 675 I 92×756 69552 L 42×197 8274

8. From the set {32, 19, 51, 97, 79, 11}, choose a number for the missing factor. Find the product to check your work.

19 A $n \times 5 = 95$
11 B $a \times 11 = 121$
97 C $6 \times b = 582$
51 D $t \times 8 = 408$
97 E $s \times 8 = 776$
11 F $608 \times d = 6688$
19 G $r \times 21 = 399$
32 H $22 \times m = 704$
51 I $49 \times b = 2499$

think The dots below show why 4, 9, and 16 are square numbers.

Give the next 10 square numbers.
Find a 2-digit number that is both square and triangular.

Next 10 square numbers: 25, 36, 49, 64, 81, 100, 121, 144, 169, 196; 36 is both square and triangular.

Using the Exercises
On page 133, you may want to work one or two of the exercises with the class, stressing the relationship between the inequalities and the division, before assigning the remaining exercises to be completed independently. Remind the children of the form to use when writing a quotient and remainder:

$$\begin{array}{r} 3\text{ R }3 \\ 4\overline{)15} \end{array}$$

Exercises 7 and 8 provide children with practice in making estimates so that they will be able to do this with division exercises in subsequent lessons.

In order to complete the *Think* problem, children must remember what a triangular number is. Those who have forgotten might find it helpful to review pages 12 and 13.

Assignments (page 133)
Minimum: 1–3, 7A–J.
Average: 1–7. Maximum: 1–8.

PAGES 134–135

Objective

Given a division exercise (with a single-digit divisor), the child will be able to find the quotient by using inequalities to estimate it.

Preparation

To prepare for this lesson, you might give children practice in completing inequalities. For example, you could write the following on the chalkboard:

$n \times 25 < 90$	$32 \times r > 150$
$40 \times s > 150$	$46 \times q < 460$
$t \times 70 < 600$	$86 \times m > 240$

Ask children to substitute for each letter some number which would make the inequality true. There will be several correct choices, but you might point out that one number for each inequality is the greatest number or least number which might be used. For instance, in $n \times 25 < 90$, n may be 1, 2, or 3, but 3 is the *greatest* number that will make the inequality statement true. Likewise, in $32 \times r > 150$, r may be any number greater than 4, but 5 is the *least* number which may be used.

How can estimation help you find quotients?

Discussing the Ideas

1. To find $262 \div 6$, you find the greatest number of sixes that can be subtracted from 262.
   A Can you subtract 10 sixes? Yes B Can you subtract 100 sixes? No

2. $40 \times 6 < 262$
   $50 \times 6 > 262$
   The quotient must be between ? and ?.
   40 50

3. Study each step below. Then try this one on your own: $5\overline{)212}$ 42 R2

Step	Think:	Write:
Step 1 $6\overline{)262}$ $240 \leftarrow 40 \times 6$	I can subtract 40 sixes from 262.	$6\overline{)262}$ 240 (40) 22
Step 2 $6\overline{)262}$ 240 22 $18 \leftarrow 3 \times 6$ 4	Then I can subtract 3 more sixes.	$6\overline{)262}$ 240 (40) 22 18 (3) 4
Step 3 43 $6\overline{)262}$ $240 \leftarrow 40$ 22 $18 \leftarrow 3$ 4	The **quotient** is 43. The **remainder** is **4.**	43 $6\overline{)262}$ 240 (40) 22 18 (3) 4

4. Explain this check for $262 \div 6$. Check: $43 \times 6 = 258$; $258 + 4 = 262$
   Multiply quotient by divisor and add remainder; result should be the same number as the dividend.

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Discussion

Study carefully with the children the material in discussion exercises 1 and 2. Exhibit on the chalkboard the division $6\overline{)262}$ and go through each of the steps illustrated in the three boxes, giving the children an opportunity to explain what is happening in each step. Be sure that you make it clear to the children that on the left we show what they think about and on the right we show what they write on their papers.

Use other examples to show how the inequalities are related to the division problem.

Ask: How many fives are in 437?

Think:	*Write:*
$80 \times 5 < 437$	$5\overline{)437}$ 400 (80)
$90 \times 5 > 437$	37 35 (7) 2

You may need to demonstrate other problems to make the relation clear. Emphasize that the first estimate should be less than the dividend. Then have the children try the division $5\overline{)212}$ on their own.

Using the Ideas

1. Copy the two inequalities in each part. Put in the correct number pair. Then find the quotient and remainder.

Pairs:	10	20	30	40	50	60	70	80	90
	20	30	40	50	60	70	80	90	100

Example

$\square \times 5 < 173$
$\square \times 5 > 173$ → $5\overline{)173}$

Answer

$30 \times 5 < 173$
$40 \times 5 > 173$

$$\begin{array}{r} 34 \\ 5\overline{)173} \\ 150 \quad (30) \\ \hline 23 \\ 20 \quad (4) \\ \hline 3 \end{array}$$

A 30 $\times 4 < 143$; 40 $\times 4 > 143$ → $4\overline{)143}$ 35 R3

B 80 $\times 3 < 258$; 90 $\times 3 > 258$ → $3\overline{)258}$ 86 R0

C 40 $\times 6 < 273$; 50 $\times 6 > 273$ → $6\overline{)273}$ 45 R3

D 70 $\times 4 < 299$; 80 $\times 4 > 299$ → $4\overline{)299}$ 74 R3

E 50 $\times 5 < 280$; 60 $\times 5 > 280$ → $5\overline{)280}$ 56 R0

F 60 $\times 7 < 456$; 70 $\times 7 > 456$ → $7\overline{)456}$ 65 R1

G 30 $\times 9 < 293$; 40 $\times 9 > 293$ → $9\overline{)293}$ 32 R5

2. From the set {10, 20, 30, . . .}, find the largest number that will make the inequality true. Then find the quotient and remainder.

A r (30) $\times 4 < 143$ → $4\overline{)143}$ 35 R3

B m (80) $\times 3 < 258$ → $3\overline{)258}$ 86 R0

C t (40) $\times 6 < 273$ → $6\overline{)273}$ 45 R3

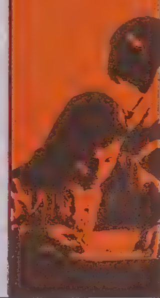
D a (70) $\times 4 < 299$ → $4\overline{)299}$ 74 R3

E q (60) $\times 5 < 324$ → $5\overline{)324}$ 64 R4

F s (40) $\times 8 < 324$ → $8\overline{)324}$ 40 R4

G n (20) $\times 7 < 203$ → $7\overline{)203}$ 29 R0

H d (30) $\times 9 < 306$ → $9\overline{)306}$ 34 R0



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Using the Exercises

Depending on the background and ability of the children, you may want to work through some examples on page 135 as a class activity. You may also choose to have children work together in small groups; they should ask you for assistance only if no one in the group can explain to the others how to do the exercises.

In exercise 2, make sure children realize that they are to work with multiples of 10. Help the children see that finding the *largest* multiple of 10 for the inequality is similar to their work with the number pairs in exercise 1.

Assignments (page 135)
Minimum: 1A–D, 2A–D.
Average: 1–2D. Maximum: 1–2.

Workbook, page 40

PAGES 136–137

Objective

Given a division problem with a 1-digit divisor and 4-digit dividend, the child will be able to find the quotient.

Preparation

To prepare for this lesson, you might provide children with practice with inequalities. For example, write several true and false inequalities on the chalkboard and have children determine which are true and which are false. You might have them replace numbers in the false statements to make them true.

$800 \times 6 < 4950$ (T)
$400 \times 5 < 1819$ (F)
$600 \times 3 > 2035$ (F)
$500 \times 7 < 3642$ (T)
$700 \times 9 > 6482$ (F)
$300 \times 8 < 2571$ (T)

Let's explore 3-digit quotients.

Discussing the Ideas

1. To find $1428 \div 4$, you find the greatest number of fours that can be subtracted from 1428.
   A Can you subtract 100 fours? Yes
   B Can you subtract 1000 fours? No

2. $300 \times 4 < 1428$
   $400 \times 4 > 1428$
   The quotient must be between __?__ and __?__. (300, 400)

3. Study the example below. Then try this one on your own: 5)1733 (346 R3)

Think:		Write:	
357 ←		357	
4)1428	I can subtract	4)1428	
1200 ← 300 × 4	300 fours from 1428.	1200	300
228	Then I can subtract	228	
200 ← 50 × 4	50 more fours.	200	50
28	Finally, I can subtract	28	
28 ← 7 × 4	7 more fours.	28	7
0		0	

4. Find and check the quotient for this example.

```
  307 R5         307
6)1847            ×6
  1800 ← 300 × 6 1842
    47            +5
    42 ←   7 × 6 1847
     5
```

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Discussion

Discussion exercises 1 and 2 stress the importance of using multiples of 100 and 1000 to estimate quotients. The use of inequalities should simply be considered a tool for this purpose. When you discuss exercise 3, work through the division problem 4)1428 step by step on the chalkboard, as illustrated in the text. Be sure the children see that on the left we show what they might be thinking about and on the right we show what they actually write on their paper. For instance, for the first step, they may think 300 is the largest multiple of 100×4 that will go into 1428; hence, they try 300. This gives them 1200 and they subtract to get 228; then they try to find the largest multiple of 10 fours that can be subtracted from 228, and so on.

Work through other examples similarly. As the children work such examples, do not expect them always to make the proper guess the first time. Of course, if a guess is too large, they will have to erase and start over; if their guess is too small, however, they should be encouraged to continue with further subtraction rather than erase and begin again.

Using the Ideas

1. For each part, copy the two inequalities. Put in the correct number pair. Then find the quotient and remainder.

Pairs:

100	200	300	400	500	600	700	800	900
200	300	400	500	600	700	800	900	1000

A [illegible] × 3 < 738; 300 × 3 > 738 ⟶ 3)738 — 246 R0

B 300 × 6 < 2074; 400 × 6 > 2074 ⟶ 6)2074 — 345 R4

C 700 × 5 < 3963; 800 × 5 > 3963 ⟶ 5)3963 — 792 R3

D 700 × 4 < 3008; 800 × 4 > 3008 ⟶ 4)3008 — 752 R0

E [illegible] × 7 < 1372; [illegible] × 7 > 1372 ⟶ 7)1372 — 196 R0

F 900 × 3 < 2820; 1000 × 3 > 2820 ⟶ 3)2820 — 940 R0

G 600 × 8 < 5030; 700 × 8 > 5030 ⟶ 8)5030 — 628 R6

H 700 × 9 < 7134; [illegible] × 9 > 7134 ⟶ 9)7134 — 792 R6

2. From the set (100, 200, 300, . . .), find the largest number that will make the inequality true. Then find the quotient and remainder.

A n (300) × 4 < 1427 ⟶ 4)1427 — 356 R3

B y (500) × 6 < 3125 ⟶ 6)3125 — 520 R5

C r (300) × 5 < 1826 ⟶ 5)1826 — 365 R1

D m (700) × 3 < 2256 ⟶ 3)2256 — 752 R0

E t (600) × 7 < 4488 ⟶ 7)4488 — 641 R1

F p (600) × 8 < 5000 ⟶ 8)5000 — 625 R0

think

The missing digits are all different. Find them.

▯▯▯ ▯▯▯ ▯▯▯
× 9
111 111 111

137

Using the Exercises

The exercises on page 137 are intended to direct the children to use estimation to make the best guess on their first try. Note that each pair of inequalities relates to a division problem. You might use one or two exercises as a basis for discussion. Children should do the actual multiplication to verify the inequalities. For example, in part 1A, children should verify that 200–300 is the correct pair by observing that 200 × 3 < 738 and that 300 × 3 > 738.

Assignments (page 137)
Minimum: 1A–D, 2A–C.
Average: 1–2. Maximum: 1–2.

Follow-up

If children enjoyed the *Think* problem, you might have them try some of the following examples to see whether they can discover the pattern and write others in which multiples of 9 are the multipliers.

```
 97654321      12345679
       ×9          × 18
---------     ---------
888888889      98765432
              12345679
              ---------
              222222222

 12345679      12345679
      ×63          × 72
---------     ---------
 37037037      24691358
74074074      86419753
---------     ---------
777777777     888888888
```

Resources for Active Learning

Mathematics in Modules, WN17, Addison-Wesley.

Workbook, page 41

PAGES 138–139

Objective

Given an appropriate set of numbers, the child will be able to find their average.

Preparation

Materials

colored strips

Review with the children how the strips can be assigned number names according to a particular chosen unit. For example, ask them to place the strips in a "stair-step" fashion, ranging from the shortest to the longest. Suggest that they think of the white strip as having the value of one. Then ask what number would be assigned to the red strip, to the yellow strip, and so on until you have reviewed the correspondence between these strips and the numbers 1 to 10, when the white strip is the unit.

Investigation

Some children may need guidance in beginning this investigation. You might explain that here the phrase "matching train" means a train which has the same length as another train. They are to find pairs of trains which have the same length and the same number of strips but which differ in the particular strips used for each train. One train of the pair must consist of one strip repeatedly; the other train must contain at least two strips that are different.

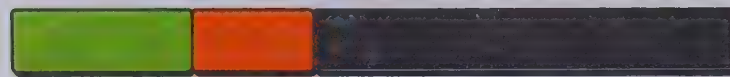
It is important for children to record the results of their investigation. Have them write down the strips used for each pair. Sample pairs of matching trains follow:

3,3,3; 4,2,3
2,2,2; 4,1,1
3,3,3; 1,4,4
4,4,4,4; 6,3,3,4

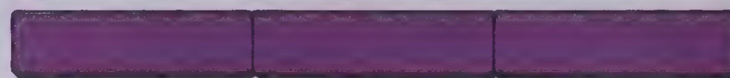
What is the average of a set of numbers?

Investigating the Ideas

Here is a "train" of three strips, **not all the same.**



Here is a "matching train" of three strips, **all the same.**



?

Can you make some pairs of "matching trains" so that
1 matching trains have the same number of strips,
2 one train has strips all the same, and
3 one train has strips not all the same? See Investigation.

Discussing the Ideas

1. How are these two equations like the trains above?
   A $3 + 2 + 7 = 12$
   B $4 + 4 + 4 = 12$
   See Discussion.

We say that **4** is the **average** of the numbers 3, 2, and 7.

2. Give the averages you found for your matching trains.
   Answers will vary.

3. Can you show with your strips that 3 is the average of 5, 4, 2, and 1?
   Match four 3-strips with a train made of one 5-strip, one 4-strip, one 2-strip, and one 1-strip.

Discussion

Exercise 1 relates the matching trains to addition and develops the idea of average. During your discussion, emphasize how the strips of the same length can be used to replace an equal number of unlike strips to build a train of the same length. Explain that the number corresponding to the strip which can replace the unlike strips is the average of that set of numbers. Emphasize that the word *average* refers to that number which can be substituted for each addend in an addition problem and still give the same sum. Observe with the children that in some cases there is no whole number that can be substituted for a given set of addends which will give the same sum.

For example, if they start with strips 4, 7, and 2, they will not find three strips of equal length which would form a matching train. This is because 13 is not evenly divisible by any whole number; the average in this case, then, involves the idea of fractions. You need not elaborate on this point, however, because most of the exercises given to the children at this time involve only whole-number averages.

Note: Question I may appear incorrectly in the student's book. The correct form is given here.

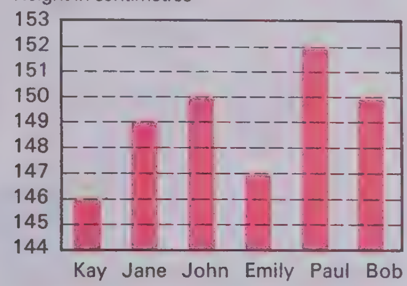
Using the Ideas

1. Find the sums. Then find the **average** of the addends.
   - A $12 + 8 = r$ 20, 10
   - B $3 + 8 + 4 = n$ 15, 5
   - C $5 + 9 + 10 + 4 = t$ 28, 7
   - D $6 + 9 + 12 + 5 + 8 = m$ 40, 8
   - E $20 + 32 + 35 = y$ 87, 29
   - F $63 + 75 + 47 + 27 = p$ 212, 53

2. Complete the sentences. Then find the average.
   - A To find the average of 4, 5, and 9, divide _?_ by 3. 18, 6
   - B To find the average of 6, 10, 13, and 3, divide _?_ by 4. 32, 8
   - C To find the average of 55 and 29, divide _?_ by _?_. 84, 2, 42

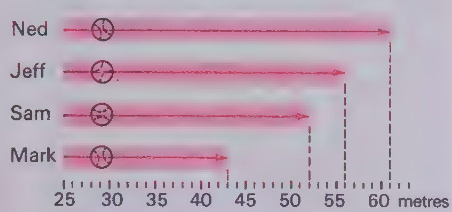
3. Give the missing numbers.
   - A The average of 5, 7, and 12 is ||||. 8
   - B The average of 4, 12, 7, 9, 13, and 3 is ||||. 8

4. The bar graph shows the heights in centimetres of 6 children. Find the average height of these children. 149 cm



5. Jan is 3834 days old. Fran is 4015 days old. Nan is 3923 days old. Find the average age (in days) of the 3 girls. 3924 days

6. This graph shows how far some boys threw a softball. Find the average distance. 53 m



★ 7. A Find the average weight (to the nearest whole number) of the children in your class. Answers will vary from class to class.
   B Find their average height in centimetres.

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More practice, page A-11, Set 21

Follow-up

Encourage the children to work on starred exercise 7 as a group or class project. They might also study other data, such as average age (in days); average daily attendance for a week; average pulse rates for boys, for girls, and for the whole class before, during, and after exercise; average breathing rate; average body temperature; and so on. Linking practice to some problem of current interest to the children can provide motivation that might otherwise be lacking.

Resources for Active Learning

Developmental Math Cards 1-2 13, Addison-Wesley.
Freedom to Learn, "Statistics," p. 148, Addison-Wesley.
Mathematics in Modules, S2, S8, Addison-Wesley.
Mathex: Graphing and Probability No. 6, "Averages," pupil pages 34–37, Encyclopaedia Britannica Publications Ltd.
Nuffield Project: *Probability and Statistics*, "Simple Average," pp. 38–45, Wiley.

Duplicator Masters, page 29
Workbook, page 42

Using the Exercises

Have the children do the exercises on page 139 independently. We leave it to your discretion to decide when the children should generalize the rule for finding averages. Many children will discover the rule as they work through exercises 1, 2, and 3. You will want to provide most children ample opportunity to make the correct discovery on their own. After they complete exercise 3, however, it might be helpful to have children generalize in their own words the rule for finding averages.

Assignments (page 139)
Minimum: 1–4. Average: 1–6.
Maximum: 1–7.

Objective

Given a division problem with a single-digit divisor, the child will be able to find the quotient by using a shortcut method.

Preparation

To prepare for this lesson, give children practice in finding averages of three or four single-digit numbers whose sum is related to one of the basic facts. For example, ask them to find the average of each group of numbers below.

3, 4, 5 $(12 \div 3 = 4)$
7, 9, 5 $(21 \div 3 = 7)$
12, 8, 9, 7 $(36 \div 4 = 9)$
2, 6, 5, 7 $(20 \div 4 = 5)$

Such an oral activity would review not only the concept of finding averages but also some basic division facts.

- Can we shorten the work in dividing?

Discussing the Ideas

Study the steps in the example below. Use the same steps to find this quotient: $4\overline{)1815}$ 453 R 3

Long Way	1st Shortcut	2nd Shortcut
Step 1 $8\overline{)2915}$ 2400 (300) 515	3 $8\overline{)2915}$ 2400 515 This quotient has **3 hundreds.**	3 $8\overline{)29^{5}15}$ 29 ÷ 8 The quotient is **3.** The remainder is **5.**
Step 2 $8\overline{)2915}$ 2400 (300) 515 480 (60) 35	36 $8\overline{)2915}$ 2400 515 480 35 This quotient has **6 tens.**	36 $8\overline{)29^{5}1^{3}5}$ 51 ÷ 8 The quotient is **6.** The remainder is **3.**
Step 3 364 $8\overline{)2915}$ 2400 (300) 515 480 (60) 35 32 (4) 3	364 $8\overline{)2915}$ 2400 515 480 35 32 3 This quotient has **4 ones.**	364 R3 $8\overline{)29^{5}1^{3}5}$ 35 ÷ 8 The quotient is **4.** The remainder is **3.**

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Discussion

To benefit fully from this discussion section, the children should work along on their papers as the steps are discussed. Note that the second shortcut encourages children to think first of dividing the number represented by the first or the first two digits of the number being divided. Thus, for $8\overline{)2915}$, they are encouraged to think of first dividing $8\overline{)29}$, then dividing $8\overline{)51}$, and finally $8\overline{)35}$. Besides this slightly different way of thinking, the second shortcut also introduces the technique of short division. You might use the following chalkboard illustration as you work through this shortcut.

1. Instead of writing this, write this

$7\overline{)2394}$ 2100	300	3 $7\overline{)2394}$ 2100

2. Instead of writing this, write this

$7\overline{)2394}$ 2100 294 280	300 40	34 $7\overline{)2394}$ 2100 294 280

3. Instead of writing this, write this

$7\overline{)2394}$ 2100 294 280 14 14	300 40 2	342 $7\overline{)2394}$ 2100 294 280 14 14

The second shortcut applies the child's ability to think in the manner developed in the first shortcut. The key to the short-division techniques is found in the three "think clouds" for dividing hundreds, tens, and ones. The rest of the material is intended to help the children see how this convenient shortcut is related to the long method they have used previously.

Work through the problem $4\overline{)1815}$ similarly, and present other examples as needed.

Using the Ideas

1. Find each quotient and remainder.

A 7)58 8 R2 → 7)581 83 RO
B 8)34 4 R2 → 8)340 42 R4
C 7)44 6 R2 → 7)4494 642 RO
D 6)33 5 R3 → 6)3366 561 RO
E 8)42 5 R2 → 8)4296 537 RO
F 4)25 6 RI → 4)257 64 RI
G 6)38 6 R2 → 6)384 64 RO
H 7)67 9 R4 → 7)6776 968 RO
I 9)29 3 R2 → 9)2943 327 RO
J 9)57 6 R3 → 9)5769 641 RO

2. Find the quotients and remainders.

A 4)329 82 RI
B 8)969 121 RI
C 8)47,562 5945 R2
D 9)6498 722 RO
E 6)4356 726 RO
F 6)2738 456 R2
G 3)1824 608 RO
H 8)31,254 3906 R6
I 9)4635 515 RO
J 9)6666 740 R6
K 5)2437 487 R2
L 7)5000 714 R2
M 7)3246 463 R5
N 5)23,647 4729 R2
O 4)2603 650 R3
P 6)56,213 9368 R5

3. The table gives weights of 8 children. Find the average weight of the children. 37 kg

Jay	76	Ann	79
Nancy	64	Ted	85
Susan	68	Bill	98
Steve	96	Mary	82

4. Here are the distances for 6 boys in the standing broad jump:
165 centimetres
140 centimetres
185 centimetres
161 centimetres
145 centimetres
180 centimetres
173 centimetres
170 centimetres
Find the average length of the jumps. 165 cm
(Actually $164\frac{7}{8}$ cm)

think

The numbers in the sets are consecutive numbers.

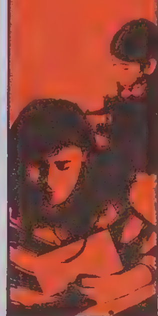
{5, 6, 7} {58, 59, 60, 61}

Now find the three consecutive numbers in this riddle.

A fine trio are we.
Consecutive numbers, too.
Our sum is sixty-three.
The rest is up to you. 20, 21, 22

More practice, page A-11, Set 22

141



Follow-up

If children enjoyed the riddle in the *Think* problem, encourage them to make up similar riddles of their own. Their riddles may follow the given pattern closely or they may be more original.

A fine trio, it's true.
Consecutive numbers, too.
Our sum is seventy-two.
The rest is up to you.
(23, 24, 25)

Duplicator Masters, pages 30–31
Workbook, page 43
Skill Masters, pages 30-31

Using the Exercises

On page 141, children should work some parts of exercise 1 by the long method and some by one of the short methods. Encourage them to use short division for exercise 2. You might work through some of the problems together.

Frequent reference to the long method, where the estimates are written at the side, will help the children understand not only the basic working of the algorithm but also such integral problems as zeros in the quotient. You will want to emphasize parts G, H, J, and O of exercise 2 since there are zeros in the quotient.

Assignments (page 141)

Minimum: 1A–E, 2A–H.
Average: 1A–C, 2A–L, 3.
Maximum: 1–4.

PAGES 142–143

Objective

Given division problems and related data, the child will be able to solve them by using division with single-digit divisors and by applying their understanding of averages.

Preparation

Your preparation period might well centre on a review of the short-division algorithm covered in the last lesson. Much of the work on these two pages involves using the algorithm for single-digit divisors. Therefore, if the children are proficient in using the division algorithm, they will be able to work most of these problems readily.

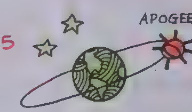
Short Stories

1 688 tractors.
8 on each flatcar.
How many flatcars? 86

2 52 weeks → 1 year.
19 years.
How many weeks? 988

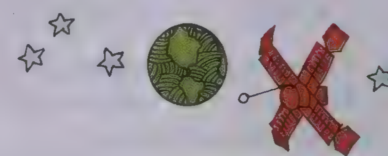
3 4 pitchers fill 1 jug. 2344 pitchers. How many jugs? 586

4 9 tiles cover 1 square metre.
6615 tiles. How many square metres? 735



5 Satellite.
Apogee (farthest distance from earth), 3304 kilometres.
Perigee (closest distance from earth), 554 kilometres.
- A What is the difference in these distances? 2750 km
- B What is the average of these distances? 1929 km

6 Satellite makes 867 orbits. 9 orbits each day. How many full days? How many extra orbits?
96 full days; 3 extra orbits



7 Car wheel.
One revolution → 2 metres
How many revolutions for 1 kilometre? 500

8 6 wheels on a truck. 35 958 wheels.
How many trucks? 5993

9 Relay team → 4 runners. 112 runners. How many teams? 28

10 Drove 3 hours. Average speed, 104 km/h. Then drove 3 more hours. Average speed, 88 km/h. Travelled how far? 576 Km

11 Neon sign blinks 7 times each minute.
- A How many blinks in an hour? 420
- B How many blinks in a 24-hour day? 10 080

12 Satellite into orbit.
Speed, 8 km per second.
- A How many km per hour?
- B How many km per day?

A. 28 800 km B. 691 200 km

142

Discussion

Before assigning the problems on pages 142 and 143, allow discussion of the material at the top of page 143. Observe the various symbols for the planets. (Note, however, that the children are not expected to memorize these symbols or the names of the planets.) As we have frequently noted, this type of lesson can play a significant role in arousing children's interest in arithmetic and its application in other areas.

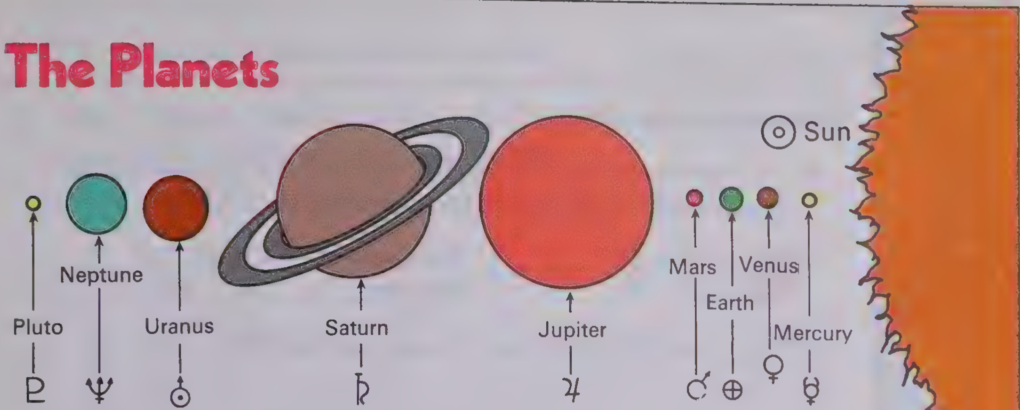
Assignments (page 142)

Minimum: Odd-numbered problems.

Average: Even-numbered problems.

Maximum: All.

The Planets



The relative sizes of the nine planets are shown above. The ancient symbols shown below the names of the planets are still used by many astronomers to represent these planets. The word "planet" comes from a Greek word meaning "wanderer." Can you explain why a word with this meaning was used?

Planets	Millions of kilometres from the sun
Mercury	58
Venus	108
Earth	150
Mars	228
Jupiter	778
Saturn	1427
Uranus	2870
Neptune	4500
Pluto	5909

1. The average distances of the planets from the sun are given in the table.

- **A** How much farther from the sun is Mars than Earth? 78 million km
- **B** Which planet is about 100 times as far from the sun as Mercury? Pluto
- **C** Is the distance of Neptune from the sun more or less than four times the distance of Jupiter from the sun? How much more or less? 1388 million km more

2. The diameters of the four planets closest to the sun are given beside the symbols for the planets. Find the average diameter of these planets. 9118½ km or 9119 km

☿ 4866 km ♀ 12 106 km ♂ 6760 km ⊕ 12 742 km

★ **3.** Find the average diameter in kilometres of the four largest planets. 87 097½ km or 87 098 km

♆ 45 430 ⛢ 46 940 ♄ 116 440 ♃ 139 580

143

Follow-up

Other space facts like those provided in this lesson might give children the incentive to write and solve problems of their own. For example, the weight of something is an indication of the pull of gravity on it. The pull of gravity on Earth is about 6 times that on the moon. This means that a boy who weighs 48 kilograms on Earth would weigh only 8 kilograms on the moon. This information could provide the basis for such problems as these:

1. About how much would a sack of cement weigh on the moon if it weighs 54 kilograms on Earth?
2. By Earth standards, would a woman be overweight if she weighed 28 kilograms on the moon? Why?
3. Suppose the Earth weights of three scientific instruments were as shown below. About how much would each of these weigh on the moon?
   (a) 81 kilograms
   (b) 16 kilograms
   (c) 426 kilograms
4. About how much would you weigh on the moon?
5. If you could high jump 150 centimetres on Earth, about how high might you be able to jump on the moon?

Resources for Active Learning

Measure and Find Out, Book 3, "Here Come the Planets," Activity 1/10, Scott Foresman. (Available from Gage Educational Publishing)

Workbook, page 44

Using the Exercises

Have the children do the problems on both pages and when they have finished, allow time for further discussion and checking papers.

Some children may be helped by writing a division equation or a multiplication equation with a missing factor for any problems on page 142 which may have caused difficulty. Also, note that the terms *apogee* and *perigee* in problem 5 may need further elaboration.

Assignments (page 143)

Minimum: 1. Average: 1–2.
Maximum: 1–3.

PAGES 144–145

Objective

Given division exercises involving a divisor that is a 2-digit multiple of ten, the child will be able to estimate and find the quotient.

Preparation

It would be suitable to provide practice with inequalities which involve multiples of ten. For example, you might write the following on the chalkboard, and have the children respond true or false for each.

$6 \times 40 > 328$ (F)
$70 \times 30 < 3152$ (T)
$50 \times 5 > 241$ (T)
$40 \times 80 < 3074$ (F)
$6 \times 30 > 211$ (F)
$80 \times 70 > 5420$ (T)

Let's divide with multiples of ten.

Discussing the Ideas

1. To find the quotient for $30\overline{)287}$, we find the greatest number of thirties that can be subtracted from 287.

$9 \times 30 < 287$
$10 \times 30 > 287$

The quotient must be between __?__ and __?__. 9, 10

2. Study the example. Then try this one: $40\overline{)256}$ 6 R16

Think:		Write:
$\begin{array}{r} 9 \\ 30\overline{)287} \\ \underline{270} \leftarrow 9 \times 30 \\ 17 \end{array}$	We can subtract 9 thirties from 287. The quotient is 9. The remainder is 17.	$\begin{array}{r} 9 \\ 30\overline{)287} \\ \underline{270}\ \ 9 \\ 17 \end{array}$

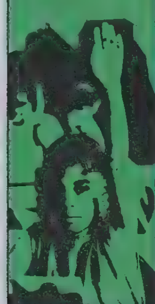
3. To find the quotient for $40\overline{)3168}$, we find the greatest number of forties that can be subtracted from 3168.
   - A Can you subtract 10 forties? Yes
   - B Can you subtract 100 forties? No

4. $70 \times 40 < 3168$
$80 \times 40 > 3168$

The quotient must be between __?__ and __?__. 70, 80

5. Study the example. Then try this one on your own: $60\overline{)2145}$ 35 R45

Think:		Write:
$\begin{array}{r} 79 \\ 40\overline{)3168} \\ \underline{2800} \leftarrow 70 \times 40 \\ 368 \\ \underline{360} \leftarrow 9 \times 40 \\ 8 \end{array}$	We can subtract 70 forties from 3168. Then we can subtract 9 more forties. The quotient is 79. The remainder is 8.	$\begin{array}{r} 79 \\ 40\overline{)3168} \\ \underline{2800}\ \ 70 \\ 368 \\ \underline{360}\ \ 9 \\ 8 \end{array}$



144

Discussion

Observe with the children that in this lesson they are again studying how inequalities can be used to help estimate quotients. The estimations should help children arrive at the correct quotients on their first guess. Though it is not essential that children make the best estimate on their first guess, the ability to do so will greatly increase their computational efficiency.

In exercises 2 and 5, point out that on the left we show what we are thinking about and on the right we show how this should be written.

It would be helpful to work through other examples similar to $40\overline{)3168}$. For example, illustrate $90\overline{)6389}$:

$90 \times 70 < 6389$
$90 \times 80 > 6389$

Think:

$$\begin{array}{r} 70 \\ 90\overline{)6389} \\ \underline{6300} \leftarrow 70 \times 90 \\ 89 \\ \underline{0} \leftarrow 0 \times 90 \\ 89 \end{array}$$

We can subtract 70 nineties from 6389. We can subtract 0 nineties from 89.

The quotient is 70. The remainder is 89.

Write:

$$\begin{array}{r} 70 \text{ R } 89 \\ 90\overline{)6389} \\ \underline{6300}\ \ 70 \\ 89 \end{array}$$

As you work through such an example, stress the relationship between the type of division here and division using only a 1-digit divisor.

Using the Ideas

1. Find the largest whole number that will make the inequality true. Then find the quotient and remainder.

A $n \times 30 < 287 \longrightarrow 30\overline{)287}$ 9 | 9 R17
E $c \times 60 < 317 \longrightarrow 60\overline{)317}$ 5 | 5 R17

B $a \times 70 < 371 \longrightarrow 70\overline{)371}$ 5 | 5 R21
F $k \times 20 < 197 \longrightarrow 20\overline{)197}$ 9 | 9 R17

C $b \times 40 < 213 \longrightarrow 40\overline{)213}$ 5 | 5 R13
G $f \times 90 < 563 \longrightarrow 90\overline{)563}$ 6 | 6 R23

D $r \times 80 < 650 \longrightarrow 80\overline{)650}$ 8 | 8 R10
H $s \times 30 < 284 \longrightarrow 30\overline{)284}$ 9 | 9 R14

2. From the set {10, 20, 30, . . .}, find the largest number that will make the inequality true. Then find the quotient and remainder.

A $s \times 40 < 3168 \longrightarrow 40\overline{)3168}$ 70 | 79 R8
E $p \times 50 < 4111 \longrightarrow 50\overline{)4111}$ 80 | 82 R11

B $t \times 60 < 1378 \longrightarrow 60\overline{)1378}$ 20 | 22 R58
F $q \times 70 < 2971 \longrightarrow 70\overline{)2971}$ 40 | 42 R31

C $n \times 80 < 3396 \longrightarrow 80\overline{)3396}$ 40 | 42 R36
G $f \times 90 < 8406 \longrightarrow 90\overline{)8406}$ 90 | 93 R36

D $a \times 30 < 1008 \longrightarrow 30\overline{)1008}$ 30 | 33 R18
H $r \times 20 < 297 \longrightarrow 20\overline{)297}$ 10 | 14 R17

3. Find the quotients and remainders.

A $20\overline{)130}$ 6 R10 B $40\overline{)276}$ 6 R36 C $30\overline{)187}$ 6 R7

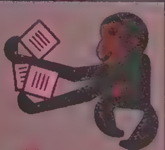
D $70\overline{)513}$ 7 R23 E $60\overline{)269}$ 4 R29 F $50\overline{)420}$ 8 R20

G $20\overline{)646}$ 32 R6 H $40\overline{)2500}$ 62 R20 I $40\overline{)999}$ 24 R39

J $80\overline{)7580}$ 94 R60 K $40\overline{)3700}$ 92 R20 L $70\overline{)4970}$ 71 R0

M $50\overline{)3863}$ 77 R13 N $90\overline{)8834}$ 98 R14 O $50\overline{)2507}$ 50 R7

think



▒ × ▒ × ▒ = 720

The red screens cover three consecutive whole numbers whose product is 720.

Find these numbers.

Find three other numbers whose product is 720.

Sample answer: 4, 12, 15

More practice, page A-12, Set 23

Using the Exercises

Assign the exercises on page 145 as independent work, but observe and help those children who still have difficulty. Note that the quotients in exercise 1 will be single-digit answers. In exercise 2, make sure children realize that their estimates should be multiples of 10, and the quotients should be 2-digit quotients.

When children try the *Think* problem, if they realize that ten is one factor of the product 720, they should have little difficulty choosing 8, 9, and 10 as their three consecutive numbers.

Assignments (page 145)
Minimum: 1A–D, 2A–D, 3A–C.
Average: 1A–D, 2A–D, 3A–I.
Maximum: 1–3.

Duplicator Masters, page 32
Workbook, page 45
Skill Masters, page 32

PAGES 146–147

Objective

Given a division problem with a 2-digit divisor, the child will be able to estimate the quotient by rounding the divisor to the nearest multiple of ten.

Preparation

To prepare for this lesson, briefly review rounding numbers to the nearest multiple of 10. For example, list several 2-digit numerals on the chalkboard and ask children to give the nearest multiple of ten for each. Remind them of the guide: if the numeral ends in 1, 2, 3, or 4, round down; if it ends in 5, 6, 7, 8, or 9, round up.

Let's explore 2-digit divisors.

Discussing the Ideas

1. To find the quotient for 52)378, think → ? × **50** < 378
   - A What is the largest number that will make the sentence true? 7
   - B Explain how the dividing is completed when you use the number found in exercise 1A. Subtract the product 7×52 from the dividend. The quotient is 7 with remainder 14.

$$\begin{array}{r} 7 \\ 52\overline{)378} \\ \underline{364} \\ 14 \end{array} \quad ⑦$$

2. To find the quotient for 63)368, think → ? × **60** < 368
   - A What is the largest number that will make the sentence true? 6
   - B Why is the number you found in exercise 2A not the correct quotient? Because 6×(60+3)>368
   - C Explain how to choose the correct quotient and complete the dividing. See Discussion.

$$\begin{array}{r} 5 \\ 63\overline{)368} \\ \underline{315} \\ 53 \end{array} \quad ⑤$$

3. To find the quotient for 35)218, think → ? × **40** < 218
   - A What is the largest number that will make the sentence true? 5
   - B Multiply the divisor by the number you found in exercise 3A; then subtract. Is the remainder less than the divisor? No
   - C Explain how the dividing is completed. Subtract 1×35 from remainder 43. The quotient is 6 with remainder 8.

$$\begin{array}{rl} 6 & \\ 35\overline{)218} & \\ \underline{175} & ⑤ \\ 43 & \\ \underline{35} & ① \\ 8 & \end{array}$$

4. Find the quotients and remainders. Be ready to explain your work.

   A 81)486 6 R0   B 59)420 7 R7   C 42)265 6 R13

   D 65)500 7 R45   E 74)289 3 R67   F 26)209 8 R1

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Discussion

Work through page 146 very carefully as a class activity. Note in particular that in exercise 1 the estimation gives a quotient that is just right. In exercise 2, the rounding of 63 to 60 gives an estimate that is too large. That is, when considering $60\overline{)368}$ or $6\overline{)36}$, we arrive at the quotient 6. Of course, a quick test will show that 6 × 63 is actually greater than 368; hence, we must select 5 as the quotient. In exercise 3, we observe that rounding 35 up to 40 gives an estimate of the quotient as 5, and this, of course, is too small.

Following a discussion of the three examples shown here, continue the discussion by exhibiting on the chalkboard the problems for exercise 4. Have the children make their estimates and observe with them whether the estimate comes out just right, too large, or too small. Keep in mind again that one of the most important objectives of this lesson is to help the children see that rounding up or rounding down may give them a first guess that is not the best guess. Point out to the children that if their quotient is too large, they must erase and start over, but if it is too small they can continue as shown in exercise 3.

Using the Ideas

1. Copy the two inequalities with the correct number pair. Then find the quotient and remainder.

Pairs: 1, 2   2, 3   3, 4   4, 5   5, 6   6, 7   7, 8   8, 9   9, 10

Think: ? × 40 < 218

Think: ? × 50 < 378

A 6 × 35 < 218
7 × 35 > 218 → $35\overline{)218}$ 6R8

B 5 × 61 < 327
6 × 61 > 327 → $61\overline{)327}$ 5R22

C 8 × 43 < 371
9 × 43 > 371 → $43\overline{)371}$ 8R27

D 6 × 37 < 223
7 × 37 > 223 → $37\overline{)223}$ 6R1

E 7 × 52 < 378
8 × 52 > 378 → $52\overline{)378}$ 7R14

F 7 × 49 < 368
8 × 49 > 368 → $49\overline{)368}$ 7R25

G 8 × 72 < 585
9 × 72 > 585 → $72\overline{)585}$ 8R9

H 6 × 89 < 555
7 × 89 > 555 → $89\overline{)555}$ 6R21

2. Find the largest whole number that will make the inequality true. Then find the quotient and remainder.

A n × 29 < 178 → $29\overline{)178}$ (n = 6) 6R4

B p × 61 < 253 → $61\overline{)253}$ (p = 4) 4R9

C q × 42 < 137 → $42\overline{)137}$ (q = 3) 3R11

D a × 39 < 226 → $39\overline{)226}$ (a = 5) 5R31

E n × 51 < 423 → $51\overline{)423}$ (n = 8) 8R15

F r × 19 < 173 → $19\overline{)173}$ (r = 9) 9R2

think

Give the missing numbers so that this will be a magic square.

93	79	80	90
82	88	87	85
86	84	83	89
81	91	92	78

Using the Exercises

Before assigning the exercises on page 147, emphasize for the children that the "think cloud" and the recommended rounding of the divisor is only an aid in finding the correct number pair. In each case, they must find two products to be sure that they have found the correct number pair. In exercise 1A, a child should think 5 × 40 < 218. Then, since 6 × 40 > 218, it is tempting to use the number pair 5, 6 for the answer to exercise 1A. However, upon careful observation and actual computation of the two products, the child will discover that both 5 × 35 < 218 and 6 × 35 < 218. In other words, for the number pair 5, 6 the first inequality is true, but the second is not. From this, the child should see that the number pair 6, 7 is the correct answer.

Assignments (page 147)
Minimum: 1A–D, 2A–C.
Average: 1A–D, 2.
Maximum: 1–2.

Workbook, page 46

PAGES 148–149

Objective

The child will demonstrate his ability to work with the concepts presented in this chapter.

Preparation

Review with the children any topics from the chapter with which they may have had special difficulty, such as finding averages, or time, rate, and distance. Since a considerable amount of the chapter is devoted to division, you will probably want to spend most of the period reviewing this.

Reviewing the Ideas

1. Find the sums.

A	B	C	D	E
594	976	898	9864	8765
637	86	407	387	3479
+868	+368	+600	+98	+2698
2099	1430	1905	10 349	14 942

2. Find the differences.

A	B	C	D	E
6007	5009	3070	6508	6000
−4387	−4923	−1487	−2346	−1374
1620	86	1583	4162	4626

3. Find the total amounts.

A	B	C	D	E
\$2.79	\$12.50	\$0.69	\$2.84	\$18.95
1.39	23.75	1.58	9.76	23.50
			8.41	11.66
\$4.18	\$36.25	\$2.27	\$21.01	\$54.11

4. Find the products.

A	B	C	D	E
76	372	58	15,365	8734
×9	×6	×47	×29	×263
684	2232	2726	445 585	2 297042

5. Find the quotients and remainders.

A 9)414 46 R0   C 30)195 6 R15   E 50)4350 87 R0   G 69)569 8 R17

B 4)345 86 R1   D 21)189 9 R0   F 70)4700 67 R10   H 7)6769 967 R0

think

Here is an interesting calendar puzzle. Have a friend choose a 3-by-3 "square" of dates on any calendar. Tell him you can find the sum of these dates faster than he can if he will just give you the smallest date in the square. Suppose he chooses the dates colored in the calendar shown. He would say 9. You would add 8 and then multiply by 9. This will always give you the sum of the dates. Can you explain why?

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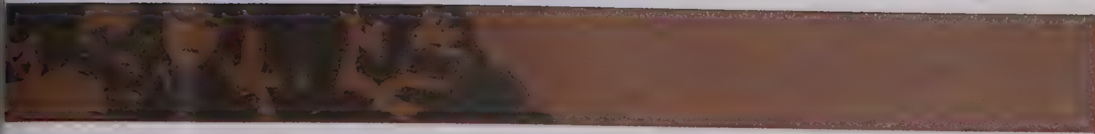
Discussion

Have the children do the exercises. You may suggest to some children that they do only part of exercises 1–5, since there is a considerable amount of work here.

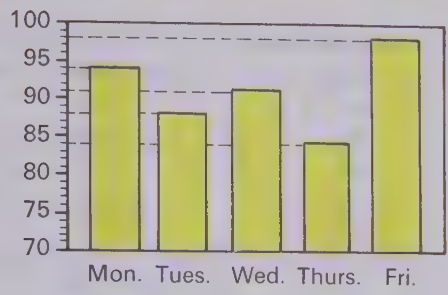
In exercise 8, the children are expected first to find the average number of points scored by each team during the six games (72 and 68) and then to find the average number of points scored per game by both teams, 140.

Children who finish quickly should attempt the *Think* problem.

When the children have finished the exercises, allow time for discussion and checking papers. You will find that all the children will benefit from a discussion of the *Think* problem. Give those who successfully completed the problem an opportunity to explain why the suggested shortcut for finding the sum works. Some children may say that the "centre" date of the nine dates chosen will always be one week and one day after the smallest date chosen. Therefore, they are really finding the average by adding 8 to the number given. Then, multiplying this average by 9, they get the sum of the nine dates. Be prepared for several different approaches to the question of why this method works.



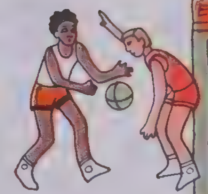
6. The bar graph shows Tom's scores on 5 spelling tests. Give his average score. 91



7. The population of the city of Winnipeg was 246 246 in 1971. The population of Winnipeg's Metropolitan area was 540 262.
   A Was the metropolitan population more than twice the population of the city? Yes
   B How many more than twice the population of the city? 47 770

8. The Eagles played the Stars in 6 basketball games during the season. Here is a record of the scores. Give the average number of points scored per game by the Eagles and by the Stars. Give the average number of points scored per game.
Average per game: 140

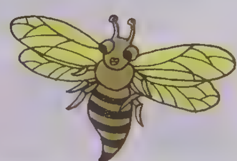
Eagles: 72
Stars: 68



EAGLES	STARS
68	55
82	84
53	48
94	67
60	69
75	85

9. There are 52 weeks in one year. A rabbit is considered fairly old when it has lived 312 weeks. How many years is this? 6

10. A package of 24 sheets of paper is called 1 **quire** of paper. If you buy 212 sheets of paper, how many quires have you bought? 8
How many extra sheets? 20



11. About 168 hours after a specially fed grub worm spins a cocoon about itself, a queen bee is formed inside the cocoon. How many days is this? 7

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Follow-up
More capable or interested children can be encouraged to organize data into a graphic framework as well as to interpret and analyze data presented on a graph. Suitable types of data might include velocity and acceleration rates, space and satellite materials, physical data about members of the class, and the outcomes obtained from probability experiments such as tossing coins or colored dice. Your current class work might also provide many pertinent ideas.

Workbook, page 47

PAGES 150–151

Objective

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

Provide the children with a review of any of the material in the text with which they may have had difficulty. Since it has been some time since the children have worked with the long multiplication algorithm, you might wish to centre much of your preparation activities on work with 3-digit multipliers. We suggest that you present on the chalkboard exercises similar to 2C and 2H, page 150, and have the children explain each step as they work through the problem.

Keeping in Touch with Addition, Subtraction, Multiplication, Division, Measurement, Functions

1. Find the sums.

A	B	C	D
29	865	5764	2463
87	976	987	172
96	493	8368	5432
54	976	476	1621
+32	+487	+98	+8464
298	3797	15 693	18 152

2. Find the sums, products, or differences.

A 987 +643 = 1630	D 635 ×31 = 19 685	G 807 −499 = 308	J 5869 ×54 = 316926
B 8003 −69 = 7934	E 6597 +9886 = 16 483	H 385 ×267 = 102 795	K 8309 −2847 = 5462
C 8596 ×233 = 2 002 868	F 4020 −1976 = 2044	I 6000 −3986 = 2014	L 6517 ×436 = 2 841 412

3. Solve the equations.

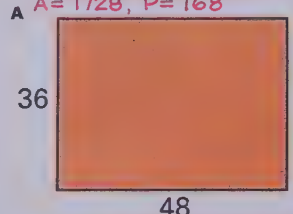
A $(54 - 9) + 9 = y$ 54 D $(90 - d) + 13 = 90$ 13 G $(7 \times 4) \div r = 7$ 4

B $(91 - 7) + m = 91$ 7 E $(320 + 24) - n = 320$ 24 H $(6 \times c) \div 8 = 6$ 8

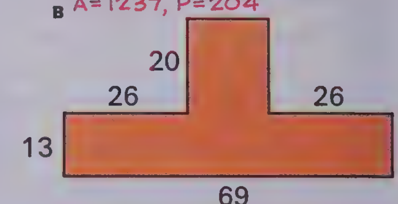
C $(107 + 6) - 6 = b$ 107 F $(9 \times 6) \div 6 = p$ 9 I $(56 \div 8) \times 8 = t$ 56

4. Find the area and perimeter of each region.

A A= 1728, P= 168



B A= 1237, P= 204



You are invited to explore

ACTIVITY CARD 6
Page 336

150

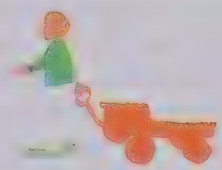
Discussion

Have the children do the exercises. When they have finished, allow time for checking papers and presentation of some of the exercises on the chalkboard. You should allow some of the children to explain their thinking in arriving at the areas and perimeters for exercise 4 on page 150.

Short Stories ECOLOGY

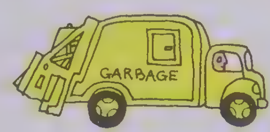
1 School Ecology Club. Collected 1840 aluminum cans. One cent for every 2 cans. How much money? $9.20

2 30 students. Collected 840 kilograms of newspapers for recycling. How many kilograms per person? 28



3 Aluminum cans: 96 kilograms. 10¢ a kilogram. How many dollars' worth? $9.60

4 City: 25 000 people. 2 kilograms garbage per person each day. How many tonnes of garbage each day? 50



5 Used newspapers: 40¢ for 50 kilograms. How much for one tonne? $8.00

6 Estimated world population. Today: 4 billion people. Year 2000: 2 billion more than today. How many people in 2000? 6 billion

7 World population in 1925: 2 billion. Today: 4 billion. How many years for population to double? Answer depends on current year.

8 Provincial Parks in Canada. 1964: 227 320 square kilometres. 1971: 246 530 square kilometres. How much more space in 1971? 19 210 km²

9 City population: 750 000. Cost of new sewage equipment: $300 per person. Total cost? $225 000 000

10 Pollution in the atmosphere each year. Cars: 86 million tonnes. Factories: 43 million tonnes. Heating and burning: 13 million tonnes. How many tonnes in all? 142 million

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Using the Exercises

An important point of the exercises on page 151 is to give the children an opportunity to discuss the theme of the lesson in some detail. You may find that these ecology problems will spark enough interest in your children to warrant further study and, possibly, will suggest special projects related to ecology and the state of our environment.

Follow-up

As a conclusion to this chapter, you might prepare a duplicator worksheet of reconstruction problems for the four basic operations. Below are sample problems of this kind.

Complete the following problems.

```
  ▯7▯        ▯76        ▯74
  4▯1      − 2▯8      − 26▯
  648        27▯        3▯5
 ▯372

      ▯       25          65▯
 63)49▯     × 9▯     5▯)▯8,562
    4▯1      100        35▯
     54     ▯2▯          ▯16
            2▯▯▯         2▯▯
                          212
                          1▯7
                           3▯
```

CHAPTER 7 | Dividing

Pages 152–169

General Objectives

To extend skills involving division to problems with 2-digit divisors and quotients

To develop skill in estimating quotients when the divisors are 2-digit numerals

To apply understanding of the division algorithm to word problems

This chapter should be considered a continuation of Chapter 6. After some applications of the skills studied in Chapter 6, estimation ideas are extended to include divisors and quotients between 10 and 100. The traditional long division algorithm is presented as a shortcut of the method developed previously. Finally, after exploring problems with larger quotients and with amounts of money, the chapter concludes with the usual chapter and cumulative reviews.

Teaching the Chapter

Materials

Watches with second hands

Vocabulary

distance	speed
odometer	speedometer
pedometer	time
rate	

Since this chapter extends development of basic algorithmic skills, few of the investigations require concrete materials. Many of the lessons are appropriate for group study, and the content also lends itself to chalkboard demonstrations conducted either by you or by some of the children.

Lesson Schedule

The suggested time allotment for this chapter is two weeks. This should be flexible, however, depending upon the speed with which children can extend their understanding of division to use of larger quotients and the shortcut method. More capable children may adapt quickly to these ideas and need less time for this development.

Evaluation of Progress

Note that our general approach to the division algorithm centres on an understanding of the ideas and on work with numbers rather than on quotient figures. We therefore urge you to permit considerable flexibility in the way the children work with these algorithms, especially the division algorithm. For example, some of the less able children may have mastered some of the simpler ideas of the division algorithm in terms of making guesses for the quotient and subtracting certain multiples of the divisor until they find a remainder that is less than the divisor, and then adding their estimates to get the final quotient. It is better that they be allowed to continue to use this method than that they be forced into a shortcut which they are unlikely to understand. The shortcut is primarily for the children who are capable of using it with understanding; it should not be treated as something to be memorized by all the children.

Resources for Active Learning

GENERAL ACTIVITIES

Freedom to Learn, "Time," p. 82; "Time and Speed," pp. 134–139, Addison-Wesley

Developmental Math Cards, J‡9, K‡4, Addison-Wesley [Games]

Modern Math Games . . ., pp. 55–56, Fearon [Multiplication and division puzzles] (Available from Clarke, Irwin)

MANIPULATIVE DEVICES

Pedometer and compass (Edmund Scientific; Math Media)

Stopwatch (Creative Playthings; Edmund Scientific)

PAGES 152–153

Objective

Given time, rate, and distance problems, the child will be able to find the time when given the rate and distance, the rate when given time and distance, and the distance when given the time and rate.

Preparation

Materials

stopwatches, second hands (if available)

To lead into the investigation, ask the children what they know about these terms:

speedometer (indicates speed of travel)
odometer (measures distance travelled by a vehicle)
pedometer (records distance travelled by a person walking, by responding to his body motion at each step)

Investigation

Have the children work in groups of three, four, or five, but, before they begin, discuss the procedures to be used. For example, if timing instruments are not available, children will have to decide on another method for counting seconds. You might teach them to count, "one thousand one, one thousand two, one thousand three," . . . up to one thousand ten, as a means of estimating 10 seconds. Make sure that each child in the group is given an opportunity to have his speed measured.

After children have measured the number of metres they travel in 10 seconds, they must try to figure their speed by studying the graph. Encourage them to figure out among themselves how to use the graph, but help any children who become confused. Since the one axis of the graph is marked in metres per 10 seconds, they need simply find along this axis the number of metres they travelled in the 10 seconds and read from the graph their rate in kilometres per hour. Direct them to record the rate of each child in their group and for each activity they measured.

7 Dividing

● *How fast do you travel?*

Investigating the Ideas

Answers will vary.
See Investigation.

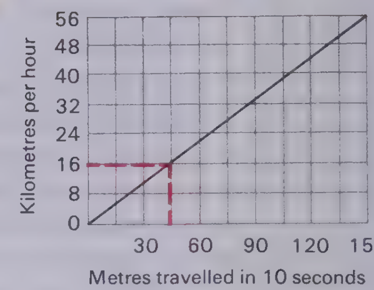
How far do you go in 10 seconds ? Measure one of these in metres.

WALKING

RUNNING

BICYCLING

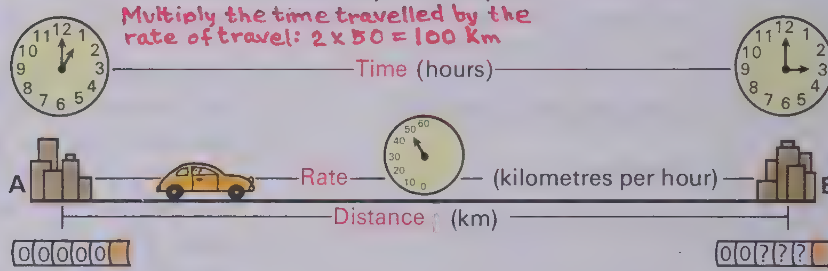
? Can you use the graph to find your **speed** in kilometres per hour ?



Note: The red dashed line shows that 45 metres in 10 seconds is about 16 kilometres per hour.

Discussing the Ideas

1. If you know how many metres you travel in 10 seconds, how can you use the graph above to find your rate in km / h ?
Find intersection of Metres travelled in 10 seconds" and the diagonal. Then find the "Kilometres per hour" figure for that point.
2. Explain how to use the information in the picture below to find the distance from city **A** to city **B**.
Multiply the time travelled by the rate of travel: 2 x 50 = 100 Km



Discussion

Have several children explain their rate of travel and how they found it from the graph. During this discussion of the investigation and the exercises, use the terms *time, rate,* and *distance* appropriately. To show the relationships between rate, distance, and time, you might suggest a problem such as the following:

Tom's rate of walking is 8 kilometres an hour. If he could walk at this rate for 3 hours, how far would he travel?

After the children have worked through the problem, help them reach the following generalization.

distance = rate × time

Similarly, you might suggest:

Susie's rate of travel is 7 kilometres an hour. How long will it take her to walk 28 kilometres?

Again, after children solve the problem, help them see that

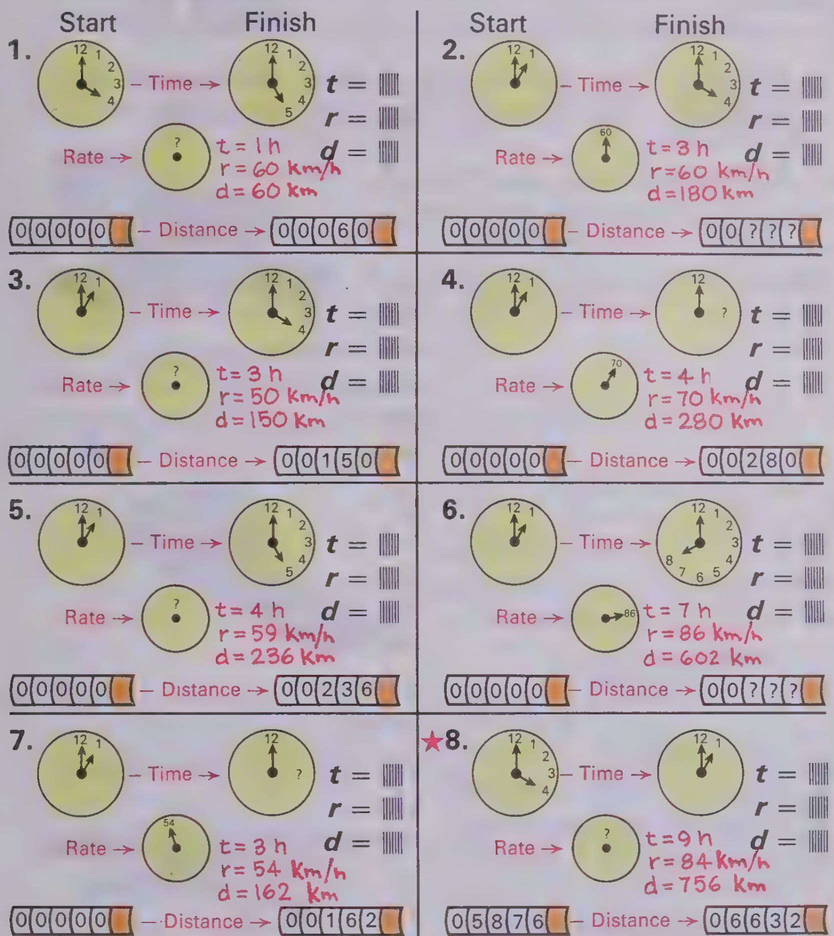
time = distance ÷ rate.

You might summarize these rules as shown below, but children should not be expected to memorize them.

$d = r \times t \qquad r = d \div t \qquad t = d \div r$

Using the Ideas

In the exercises the **time** is the number of hours that have passed from the time the trip started until the end of the trip. The **rate** is the number of kilometres travelled per hour. The **distance** is the number of kilometres from where the trip started to where it ended. Give the **time** (*t*), **rate** (*r*), and **distance** (*d*), for each trip.



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Resources for Active Learning

Applied Mathematics Cards, "Time—The Stop Watch," Group 1/17: "Time . . .," Group 2/12, 13, Schofield and Sims. (Available from Mafex Associates, Willowdale)

Maths Mini-lab, Cards 94–96, Selective Educational Equipment.

Using the Exercises

It would be helpful to use a few of the problems on page 153 as a basis for discussion. Children may rely on the generalizations developed in the discussion section, or they may simply work through each problem logically. Note that, in the starred exercise, the odometer on the left has a reading other than all zeros; you may need to point out to the children that they will have to subtract in order to find the distance.

Assignments (page 153)

Minimum: 1–4. Average: 1–6. Maximum: 1–8.

PAGES 154–155

Objectives

Given problems involving time, rate, and distance, the child will be able to write equations and solve the problems.

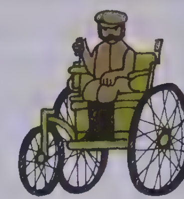
Given incomplete pairs of inequalities involving 2-digit factors, the child will be able to use estimation and choose a pair of multiples of ten to complete the inequalities.

Preparation

Depending on the children's needs and abilities, review either the relationships among time, distance, and rate or the procedure of using estimation to complete inequalities.

Finding Time, Rate or Distance

Write equations for exercises 1 through 6. Solve each equation to find the **time**, **rate**, or **distance**.



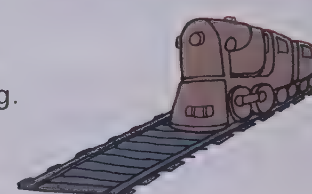
1. The first successful gas-powered car was a three-wheeler built in 1886 by Carl Benz. It went 84 kilometres in six hours. How fast did it go? $84 \div 6 = 14$ Km/h

2. The winner of a bicycle race travelled 272 km in 8 hours. What was his average speed? $272 \div 8 = 34$ km/h

3. The Queen Elizabeth was one of the largest passenger ocean liners ever built. She travelled about 51 km/h. How far could the Queen Elizabeth travel in a day? $51 \times 24 = 1224$ km

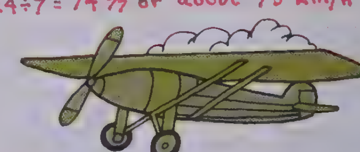
4. How many hours would it take the Queen Elizabeth to travel 408 km at the rate of 51 km per hour? $408 \div 51 = 8$ h

5. The longest straight railroad in the world is in Australia. This railroad is 524 kilometres long. If a train could average 131 km/h, how long would it take to cover this distance? $524 \div 131 = 4$ h



6. If a train travelled the 524 km in 7 hours, about what was the average speed? $524 \div 7 = 74\frac{6}{7}$ or about 75 km/h

★ **7.** During his historic flight across the Atlantic Ocean, Charles A. Lindbergh flew his plane 5812 km in 33 hours. About what was his average rate? $5812 \div 33 = 176\frac{4}{33}$ or about 176 km/h



154

Discussion

Unless children need particular help, assign the exercises on page 154 as independent work. These exercises are an extension of the previous lesson and are not related to the exercises on page 155. When children finish the problems, allow time for discussion and checking papers. Some interesting statistics and other information are given on this page, and the children may show considerable interest in discussing some of these ideas. You should keep in mind during such a discussion that one of the important goals of such a lesson is to stimulate the children's interest in arithmetic; therefore, encourage work with related topics.

Assignments (page 154)

Minimum: 1–4. Average: 1–6. Maximum: 1–7.

Let's practice estimation.

These exercises will help you find quotients in division problems such as $62\overline{)2689}$.

1. Give the number pair for each gray space. Some estimation hints are given to help you find the number pair for exercises A through C. Multiply to be sure you have selected the correct number pair.

Pairs: 10/20, 20/30, 30/40, 40/50, 50/60, 60/70, 70/80, 80/90, 90/100

Think: $? \times 60 < 2689$

A $\square \times 62 < 2689$
$\square \times 62 > 2689$

(Answer: 40, 50)

Think: $? \times 50 < 2734$

B $50 \times 47 < 2734$
$60 \times 47 > 2734$

Think: $? \times 70 < 5160$

C $70 \times 65 < 5160$
$80 \times 65 > 5160$

D $60 \times 31 < 2000$
$70 \times 31 > 2000$

E $60 \times 59 < 3874$
$70 \times 59 > 3874$

F $90 \times 28 < 2689$
$100 \times 28 > 2689$

G $80 \times 45 < 3760$
$90 \times 45 > 3760$

H $50 \times 67 < 3642$
$60 \times 67 > 3642$

I $90 \times 81 < 7438$
$100 \times 81 > 7438$

J $90 \times 36 < 3476$
$100 \times 36 > 3476$

K $70 \times 92 < 6986$
$80 \times 92 > 6986$

L $60 \times 74 < 4835$
$70 \times 74 > 4835$

2. From the set {10, 20, 30, 40, 50, 60, 70, 80, 90}, find the largest number that will make each sentence true.

A $n \times 62 < 2689$ 40
B $n \times 59 < 5874$ 90
C $n \times 31 < 2000$ 60
D $n \times 78 < 5160$ 60
E $n \times 48 < 3760$ 70
F $y \times 63 < 5538$ 80
G $y \times 29 < 2600$ 80
H $y \times 18 < 1307$ 70
I $y \times 66 < 6004$ 90
J $y \times 42 < 1837$ 40

think

To tell you who I am,
I'll give you just one clue.
I'm as much more than 8
As I'm less than 52

WHO AM I? 30

Resources for Active Learning

Measure and Find Out, Book 2, "Your Heart is a Clock," and "A Breath-taking Exercise," Activities 2/5, 6, Scott Foresman. (Available from Gage Educational Publishing)

Developmental Math Cards, H[1]6, J[2]7, J[2]10, Addison-Wesley. [Simple rate problems]

Using the Exercises

The exercises on page 155 deal with a concept which will be further developed in the next lesson. The estimation process here is simply a development of the same concept treated earlier when only one factor had two digits. It would be helpful to read the material at the top of the page with the children, pointing out that the "cloud" suggests the rounding process used in estimating quotients.

The solution of the *Think* problem requires finding the number halfway between 8 and 52. Some children may find the average; some may use a model, or illustration, and trial and error; and others may subtract 8 from 52 and add half of the difference (in this case 22) to 8. The many possible approaches make discussion of the problem worthwhile for the whole class.

Assignments (page 155)

Minimum: 1A–F, 2A–E.
Average: 1A–I, 2A–G.
Maximum: 1–2.

PAGES 156–157

Objective

Given division problems involving 2-digit divisors and quotients, the child will be able to find the quotients by estimating and using the long method.

Preparation

To prepare for this lesson, review some inequalities similar to those in exercise 2 on the previous page. Ask children to give the largest multiple of ten which will make the inequality true for examples such as:

$n \times 74 < 3421$ (40)
$n \times 31 < 2256$ (70)
$n \times 64 < 5720$ (80)

Let's explore divisors and quotients between 10 and 100.

Discussing the Ideas

1. To find the quotient for **62**)2689, think, → ? × **60** < 2689

A What is the largest multiple of 10 that will make the inequality true? 40

B Explain how you can complete the dividing when you use the number found in part A. Subtract 40x62 from the dividend and then find the next greatest multiple of 62 that can be subtracted from that difference.

C Try this one on your own: 61)2878 47 R 11

```
62) 2689
   -2480   (40)
     209
    -186   (3)
      23
```

2. Sometimes your estimate may be too large. ? × **80** < 5796

A What is the largest multiple of 10 that will make the inequality true? 70

B Why is this multiple of 10 not correct for the quotient? 70 x (80+4) > 5796

C Explain how to choose the correct multiple of 10 and complete the dividing.

D Try this one on your own: 64)3130 48 R 58

```
      69
84) 5796
   -5040   (60)
     756
    -756   (9)
       0
```

3. Sometimes your estimate may be too small. ? × **80** < 4668

A What is the largest multiple of 10 that will make the inequality true? 50

B Why is that multiple of ten not correct for the quotient? Because it is not the largest multiple of 10 that can be subtracted: 60 x 75 < 4668

C Explain how to complete the dividing.

D Try this one on your own: 86)5240 60 R 80

```
      62
75) 4668
   -4500   (60)
     168
    -150   (2)
      18
```

156

Discussion

The important point in this lesson is that the rounding process can be used as an aid in finding the proper quotient. This process, however, sometimes results in an estimate that is too large or too small, and children must learn how to proceed in finding the answer to such a problem.

Carefully work through the development in exercises 1–3. You might help the children work through the problems suggested in each exercise for independent work and provide additional examples also. For example, the problem in exercise 3D might be treated as follows:

```
86)5240   Think: ? × 90 < 5240
   4300  (50)     Try 50
    940
```

Since another multiple of ten may now be used, it is obvious that 50 was too small; hence, the children may erase and write 60, multiplying 60 × 86, or they may continue by subtracting 10 × 86.

```
   60 R 80                60 R 80
86)5240          or    86)5240
   4300  (50)             5160  (60)
    940                     80
    860  (10)
     80    0
```

Because children will not ordinarily learn to divide simply by studying the printed page, it is essential that you give carefully guided practice and have them work through several examples both at their seats and at the chalkboard. These examples and discussions are intended only as models for such guided practice.

Using the Ideas

1. In each exercise, find the missing tens' digit in the quotient.

A 9)288 — 3 ▮2	B 7)266 — 3 ▮8	C 28)868 — 3 ▮1	D 41)2173 — 5 ▮3
E 32)2240 — 7 ▮0	F 17)442 — 2 ▮6	G 53)2226 — 4 ▮2	H 71)3692 — 5 ▮2
I 48)1584 — 3 ▮3	J 82)5658 — 6 ▮9	K 25)625 — 2 ▮5	L 69)1035 — 1 ▮5
M 94)2444 — 2 ▮6	N 88)4400 — 5 ▮0	O 76)4712 — 6 ▮2	P 55)5445 — 9 ▮9

2. Find the quotients and remainders. Check your work.

A 8)236 — 29 R4	B 39)337 — 8 R25	C 30)1642 — 54 R22	D 62)5831 — 94 R3
E 19)158 — 8 R6	F 60)500 — 8 R20	G 80)605 — 7 R45	H 7)2304 — 329 R1
I 38)1420 — 37 R14	J 50)2970 — 59 R20	K 6)47,653 — 7942 R1	L 44)387 — 8 R35
M 75)2986 — 39 R61	N 70)6734 — 96 R14	O 86)7316 — 85 R6	P 40)327 — 8 R7
Q 35)3163 — 90 R13	R 54)4731 — 87 R33	S 98)7685 — 78 R41	T 9)43,297 — 4810 R7

3. Solve the equations.

A $272 \div 8 = n$ 34 C $2881 \div 43 = n$ 67 E $3276 \div 7 = n$ 468

B $2940 \div 60 = n$ 49 D $182 \div 26 = n$ 7 F $53{,}487 \div 9 = n$ 5943

★ **4.** Find the **divisor** for each exercise. The quotient is given.

A 25 ▮)225 — 9 B 24 ▮)720 — 30 C 29 ▮)2175 — 75 D 33 ▮)1287 — 39

★ **5.** For each exercise, find one number that can serve as **both** quotient and divisor.

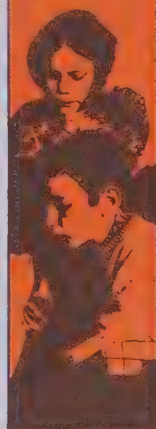
A 5 ▮)25 — ▮5 B 9 ▮)81 — ▮9

C 10 ▮)100 — ▮10 D 30 ▮)900 — ▮30

think

Give the digit that should go in the ▮. 9
The ⋆'s are for other digits that you need not find.

```
     ⋆⋆
 ▮)⋆⋆⋆
   ⋆⋆⋆
    8⋆
    ⋆⋆
     0
```



More practice, page A-12, Set 24

Using the Exercises

On page 157, explain representative problems from exercises 1 and 2 before you have the children do the exercises. You may choose to assign only part of exercise 2 to the slower children, inasmuch as there is a considerable amount of work on this page.

The children who complete the page early should also find the *Think* problem quite challenging. Be sure to give them an opportunity to explain the reasoning they use in attempting to solve these problems.

Assignments (page 157)

Minimum: 1A–H, 2A–H.
Average: 1A–L, 2A–L, 3.
Maximum: alternate parts of 1–2; 3–5.

Resources for Active Learning

Mathematics in Modules, WN21, Addison-Wesley.

Duplicator Masters, page 33
Workbook, page 48
Skill Masters, page 33

PAGES 158–159

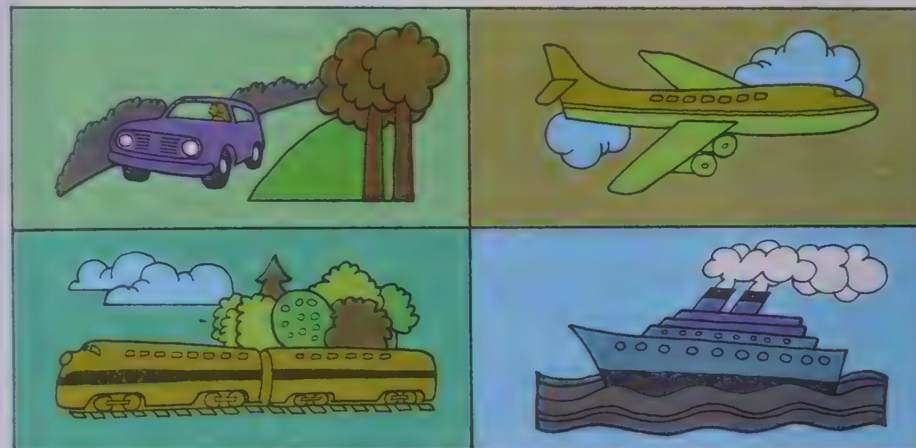
Objective

Given word problems involving division, the child will be able to solve the problems by applying his understanding of division with 2-digit divisors.

Preparation

Depending on the children's ability and needs, you might review the division algorithm as presented in the previous lesson, or you might arouse interest in the problems by talking about transportation and similar areas of interest.

Solving Story Problems — Travel



1. A large ocean liner averages about 55 km/h. About how many hours would it take this ship to travel 2970 km? 54 h
2. An express train averages 120 kilometres per hour. About how long would a 1380-kilometre trip take? $11\frac{1}{2}$ h
3. If a car travels at an average speed of 72 km/h, how long would it take to go 705 kilometres? $9\frac{19}{24}$ h
4. A large jet airliner travels 2792 km in 3 hours. About how far does it fly in 1 hour? About 931 km
5. How far would an ocean liner travel in 78 hours if it averages 49 kilometres per hour? 3822 km
6. Suppose a freight train averages about 42 kilometres per hour. About how long would it take the train to go 1794 kilometres? About 43 h
7. A man drove 616 km at a speed of 88 km/h and 480 km at a speed of 96 km/h. How far did the man drive? How long did it take? 1096 km
8. A captain wants his ship to go 3612 km in 3 days and 14 hrs. How fast (how many km per hour) should he plan to travel? 42 km/h

158

Discussion

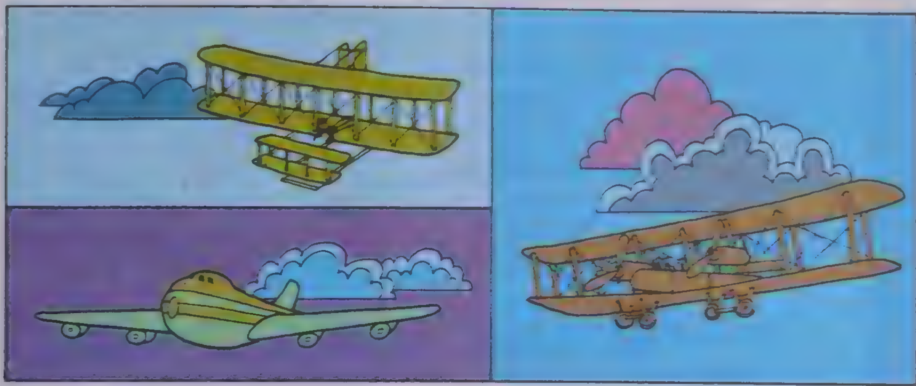
Although the problems on page 158 are intended to be worked independently, some groups of children will need guidance in working through them. If your discussion takes place as the children work or after they have worked independently, have volunteers exhibit on the chalkboard the solutions and algorithms used for several problems. It would be helpful to review the terms time, rate, and distance as you discuss this page. Since one of the main purposes of these problems is to provide practice with the division algorithm with 2-digit divisors, you should stress both the estimation skills and the procedure of the algorithm appropriate for each problem.

The answers to exercises 4 and 6 are approximate. Starred exercise 7 is simply a problem of combining distances for the first question and totalling the two separate times for the second question. Exercise 8 should present no special difficulty for capable children once they realize that they should figure out how many hours in 3 days; the problem then becomes rate = 3612 ÷ 86, and the answer is given in kilometres per hour.

Assignments (page 158)

Minimum: 1–4. Average: 1–6. Maximum: 1–8.

AIRPLANES



1. If a jet plane flies 4044 kilometres in 4 hours, how fast is it travelling ? 1011 km/h

2. If a jet plane carried 98 passengers on each flight, how many flights would it take to transport 3430 passengers from Toronto to Chicago ? 35

3. We would say that a jet plane travelling 2576 kilometres per hour is going very fast. In 1904 the Wright brothers flew a plane at a top speed of about 56 kilometres per hour. How many times faster is the jet than the plane flown by the Wright brothers ? 46

4. In 1910 an airplane flying at an altitude of 1800 metres would have been close to the record altitude. In 1960, 34 200 metres was close to record altitude. How many times higher could airplanes fly in 1960 than in 1910 ? 19

5. A "747" jet airliner measures 59 m 50 cm from one wing tip to the other.
   - A Give in centimetres the wing span of a "747". 5950 cm
   - B The smallest airplane has a wing span of 2m 15 cm. Give the wing span in centimetres. 215 cm
   - C Use centimetres to calculate the number of the small airplanes that could fit within the wing span (wing tip to wing tip) of the 747 jet airliner.
     27 (with 145 cm of wing span as remainder) (See Using the Exercises.)

159

Using the Exercises

Assign the problems on page 159 as independent work. However, it would be helpful to ask for volunteers to work some solutions on the chalkboard. You might also use problems such as 3 and 4 to observe that some of the information given is not needed in finding the answers. Remind the children how important it is to read the questions carefully so that they are not confused by superfluous data.

You may want to call children's attention to the fact that in exercise 5C the answer should be given simply as a whole number (27). Although there is a remainder when 5950 is divided by 215, the problem concerns the greatest number of the small planes that could fit within the wing span of the large plane, so the remainder is irrelevant in this context.

Assignments (page 159)
Minimum: 1–3. Average: 1–4.
Maximum: 1–5.

Follow-up
More capable children may be interested in extending these problems about travel and speeds to satellites and planets. Provide them with data such as that below or give them source materials for research, such as *What's Up There?* (prepared by the National Aeronautics and Space Administration, and available from the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C., 20402).

Space Facts

1. The moon travels about 12 100 000 kilometres in its 28-day orbit of earth. To the nearest thousand kilometres, what is its approximate speed in
   (a) kilometres per day?
   (b) kilometres per hour?
2. TIROS 1 (short for Television Infra-Red Observation Satellite) orbited through about 45 000 kilometres of space every 99 minutes. Estimate its average speed per minute.
3. Explorer X had a perigee (point of orbit nearest to earth) of 161 kilometres and an apogee (point of orbit farthest from earth) of 299 000 kilometres. What is the average of these two distances?
4. The perigee of the moon's orbit about earth is 356 743 kilometres, while its apogee is 406 140 kilometres. What is the average distance of the moon from earth?

Duplicator Masters, page 34

PAGES 160–161

Objective

Given a division problem involving 2-digit divisors and quotients, the child will be able to find the quotient by estimating and using a shortcut method.

Preparation

To prepare for this lesson, you might give the children a few long division problems and work through them as illustrated on page 160. Or you may prefer to keep the preparation oral and review rounding numbers between 10 and 100.

Let's shorten the work in dividing.

Discussing the Ideas

See Discussion.

1. The example in the box shows a shortcut for dividing. The method you have been using is shown for comparison.
   - A Explain the estimation in part 1 of the shortcut. Think: 4)15
   - B Explain part 1 of the shortcut. Multiply 3 × 42 and subtract from 157.
   - C Explain the estimation in part 2 of the shortcut. Think: 4)31
   - D Explain part 2 of the shortcut. Multiply 7 × 42 and subtract from 315.

Shortcut	Usual method
1 3	
42)1575	42)1575
126	1260 (30)
31	315
2 37	37
42)1575	42)1575
126	1260 (30)
315	315
294	294 (7)
21	21

See Discussion.

2. In this example, if you use 22 ÷ 3 to get your estimate, your first quotient figure will be too small.
   - A Explain what you must do in each method when your first estimate is too small.
   - B Explain the estimation in part 3 of the shortcut. Think: 3)12
   - C Explain part 3 of the shortcut. Multiply 4 × 26 and subtract from 128.

Shortcut	Usual method
1 3 7	
26)2208	26)2208
182	1820 (70)
38	388
2 8	260 (10)
26)2208	128
208	84
12	26)2208
3 84	1820 (70)
26)2208	388
208	260 (10)
128	128
104	104 (4)
24	24

3. Sometimes your first estimate may be too large. You might estimate 6 for this problem: **64)3776**
   - A What must you do when your first estimate is too large? Choose smaller number.
   - B Use the shortcut to complete the dividing.

59
64)3776
320
576
576

Discussion

It is essential in this lesson that children understand the basic long division process as it has been presented in previous lessons. They should understand that their chief task in this lesson is to learn a shortcut method (the division algorithm) for finding the correct quotient. Therefore, one of the important points to stress is the relation of the shortcut method to the longer method they have been using. As you work through the examples, encourage children to verbalize this relationship at every step.

Another important point is that children should estimate how many digits their quotient will have before they actually work out the problem. The material in the text is designed to help children make such estimates. Once children learn to place the quotient (1, 2, or 3 digits), the problem becomes in effect a series of divisions involving a 2-digit divisor and a 1-digit quotient. For instance, in exercise 1, we estimate the quotient by thinking 4)15 rather than 40)157. Also, we know the quotient must have two digits because $10 \times 42 < 1575$ and $100 \times 42 > 1575$.

Note that the response for exercises 2A and 3A is simply, "Erase and start over."

Using the Ideas

1. Use the heavy black numerals to help you estimate the first quotient figure for each exercise below.

A 52)209 4	B 39)245 6 (40)	C 23)115 5	D 45)1760 3 (50)
E 88)4148 4 (90)	F 42)1575 3	G 65)28 293 4 (70)	H 73)38 574 5

2. For each of the examples above, tell whether the quotient is between 0 and 10, 10 and 100, or 100 and 1000.
A 0–10; B 0–10; C 0–10; D 10–100;
E 10–100; F 10–100; G 100–1000; H 100–1000

3. Find the quotients and remainders. Use the shortcut.

A 39)1722 44 R6	B 71)6825 96 R9	C 42)966 23 R0	D 58)2842 49 R0	E 18)967 53 R13
F 27)918 34 R0	G 63)4665 74 R3	H 44)2812 63 R40	I 65)5146 79 R11	J 89)8265 92 R77

4. Find the quotients and remainders. Use any method you choose. Check your work.

A 61)2878 47 R11	B 39)1092 28 R0	C 64)3030 47 R22	D 19)1577 83 R0
E 58)3700 63 R46	F 47)987 21 R0	G 85)4940 58 R10	H 32)1920 60 R0

think

$1 + 2 = (2 \times 3) \div 2$
$1 + 2 + 3 = (3 \times 4) \div 2$
$1 + 2 + 3 + 4 = (4 \times 5) \div 2$
$1 + 2 + 3 + 4 + 5 = (5 \times 6) \div 2$
$1 + 2 + 3 + 4 + 5 + 6 = (6 \times 7) \div 2$

Check the equations to be sure they are true. Then estimate the sum of the whole numbers 1 through 30.

More practice, page A-13, Set 25

Follow-up

Most children would benefit from further practice with the shortcut method, so you might prepare a worksheet similar to the one below.

Fill in the missing steps in each problem:

```
      9
72) 6581          72) 6581
                         0     (90)
    ____              ____
     101               101     (1)
    ____              ____
      29                29

       1
56) 2893          56) 2893
     280              2800
    ____                 9
     56                  6
```

Duplicator Masters, page 35
Workbook, page 49
Skill Masters, page 35

Using the Exercises

You may choose to work through several of the exercises on page 161 together with the children. Be sure the children note the boldface numerals, which are intended to help them make estimates and place the first digit of the quotient in the correct position.

Continue to relate the shortcut method to the familiar longer method. Also, do not force any child to use the shortcut method if he feels more comfortable with the longer method.

Assignments (page 161)
Minimum: 1–2, oral; 3A–E.
Average: 1–2, oral; 3.
Maximum: 1–2, oral; 3–4.

PAGES 162–163

Objective

Given a division problem involving a 2-digit divisor and a 3-digit quotient, the child will be able to find the quotient by estimating and using a shortcut method.

Preparation

The best preparation for this lesson is an understanding of the shortcut presented in the previous lesson. Here again, however, we stress that use of the shortcut is not absolutely essential to the working of these problems. The children can continue to work with the earlier methods. Rather than writing quotient figures, they can write their estimates down the side and subtract multiples of the divisor. However, with these larger quotients, the children will find that the shortcut method reduces their work considerably. You may also find that many of the children will begin to appreciate the shortcut now that they are working with larger quotients in this lesson.

Let's explore larger quotients.

Discussing the Ideas

1. **A** Can you show that the quotient for $69\overline{)22\,364}$ is less than 1000 and greater than 100?
1000×69 > 22 364; 100×69 < 22 364

B Explain how to estimate the number of hundreds in the quotient.
Think: 7)22

C Explain step 1.
Multiply 3×69 and subtract the product from 223.

D Explain how to estimate the number of tens in the quotient.
Think: 7)16

E Explain step 2.
Multiply 2×69 and subtract the product from 166.

F Explain how to estimate the number of ones in the quotient.
Think: 7)28

G Explain step 3.
Multiply 4×69 and subtract the product from 284.

H What are the quotient and the remainder?
324 R8

1
```
  70
       3
69) 22364
    207
     16
```

2
```
       32
69) 22364
    207
     166
     138
      28
```

3
```
       324
69) 22364
    207
     166
     138
      284
      276
        8
```

2. **A** Can you show that the quotient for $42\overline{)12\,768}$ is less than 1000 and greater than 100?
1000×42 > 12 768; 100×42 < 12 768

B Explain how to estimate the number of hundreds in the quotient.
Think: 4)12

C Explain step 1.
Multiply 3×42 and subtract the product from 127.

D Explain how you can tell that there are 0 tens in the quotient.
No 42's can be subtracted from 16.

E Explain how to estimate the number of ones in the quotient.
Think: 4)16

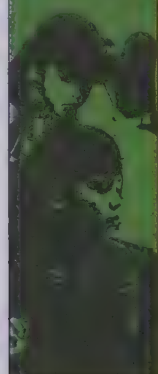
F Explain step 3.
Multiply 4×42 and subtract the product from 168.

G What are the quotient and the remainder?
304 R0

1
```
       3
42) 12768
    126
      1
```

2
```
       30
42) 12768
    126
     16
```

3
```
       304
42) 12768
    126
     168
     168
```



162

Discussion

Work through discussion exercise 1, a step at a time. You might observe with the children that in part 2 we "bring down" the 6 in the second step simply by thinking about subtracting 0 from 6. We then think "How many sevens in 16?" in order to determine our second quotient figure, that is, the number of tens in the quotient. When we arrive at the number 2, we continue the problem. Along with the shortcut method illustrated here, you could easily show the parallel method of writing the numbers at the side. For example, the long method would show 300, then 20, then 4 written at the side with the numbers subtracted, including all the zeros.

Work through exercise 2 similarly, stressing the place value in order to help the children see why they write zero in the quotient in certain places. It would be helpful to work through a third example which has zero at the end of the quotient. For example:

```
      260 R17
48) 12497
     96
     289
     288
      17
```

Using the Ideas

1. Study the first two steps in this example. Then copy the problem and see if you can complete it. Notice that the quotient is more than 100 and less than 1000.

1

```
        3
79)26897
   237
    31
```

2

```
        340 R37
79)26897
   237
    319
    316
      37
```

2. Find the quotients and remainders. Check your work.

A 51)11 934 — 234 R0	B 32)3648 — 114 R0	C 64)14 784 — 231 R0	D 39)21 879 — 561 R0
E 62)22 630 — 365 R0	F 65)40 764 — 627 R9	G 75)41 625 — 555 R0	H 23)9476 — 412 R0
I 78)30 342 — 389 R0	J 89)61 143 — 687 R0	K 56)47 376 — 846 R0	L 27)5237 — 193 R26

3. Find the quotients and remainders. Check your work.

A 31)3265 — 105 R10	B 52)20 904 — 402 R0	C 69)37 260 — 540 R0	D 83)61 918 — 746 R0
E 35)7256 — 207 R11	F 46)36 835 — 800 R35	★ G 231)1848 — 8 R0	★ H 342)7005 — 20 R165

4. A There are 28 children in a class. The sum of all their spelling scores is 2436. Find the average score. 87

B There are 3084 washers in a box. If 2 dozen washers are put into each package, how many packages will there be? 128 How many extra washers? 12

think

Give the missing digits.

```
      □□89
8□)□127
   64□0
   72□□
   72□□
      0
```

More practice, page A-13, Set 26

Using the Exercises

On page 163, it would be helpful to work through exercise 1 with the children. Help them realize that they are working with a 3-digit quotient and that when they reach the final step, they cannot subtract any seventy-nines from 37. Thus, there are zero ones in the quotient.

Assign the remaining exercises as independent work, or you may prefer that some children work together.

Encourage interested children to try the *Think* problem, with the caution that it is quite challenging.

Assignments (page 163)
Minimum: 1, 2A–H. Average: 1, 2A–H, 3A–D. Maximum: All.

Duplicator Masters, page 36
Workbook, page 50
Skill Masters, page 36

PAGES 164–165

Objective

Given multiplication and division problems which include dollar-and-cent notation, the child will be able to solve the problems by using the multiplication and division algorithms.

Preparation

To prepare for this lesson, you might work through a division problem with the usual division algorithm and then check it by using the multiplication algorithm (and adding the remainder). Such a preparation should review both algorithms which the children will use in this lesson.

Investigation

As indicated in the text, the children need no materials for this investigation. Direct the children to figure out each answer mentally and to be ready to give a reason for their choice. As with previous estimation problems they should round the amounts given as the cost of the items so that they can think in dollars rather than dollars and cents. To do this, children must understand the dollar-and-cent notation and apply their skill in estimating.

Encourage them to work independently and give only minimum guidance. If necessary, lead children to independent thinking by asking guiding questions.

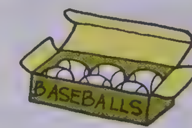
How can multiplication and division be used to solve problems about money?

Investigating the Ideas

$3.98 each

$20

Terry



6 for $28.50

$5

Jose

? Can Terry buy five records? Yes
Can Jose buy one baseball? Yes

NO pencil — **NO** paper
See Investigation.

Discussing the Ideas

1. How did you decide your answers to the Investigation questions? Think: $3.98 < $4.00 and 4×5=20, so $3.98×5 < $20.
Think: $28.50 < $30.00 and 30÷6=5, so $28.50÷6 < $5.
2. Explain the flow chart below for 6 × $3.49. See Discussion.

$3.49	→ Think of the number of cents in $3.49.	→ Multiply by 6.	→ Write in dollar notation.
$3.49	349 cents	$349 \times 6 = 2094$	$20.94

3. Explain the flow chart below for $4.65 ÷ 3. See Discussion.

$4.65	→ Think of the number of cents in $4.65.	→ Divide by 3.	→ Write in dollar notation.
$4.65	465 cents	$465 \div 3 = 155$	$1.55

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Discussion

Discussion exercise 1 is intended to stimulate conversation about the investigation. Stress that dollar-and-cent notation can be rounded to dollars simply by following the same principles of rounding which the children have been using.

One of the most important objectives of exercises 2 and 3 is to help children realize that they are multiplying and dividing not money, but numbers. For example, exercise 2 shows that they are to think of $3.49 as 349 pennies, and, hence, simply think of multiplying the number 349 by 6. They arrive at the answer 2094, and then reconvert this to dollar notation. (They observe that the 94 is the number of cents and 20 is the number of dollars.)

Using the Ideas

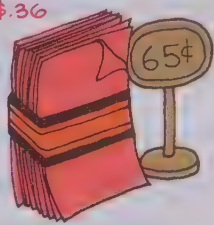
1. Find the answers to these money problems.

A $2.53 × 7 = $17.71	B $5.13 × 27 = $138.51	C $0.56 × 4 = $2.24	D $0.87 × 54 = $46.98	E $3.62 × 435 = $1574.70
F $5.64 × 65 = $366.60	G $8.00 × 98 = $784.00	H $36.50 × 7 = $255.50	I $36.98 × 32 = $1183.36	J $0.95 × 98 = $93.10
K 5)$1.15 $.23	L 3)$0.72 $.24	M 7)$6.23 $.89	N 4)$13.88 $3.47	
O 6)$272.22 $45.37	P 9)$77.67 $8.63	Q 80)$4.00 $.05	R 60)$22.80 $.38	
S 39)$234 $6	T 61)$15.25 $.25	U 45)$41.85 $.93	V 36)$20.52 $.57	

2. Solve the problems.

A Tom received $2.75 for each lawn he mowed. How much did he earn by mowing 8 lawns? $22.00

B Jill bought a package of 6 handkerchiefs for $2.16. Find the cost of one of these handkerchiefs. $.36



C The price of a package of colored paper is shown in the picture. Jan bought 8 packages. What was the total cost? $5.20

D In exercise 2c, what change did Jan receive when she gave the clerk a $10 bill? $4.80

E If 8 kilograms of nuts cost $3.92, find the cost of one kilogram of nuts. $.49

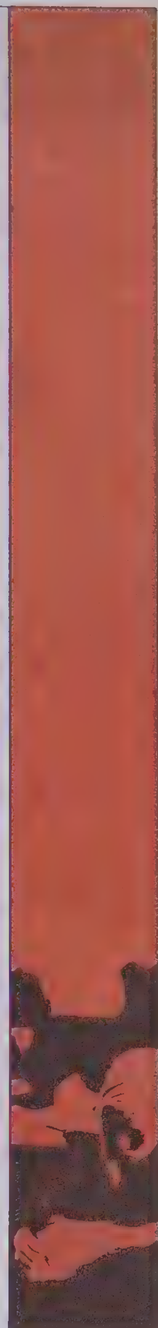
F Mr. Jones bought a TV for $288.90. He paid the same amount each month for 9 months. How much was each payment? $32.10

G One of the most expensive fabrics is a gold lace that costs $151.20 per metre (one metre long and 50 cm wide). Find the cost of 8 metres of this fabric. $1209.60

H If 51 adult tickets were sold and $38.25 was collected, how much did each ticket cost? $.75

I Which costs less per kilogram—an 8-kg package costing $3.92 or a 6-kg package costing $2.88?

More practice, page A-14, Set 27



Follow-up
It would be appropriate to have children write several money problems of their own, solve them, and then exchange them with their classmates. For this purpose, supply advertisements and catalogues to help children use realistic prices in their problems. Also, make sure the children record the answers to the problems they make so that others can check their work.

Resources for Active Learning
Math Activity Cards, "A Cook-out," D47, Macmillan.

Duplicator Masters, page 37
Workbook, page 51
Skill Masters, page 37

Using the Exercises
Before assigning the exercises on page 165, observe again with the children that they are working with numbers; thus, they think of a given problem involving money, convert this to an exercise involving numbers, and then interpret their answer in terms of the usual dollar notation. After the preliminary discussion, have the children do the exercises. When they have finished, allow time for a discussion and checking papers.

Assignments (page 165)
Minimum: 1A–J, 2A–D.
Average: 1A–N, 2A–F.
Maximum: 1–2.

PAGES 166–167

Objective

The child will demonstrate his ability to work with the concepts presented in this chapter.

Preparation

It would be helpful to review with the children the techniques developed for estimating quotients. Stress the fact that estimating is helpful, but they should remember that, when rounding the divisors, they may or may not arrive at the best guess for the quotient.

Reviewing the Ideas

1. Choose the word that completes each sentence correctly.

A To estimate the quotient when the divisor ends in 1, 2, 3, or 4 (such as 21, 42, 53, or 34), we usually use the closest multiple of ten __?__ (greater than, less than) the divisor.

B To estimate the quotient when the divisor ends in 5, 6, 7, 8, or 9 (such as 45, 26, 87, or 19) we usually use the closest multiple of ten __?__ (greater than, less than) the divisor.

2. Find the quotients and remainders. Check your work.

A 29)174 6 R0	B 35)243 6 R33	C 57)250 4 R22	D 33)296 8 R32	E 85)267 3 R1
F 47)260 5 R25	G 23)158 6 R20	H 51)423 8 R15	I 61)253 4 R9	J 29)266 9 R5
K 38)368 9 R26	L 86)518 6 R2	M 24)223 9 R7	N 75)600 8 R0	O 65)565 8 R4

3. Solve the equations.

A $301 \div 7 = n$ 43	C $3658 \div 59 = n$ 62	E $7040 \div 80 = n$ 88
B $1674 \div 62 = n$ 27	D $1143 \div 9 = n$ 127	F $841 \div 29 = n$ 29

4. Find the quotients and remainders. Check your work.

A 79)10 047 127 R14	D 39)16 602 425 R27
B 71)29 983 422 R21	E 43)41 252 959 R15
C 95)20 746 218 R36	F 84)58 225 693 R13

5. Find the answers to these money problems.

A	B	C
\$1.79	\$0.84	\$2.68
×3	×25	×19
\$5.37	\$21.00	\$50.92

D 6)\$1.14 \$.19 E 21)\$7.35 \$.35

think

Have a friend select any row on a calendar that has 7 days. Tell him you can find the sum of the dates faster than he can. Suppose he chooses a week like this:

9	10	11	12	13	14	15

Take the smallest number, add 3 and multiply by 7. Can you explain why this gives the sum?

See Discussion.

Discussion

If you choose, use exercise 1 on page 166 as a discussion exercise. However, have the children try exercise 2 on their own. When they have finished, allow time for presentation of these exercises on the chalkboard and for explanation to the class.

The explanation for the *Think* problem rests in the fact that adding 3 to the smallest of 7 consecutive numbers is actually a short way to locate average. Then, all that remains to be done is to multiply the average by the number of days to find the sum.

TRAVEL PROBLEMS

1. Solve the problems. Write an equation for each problem.
   - A A man used 24 *l.* of gasoline while travelling 336 km. What is the average number of km he can travel while using one litre of gasoline? 336 ÷ 24 = 14km

   - B If a man wants to go 195 kilometres in 3 hours, about how fast should he drive? 195 ÷ 3 = 65 km/h
   - C If a motor scooter travels about 36 kilometres per hour, about how long will it take to go 180 kilometres? 180 ÷ 36 = 5 h
   - D If Mr. Jones drives about 62 kilometres per hour, about how long will it take him to go 806 kilometres? 806 ÷ 62 = 13 h
   - E If a train averaged 83 kilometres per hour and travelled 498 kilometres, how many hours did it travel? 498 ÷ 83 = 6 h

2. Solve these money problems.
   - A Mr. Ray bought 5 tires. Each tire cost $24.95. Find the total cost of the tires. $124.75
   - B Mr. Ito paid $93.96 for 4 tires. How much did each tire cost? $23.49
   - C What is the difference in the cost of Mr. Ray's and Mr. Ito's tires? $1.46 per tire, or total of $30.79

3. On a vacation trip the Williams family travelled these distances in a car.
   - A What was the total distance for the five days? 1680 Km
   - B What was the average distance the family travelled in a day? 336 Km
   - C Mr. Williams bought a total of 420 litres of gasoline on the trip. How many kilometres for each litre? 4 Km

Monday	505
Tuesday	416
Wednesday	274
Thursday	366
Friday	119

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Resources for Active Learning

Experiments in Mathematics, Stage 3, "A Vehicle Survey," pp. 16–17, Houghton Mifflin [Graphing] (Available from Thomas Nelson and Sons)

Maths Mini-lab, Cards 58–61, Selective Educational Equipment. [More travel problems]

Workbook, page 52

Using the Exercises

You might encourage the children to form small groups and discuss the solutions for each of the problems on page 167. Point out that they should first write an appropriate equation for each and then try to find the answer. When the children have finished, discuss at least a few parts of each problem.

PAGES 168–169

Objective

The child will demonstrate his ability to work with the concepts presented for cumulative review.

Preparation

If any topic or skill has seemed particularly troublesome for the children, review it with them. Or, use material from page 169 to introduce this problem set. If possible, display thermometers. Also, discuss what is meant by the terms *boiling point* and *freezing point*.

Keeping in Touch with Addition, Subtraction, Multiplication, Division, Place value, Functions

1. Find the products.

A 10×10 100 C 40×30 1200 E 40×40 1600 G 80×60 4800
B 30×10 300 D 60×80 4800 F 90×70 6300 H 50×0 0

2. Give the number for n.

A $6386 = (63 \times n) + (8 \times 10) + 6$ 100
C $6547 = (654 \times n) + 7$ 10
B $58\,346 = (58 \times n) + (3 \times 100) + (4 \times 10) + 6$ 1000

3. Give the numbers for a and b. Then give the number for c.

A $20 \times 4 = a$ 80; $6 \times 4 = b$ 24; $26 \times 4 = c$ 104
C $280 \div 7 = a$ 40; $14 \div 7 = b$ 2; $294 \div 7 = c$ 42
B $50 \times 6 = a$ 300; $8 \times 6 = b$ 48; $58 \times 6 = c$ 348
D $240 \div 8 = a$ 30; $32 \div 8 = b$ 4; $272 \div 8 = c$ 34

4. Estimate the answers to these exercises.

A $594 + 316 + 891$ 1800 D 48×31 1500 G $903 - 396$ 500
B $5876 + 2946$ 9000 E 59×22 1200 H 198×204 40 000
C $39 + 49 + 59 + 69$ 220 F 81×99 8000 I $363 \div 92$ 4

5. Give the missing numbers in the table.

Function Rule	
$30 \times n$	
n	Input
9	270
6	A 180
10	B 300
80	C 2400
D 60	1800

★ 6. Find the missing digits.

A ☐☐6 (6 0) − 23☐ (8) = 368
B ☐705 (8) − 4☐☐9 (2 3) = 4466
C 5☐☐6 (3 0) − ☐53☐ (3, 8) = 1768
D 5☐☐4 (7 3) − ☐73☐ (2, 0) = 3004
E ☐☐☐☐ (7 0 0 0) − 3658 = 3342
F ☐8☐7 (8 0) − 3☐7☐ (9, 9) = 4828
G 7☐2☐ (7 2) − ☐664 (4) = 3058
H ☐00☐ (8 8) − 6767 = 1☐☐1 (2 4)
I 7125 − ☐☐☐☐ (3 6 4 7) = 3478

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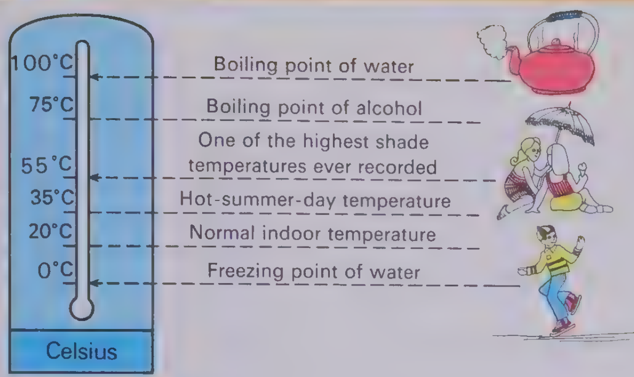
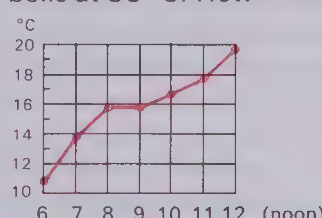
Discussion

Assign the exercises on page 168 as independent work. When the children have completed them, it would be helpful to discuss some parts of each exercise. Point out such things as the number of zeros in the factors and in the products of exercise 1. In exercise 3, point out the applications of the distributive principle. Encourage children to discuss any problems which they found difficult.

TEMPERATURE



1. A How many Celsius degrees greater is the boiling point of water than the freezing point ? 100°C
   B How many Celsius degrees greater is the boiling point of water than the highest temperature recorded in shade ? 45°C
   C How many Celsius degrees are there between the boiling points of water and alcohol ? 25°C
2. How would you write the temperature when it is seven degrees below zero ? −7°C
3. Water at the top of a certain high mountain boils at 88 °C. How much less is the boiling point of water at this height than it is at sea level ? 12°C
4. Here is a **line graph** that shows the temperature at each hour between 6 A.M. and noon. Study the graph and find the average of the hourly temperatures. 16°



You are invited to explore

ACTIVITY CARD 7
Page 336

Using the Exercises

Accompany the study of the problems on page 169 with discussion of the two common kinds of thermometers. It would also be helpful to discuss the flow chart which shows how a temperature given in one unit may be converted to the other. Help the children realize that every unit of the Fahrenheit is $\frac{5}{9}$ the unit of the Centigrade. The addition or subtraction of 32 results from the placement of the freezing point of water on the Fahrenheit scale at 32° F. and on the Centigrade scale at 0° C. The flow chart works for this value also: 0° C. → $(0 \times \frac{9}{5}) + 32 \rightarrow$ 32° F.

Follow-up

Encourage those interested in page 169 to pursue further study of thermometers. You might suggest that they find information about the Kelvin scale and its relationship to the scales studied here.

Resources for Active Learning

Applied Mathematics Cards, "Measuring Temperature," Group 2/26, Schofield and Sims. (Available from Mafex Associates, Willowdale)

CHAPTER 8 | Number Theory

Pages 170–185

General Objectives

To provide experience in working with factors and multiples
To introduce least common multiple
To introduce greatest common factor
To provide work with prime numbers
To introduce formally the ideas of union and intersection of sets
To provide experience in working with composite numbers
To introduce the complete factorization of a composite number

The first lesson of this chapter develops the concept of the set of factors of a number. Following this, factor trees are introduced to help the children find factors of a number and to provide a way to write a composite number as a product of prime numbers. The concept of prime numbers is also reviewed through the use of factor trees. Union and intersection of sets are introduced as a tool for finding common factors of two numbers and common multiples of numbers. The final pages of the chapter provide experiences with common factors, greatest common factor, common multiples, and least common multiple. A chapter review and cumulative review complete the chapter.

Mathematics

Consider the following definition.

> If a, b, and c are whole numbers such that $a \times c = b$, then a and c are whole-number factors of b.

With this definition, we are ready to define *prime number*.

> The set of prime numbers is the set of all whole numbers x such that x has exactly two factors.

Thus:

- 2 is a prime number (factors: 2, 1).
- 3 is a prime number (factors: 3, 1).
- 5 is a prime number (factors: 5, 1).
- 7 is a prime number (factors: 7, 1).
- 11 is a prime number (factors: 11, 1).

Note that:

- 1 is not prime (factor: 1).
- 6 is not prime (factors: 1, 2, 3, 6).
- 91 is not prime (factors: 1, 7, 13, 91).

Every number greater than 1 that is not prime is a product of one and only one collection of prime numbers. Such numbers are called *composite numbers*. For example, $6 = 2 \times 3$; $8 = 2 \times 2 \times 2$; $9 = 3 \times 3$; $100 = 2 \times 2 \times 5 \times 5$; and so on. Expressing a composite number as a product of primes is called the complete factorization of a number. The fact that each composite number can be expressed (without regard to order) as a product of primes in exactly one way is called the *Fundamental Theorem of Arithmetic*.

Closely connected to the idea of the factors of a number is the idea of the common factors of two numbers and the greatest common factor of two numbers. As far as its use in arithmetic is concerned, the most important application of greatest common factor is in changing fractions to lowest terms. That is, if we have a fraction which is not in lowest terms, we can change this fraction to lowest terms by dividing numerator and denominator by the greatest common factor of the two numbers. We define greatest common factor simply as the largest number in the set of factors common to both numbers.

Consider the following definition of the *multiples* of a number.

> If a and b are whole numbers and c is the product of a and b, then c is said to be a multiple of a (and of b).

Because zero is a multiple of every whole number, we generally omit the number zero in any listing of the multiples of a number. For this reason, when we speak of the least common multiple of two numbers, we mean the smallest number other than zero that is a multiple of both numbers. The concept of least common multiple is valuable in the study of fractional numbers because the least common multiple of two denominators is the least common denominator for two fractions, and we use a common denominator to find the sum of two fractional numbers.

The following examples illustrate the way in which greatest common factor and least common multiple are used. Consider the fraction $\frac{20}{30}$. Ten is the greatest common factor of the two numbers, 20 and 30, and by dividing both 20 and 30 by 10, we change the fraction $\frac{20}{30}$ to $\frac{2}{3}$. Now, suppose we wish to add the numbers $\frac{3}{4}$ and $\frac{1}{6}$. Since 12 is the least common multiple of the numbers 4 and 6, we select 12 as the common denominator, writing $\frac{3}{4}$ as $\frac{9}{12}$ and $\frac{1}{6}$ as $\frac{2}{12}$. Now we can add the numbers $\frac{9}{12}$ and $\frac{2}{12}$ to get $\frac{11}{12}$, the sum of $\frac{3}{4}$ and $\frac{1}{6}$.

Teaching the Chapter

Materials

Centimetre ruler
Colored chalk
Colored strips
String, one 30-cm piece per child

Vocabulary

common factor common multiple

composite number	least common multiple
factor	multiple
factor tree	prime factor
greatest common factor	prime number
intersection of two sets	union of two sets

Most of the words in the vocabulary list represent ideas that play vital roles in the development of topics in this chapter. However, you need not stress mastery of the terms *union* and *intersection of sets* since these words will be reviewed when the ideas are utilized later in the development of other topics. These terms are included in this vocabulary list because the intersection of two sets is useful in finding the common multiples or the common factors of two numbers.

Lesson Schedule

Two weeks should be sufficient time for covering this chapter. Of course, as for other chapters, you will want to adjust your time schedule to the needs, abilities, and interests of your children. If you have an above-average group of children, you may be able to cover the chapter in less than two weeks; if your children are slower, you may need to spend as much as two-and-one-half weeks on it. Although the topics presented here provide helpful background for certain aspects of the work with fractions and fractional numbers that will follow, they are not absolutely indispensable so we suggest that you avoid unduly extending your treatment of this chapter.

Evaluation of Progress

The children's mastery of the topics in this chapter can be evaluated readily from their achievement on tests for skills in the following areas: determining whether a given number *a* is a factor of another number *b;* determining whether a number is prime; constructing a factor tree; and finding the greatest common factor of two numbers. However, you are urged not to base your entire evaluation on these mechanical skills. You should pay particular attention, through day-to-day observation, to the children's overall understanding and ability to reason through some of the ideas as they are introduced. As pointed out in the vocabulary section for this chapter, the concept of union and intersection of sets need not be mastered at this time; it will be introduced again in later work with fractions and fractional numbers.

The chapter review and the cumulative review included on pages 182 through 185 provide a range of problems that thoroughly covers the material taught up to this point. Use them either as progress tests or for review, as you prefer.

Resources for Active Learning

General References

A Cloudburst, Vol. 2, Nos. 2223, 2313–2343, Midwest Publications

Experiments in Mathematics, Stage 1, "A Multiplication Pattern," pp. 56–57, Houghton Mifflin (Available from Thomas Nelson and Sons)

Mathex: Numeration No. 7, "Sieve of Eratosthenes," pp. 9–12 (pupil pages 6–8), Encyclopaedia Britannica Publications Ltd.

Nuffield Project: *Computation and Structure* 3, "Factors and Primes," pp. 67–69, Wiley

Nuffield Project: *Problems* – Red Set, No. 2, Wiley

Manipulative Devices

Attribute Games and Problems (Selective Educational Equipment; Webster, McGraw-Hill)

Dienes Logical Blocks (Herder and Herder)

Geoboards (Addison-Wesley)

Commercial Games

Domino Number Games – Factors and Multiples (Heath)

On-Sets (Hammett; Nasco; Wff 'N Proof)

Operational Systems Games – Union and Intersection, Multiples and Divisors (Webster, McGraw-Hill)

Prime Drag (Creative Publications)

Ranko – white cards (Midwest Publications)

PAGES 170–171

Objective

Given a composite number less than 100, the child will be able to list the factors of the number.

Preparation

Materials

colored strips

To prepare for the lesson, it would be helpful to review the terms *factor* and *product* as you review basic multiplication facts. For example, you might provide an oral activity, saying: "If my two factors are 7 and 8, what's my product?" Or: "If my product is 64 and one factor is 8, what is the other factor?" As children become familiar with the activity, you might make it more brisk by saying simply: "Factors: 7 and 9" or "Product 54, factor 6," and expect children to respond with the appropriate answer.

Investigation

In this investigation, children are to match sets of their strips with the 12-centimetre strip illustrated in the text. Before they begin, suggest that they record each "equals" train which they find. For example, the "equals" train illustrated might be recorded as 6 twos. Although children have no 12-strip in their set of strips, they can make one by combining the 10-strip and the 2-strip. However, suggest this only after children have had ample opportunity to work on their own.

8 Number Theory

What are the factors of a number?

Investigating the Ideas

The figure shows an "equals" (all strips the same) train that matches a 12-centimetre strip.

? How many other "equals" trains that match the 12-centimetre strip can you find ? See Investigation.

Twelve 1-strips (white)
Four 3-strips (light green)
Three 4-strips (purple)
Two 6-strips (dark green)

Discussing the Ideas

1. The figure above shows that 2 is a factor of 12. What other numbers did you find that are factors of 12 ? See Discussion.
2. Can you use your strips to show that 5 is not a factor of 12 ? See Discussion.
3. Study the figures below.

F F P $8 \times 6 = 48$ { 8 is a **factor** of 48. 6 is a **factor** of 48. Multiplication can help you find factors of a number.	P F F $48 \div 3 = 16$ { 3 is a **factor** of 48. 16 is a **factor** of 48. Division can help you find factors of a number.

Can you find any other factors of 48 ? 1, 2, 4, 12, 24, 48

4. Show that 4 is a factor of 48 by finding the quotient for $4\overline{)48}$. 12
What is another factor of 48 ? 12
5. Show that 5 is **not** a whole number factor of 48 by finding the quotient for $5\overline{)48}$ and by observing that the **remainder** is not zero. 9 R3

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Discussion

In exercise 1, as you discuss the investigation, have children list the strips which they were able to use in their "equals" trains. Make sure they include the 1-strip and explain how they might use a 12-strip if they had one. Thus, their list of factors of 12 should be 1, 2, 3, 4, 6, and 12. As the children discuss exercise 2, they should observe that there are no trains of the 5-strips that can be formed to equal a 12-strip. They can demonstrate this by showing that a train of two 5-strips is less than a 12-strip, while a train of three 5-strips is greater than a 12-strip.

As you discuss exercise 3, stress the fact that 1 and 48 are both factors of 48, just as 1 and 12 are factors of 12. Also, emphasize the test which can be used to determine whether or not one number is a factor of another. If we get a quotient such that the divisor times the quotient is equal to the dividend, then the divisor is a factor of the dividend. On the other hand, if the divisor times the quotient is not equal to the dividend (there is a remainder other than zero), the divisor is not a factor of the dividend.

Using the Ideas

1. Each equation shows 24 written as the product of two factors. Can you write 24 as a product of two factors that are not shown? No

$1 \times 24 = 24$
$2 \times 12 = 24$
$3 \times 8 = 24$
$4 \times 6 = 24$

2. List all the whole numbers that are factors of 24. The equations from exercise 1 should help you. 1, 2, 3, 4, 6, 8, 12, 24

3. Tell whether or not the first number is a factor of the second.
   A 3, 27 Yes B 4, 58 No C 6, 72 Yes D 7, 91 Yes E 8, 108 No F 8, 96 Yes

4. A Show all the ways of writing 18 as the product of two factors. Use each factor only once. (There are 3 ways.) $1 \times 18 = 18$, $2 \times 9 = 18$, $3 \times 6 = 18$
   B List all the factors of 18. 1, 2, 3, 6, 9, 18

5. Show all the ways of writing each number as the product of two factors. Use each factor only once. See Answers, T.E. page 171.
   A 15 (There are 2 ways.) D 30 (There are 4 ways.)
   B 45 (There are 3 ways.) E 35 (There are 2 ways.)
   C 42 (There are 4 ways.) F 60 (There are 6 ways.)

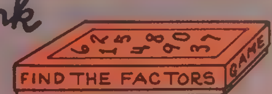
6. List all the factors for each number in exercise 5.
   See Answers, T.E. page 171.

7. Solve the equations. Then list all the factors of 72.
   A $72 \div 1 = n$ 72 D $72 \div 4 = n$ 18
   B $72 \div 2 = n$ 36 E $72 \div 6 = n$ 12
   C $72 \div 3 = n$ 24 F $72 \div 8 = n$ 9
   1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72

8. List all the factors of each number.
   A 20 1, 2, 4, 5, 10, 20 C 32 1, 2, 4, 8, 16, 32
   B 13 1, 13 D 27 1, 3, 9, 27

9. Give a number that is a factor of every number. 1

think



Some numbers have exactly 3 factors. For example, 4 has the factors {1, 2, 4} and 9 has the factors {1, 3, 9}. Find some other numbers that have exactly 3 factors.

25, 49, 121, etc.
(Squares of prime numbers)

171

Answers, exercises 5, 6, page 171

*5. A 15×1, 3×5
B 45×1, 15×3, 9×5
C 42×1, 21×2, 6×7, 3×14
D 30×1, 15×2, 6×5, 3×10
E 35×1, 7×5
F 60×1, 30×2, 10×6, 15×4, 20×3, 5×12

*6. A 15, 1, 3, 5
B 45, 1, 15, 3, 9, 5
C 42, 1, 21, 2, 6, 7, 3, 14
D 30, 1, 15, 2, 6, 5, 3, 10
E 35, 1, 7, 5
F 60, 1, 30, 2, 10, 6, 15, 4, 20, 3, 5, 12

*Order may vary within a pair or listing.

Using the Exercises

Assign the exercises on page 171 as independent work. Exercises 4A and 5 may warrant special attention. For example, in exercise 5A, notice that for the number 15, we observe that there are two ways to write this as the product of two factors. Point out to the children that 5×3 is not to be considered as different from 3×5 and that they must find two different pairs of numbers whose product is 15. Of course, the expected pairs are 3 and 5, and 1 and 15.

The chances of solving the *Think* problem by trial and error are slight. Encourage the children to look for a pattern in their solutions. Give them a hint such as:

$$4 = 2 \times 2$$
$$9 = 3 \times 3$$
$$25 = 5 \times 5$$
$$49 = 7 \times 7$$

This may help them see that the numbers they are searching for are the squares of the prime numbers.

Assignments (page 171)
Minimum: 1–5. Average: 1–7.
Maximum: 1–9.

PAGES 172–173

Objective

Given a composite number less than 100, the child will be able to form a factor tree for it.

Preparation

Unless you want to provide the children with a warm-up activity by reviewing numbers written as a product of two factors, it would be appropriate to begin immediately with the investigation.

Investigation

Factor trees provide a convenient tool for finding the factors of a number. Make sure the children understand the rules for building factor trees before they proceed to find other factor trees for 60. Since every number has itself and 1 as factors, a factor tree using the number 1 would be unending.

Rule 2 should be interpreted by the children as meaning that, as long as any number in a row can still be written as a product of two numbers, the factor tree should grow taller. You might also point out that the product of the numbers in any row is the given number. Rule 3 means that as long as the numbers in any row of one factor tree differ from the numbers in a row of another factor tree, the factor trees are different. However, a change in the order of the factors does not constitute a different factor tree.

The factor trees the children build may vary widely at the third level. To be different from those already shown, however, each should have at the second level, one of the following pairs of factors:

12 × 5
6 × 10
2 × 30

● *What is a factor tree?*

Investigating the Ideas

Here are some rules for building **factor trees**.

RULE 1 No 1's allowed.

RULE 2 Each tree grows as tall as possible.

RULE 3 Two trees are different if the factors at any level are different.

? How many other different trees can you build for 60?
See Investigation.

2 × 2 × 3 × 5
4 × 15
60

3 × 5 × 2 × 2
3 × 5 × 4
3 × 20
60

TWO DIFFERENT TREES FOR 60

Discussing the Ideas

1. What factors are in the top row of each of your factor trees for 60? 2,2,3,5

2. Does 6 have a factor tree that is different from this one? Explain. No. A change in order of factors does not make a different tree.

3 × 2
6

3. Copy each factor tree on your paper and give the missing factors. Explain how you completed the trees.

2 ▒ × ▒ 3 × ▒ 5
2 × ▒ 15
30

▒ 2 × ▒ 5 × 3
10 ▒ × ▒ 3
30

4. Find another factor tree for 30.
5 × 2 × 3
5 × 6
30

5. Show that each row of each factor tree for 30 contains numbers that are factors of 30. 2×15=30, 3×10=30, 5×6=30, 2×3×5 (in any order) = 30

6. Which row of the three trees contains the same factors of 30? Top row

172

Discussion

Have several children write different factor trees for 60 on the chalkboard. Have the children check to see that there are no duplicates. Then ask the children if they notice anything similar among the top rows of each of the factor trees. Point out that no matter what factor tree we build for a given number, we get the same result for the top row of numbers. The numbers may, of course, be in a different order, but the same numbers occur. This is evidence of the Fundamental Theorem of Arithmetic (or the Unique Factorization Theorem). Each composite number can be factored as the product of primes in exactly one way (without regard to order). Discussion exercise 1 emphasizes this idea.

The discussion exercises develop the points mentioned above. Exercise 2 stresses the fact that order is not to be regarded; exercises 3 and 4 stress both how a tree should "grow" until no number in a row can be written as a product of two numbers and how some numbers have several trees. Exercises 5 and 6 stress that the product of each row in a tree is the given number, and that the top rows for the trees of a given number always consist of the same factors.

Using the Ideas

1. Copy each factor tree and give the missing factors.

A
```
3 × ▥ × 2        (3)
  9▥  ×  2
     18
```

B
```
▥ × 2 × ▥        (3, 3)
  6  ×  ▥3
     18
```

C
```
▥ × ▥ × 5        (2, 2)
  4▥  ×  ▥5
     20
```

D
```
▥ × 5 × ▥        (2, 2)
 2▥  ×  ▥10
     20
```

E
```
2 × ▥ × ▥        (5, 5)
 2▥  ×  ▥25
     50
```

F
```
▥ × ▥ × ▥        (2, 5, 5)
 10▥  ×  5
     50
```

2. How many different trees can you draw for 24? Four are possible. Second row can have these factor combinations: 3X8, 4X6, 2X12 (Two variations are possible in 3rd row: 2X6X2 or 2X4X3.)
3. Draw a factor tree for each of these numbers.

A 45 B 28 C 16 D 70 E 36 F 75

See Answers, T.E. page 173

4. In each exercise, part of a factor tree is shown. Give the number for each ▥.

A
```
× 7 × ▥ × ▥      (2, 3)
   ×   6
```

B
```
▥ × ▥ × 7 × 5    (3, 5)
  15   ×
```

C
```
2 × 5 × 17
  ▥10
```

D
```
7 × 3 × 5
 21▥
```

5. Give the missing numbers.

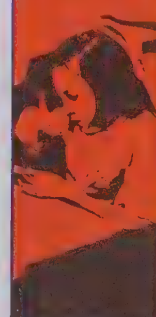
A If 6 is a factor of a number, then ▥ and ▥ are factors of that number. 2, 3

B If 15 is a factor of a number, then ▥ and ▥ are factors of that number. 3, 5

C If 2 and 5 are factors of a number, then ▥ is a factor of the number. 10

D If 7 and 3 are factors of a number, then ▥ is a factor of the number. 21

More practice, page A-15, Set 28



Answers, exercise 3, page 173
(These are sample answers.)

A
```
3 × 5 × 3
  15 × 3
    45
```

B
```
2 × 2 × 7
  4 × 7
    28
```

C
```
2 × 2 × 2 × 2
  4 × 2 × 2
    4 × 4
      16
```

D
```
2 × 5 × 7
  10 × 7
    70
```

E
```
3 × 3 × 2 × 2
  9 × 2 × 2
    18 × 2
      36
```

F
```
5 × 5 × 3
  25 × 3
    75
```

Duplicator Masters, page 38
Workbook, page 53
Skill Masters, page 38

Using the Exercises
Have the children do the exercises on page 173. When they have finished, allow time for discussion and checking papers. Because exercise 4 shows only parts of factor trees, it is not important what numbers we start with. The important idea in such exercises is illustrated in exercise 4A. If 6 is a factor of a number, we know immediately that 2 and 3 are factors of the number. In exercise 4B, we see that, if 15 is a factor of a number, then 3 and 5 are factors of the number.

Assignments (page 173)
Minimum: 1A–C, 2, 3.
Average: 1–4. Maximum: 1–5.

PAGES 174–175

Objective

Given whole numbers less than 50, the child will be able to identify the prime numbers from this set.

Preparation

To prepare for this lesson, you might give children pairs of factors and ask them if either of the factors may itself be written as the product of two numbers. For example, associate your question to the factor tree studied in the previous lesson, and ask: "Could 3 and 4 be the numbers in the top row of a factor tree?" (Since 4 can be written as 2×2, the expected response is no.) "What about 6×5? . . . 2×7? . . . $5 \times 3 \times 6$? . . . $7 \times 8 \times 2$?" and so on.

Investigation

It would be suitable to have children work together on this investigation. Encourage children to approach the question methodically and write the numbers in an ordered list. You might note with the children that they do not necessarily have to build a factor tree for each number less than 50 that has one; they are being asked only to name those numbers for which a factor tree *cannot* be built. Of course, it is likely that some children will incorrectly choose a number which can be factored. Encourage discussion among the children so that they will actually be checking each other's work.

Investigating the Ideas

The numbers 2, 3, 5, and 7 do not have factor trees. Do you see why?

							$2 \times 2 \times 2$			
		2×2		3×2			4×2	3×3	5×2	
2	3	4	5	6	7		8	9	10	. . .

? Can you find the other numbers less than 50 that do not have factor trees? See Investigation. 0, 1, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47

Discussing the Ideas

1. The prime numbers are the whole numbers greater than 1 that have no factor trees. How many factors does each prime number have? Two (itself and 1)

2. The composite numbers are the whole numbers greater than 1 that have factor trees. What are the first 5 composite numbers? 4, 6, 8, 9, 10

3. The top row of each factor tree should have only prime numbers. Explain why this is so. If the numbers are not prime, other factors can be shown, so the tree is incomplete.

4. How can you use the numbers in the top row of this factor tree to find the number for A? Find the missing numbers. Multiply: $7 \times 2 \times 5 \times 3 = 210$

$7 \times 2 \times 5 \times 3$

14 B $\times$ C 15

A 210

5. Can you use exercise 4 to help you write 210 as the product of prime factors? $210 = 7 \times 2 \times 5 \times 3$

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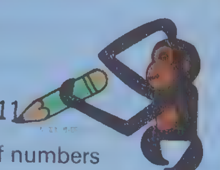
Discussion

Have some children write on the chalkboard the numbers less than 50 which have no factor trees. When the class agrees that the list is correct, use the discussion exercises to teach the meaning of prime and composite numbers. Observe with the children that each prime number has exactly two different factors, 1 and itself. Note that this definition excludes the number 1, for it has only itself as a factor. Also, note that we do not consider a number like 5 as having a factor tree, since we cannot find two numbers other than 1 which are factors of 5. In exercise 5, children need not build a new factor tree for 210. By associating exercises 4 and 5, they should identify the top row of the factor tree in exercise 4 as the answer to exercise 5.

Using the Ideas

1. List all the prime numbers between 1 and 50. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47
2. What number is both an even number and a prime number? 2
3. Find two prime numbers whose difference is 1. 2, 3
   Are there other such pairs? No
4. Find two prime numbers whose difference is 3. 2, 5
   Are there other such pairs? No

think



In the set of numbers
Less than ninety-three,
I'm the largest one
That doesn't have a tree.

WHO AM I? 89

5. Copy and complete each factor tree. Then give the number at the bottom as a product of prime factors.

A ▯ × ▯ × ▯ (7, 2, 5); 7 × ▯ (10); **70** = 7×2×5

B ▯ × ▯ × ▯ × ▯ (2, 2, 2, 2); 4 ▯ × 4; **16** = 2×2×2×2

C ▯ × ▯ × ▯ (3, 5, 7); 3 × ▯ (35); **105** = 3×5×7

D ▯ × ▯ × ▯ × ▯ (2, 3, 5, 5); 6 × ▯ (25); **150** = 2×3×5×5

E ▯ × ▯ × ▯ × ▯ (3, 5, 2, 5); 15 × ▯ (10); **150** = 3×5×2×5

F ▯ × ▯ × ▯ × ▯ (2, 5, 3, 5); 10 × ▯ (15); **150** = 2×5×3×5

6. Express each number as a product of prime factors.
   A 21 B 42 C 30
   D 27 E 72 F 60
   A 3×7 B 2×3×7 C 2×3×5
   D 3×3×3 E 2×2×2×3×3 F 2×2×3×5

★ 7. Will two different factor trees for the same number give the same factors in the top row? Yes. See Using the Exercises.

think

Twin primes are	3,5
primes that differ by	5,7
two. Here are some	11,13
pairs of twin primes.	17,19

Find a pair of twin primes that is between 100 and 110.
101, 103 or 107, 109

175

Using the Exercises

On page 175, you might choose to work through exercises 1–4 together. Note that the answers to exercises 2, 3, and 4 depend upon the fact that 2 is the only even prime number.

When children finish the remaining exercises, exhibit exercises 5A, B, and C on the chalkboard and give the children an opportunity to discuss them. Have the children attempt to show different factor trees for these three numbers. Note with the children that no matter what factor tree we show for a number (such as 70, 16, or 105), we get the same set of prime factors in the top row of the tree. Elicit from the children the fact that for each composite number there is exactly one set of prime factors which can be multiplied together to give this composite number. A discussion of starred exercise 7 will also bring out this idea.

Both *Think* problems should stimulate further discussion of the ideas in this lesson.

Assignments (page 175)
Minimum: 1–4, 5A–C.
Average: 1–6. Maximum: 1–7.

Follow-up

Children might enjoy the following game with prime numbers. Have them make 2 cubes and write on the sides of each the first six prime numbers (2, 3, 5, 7, 11, 13). Then two, three, or four players can take turns rolling the dice. For each pair of numbers a player rolls, he should record his sum and product and indicate whether each is even or odd. After each player has had ten throws (or six throws if four are playing), they should tally their scores according to the following point system:

Every odd sum is worth 5 points.

Every even product is worth 5 points.

If both the sum and the product of the same roll are even, that roll is worth 25 points. (Children should discover the uniqueness of 2 as the only prime number which is even.)

A sample chart for one player follows:

Roll	Sums	Products
1	E	O
2	O (5)	E (5) – 10 points
3	E	O
4	E	O
5	E	O
6	E	O
7	E	E – 25 points
8	E	O
9	E	O
10	O (5)	E (5) – 10 points
		Total: 45 points

If there is a tie score, the player who rolled the odd sum and even product, or the player who first rolled a pair of twos, wins.

Resources for Active Learning

Mathex: Numeration No. 7, "Prime Numbers," pp. 6–8, Encyclopaedia Britannica Publications Ltd.

Modern Math Games . . ., "Two for the Primes," p. 12, Fearon. [A game] (Available from Clarke, Irwin)

Notes on Mathematics in Primary Schools, "Prime Numbers," pp. 15-16, Cambridge University Press. (Available from Macmillan of Canada)

Nuffield Project: *Problems* – Red Set, No. 4, Wiley.

Workbook, page 54

PAGES 176–177

Objective

Given two finite sets, the child will be able to recognize both the union and the intersection of the sets.

Preparation

Materials

string, 30-cm pieces (1 per child)

To prepare for this lesson, you might review the word *set* with the children, to make sure their understanding of the term is intuitively correct. Recall that the term itself is undefined in mathematics, but an intuitive notion of its meaning is not difficult for children to develop. You might review the term in the following manner: Tell the children that you are thinking of a set; ask them to name objects which they think might be in the set. After you have responded to 8 or 9 objects, see if a volunteer can describe your set; if no one can, describe it yourself. You might then ask children to think of a set, and call on their classmates to describe it. The time spent on this preparation should not be more than about five minutes.

Investigation

In this investigation, children actually work with a number of pairs of sets and form the intersection of each pair. Suggest that they not only count how many ways they can arrange their loop, but also record each situation. You might exhibit headings for three columns they might use.

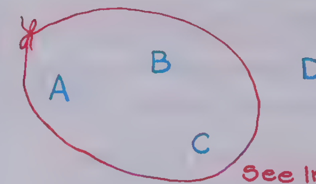
In loop of string	In red loop	In both loops
D, C	*A, B, C*	*C*
D, B	*A, B, C*	*B*
D, A	*A, B, C*	*A*
D, A, B	*A, B, C*	*A, B*
D, A, C	*A, B, C*	*A, C*
D, B, C	*A, B, C*	*B, C*
D, A, B, C	*A, B, C*	*A, B, C*

(The possibilities are listed only for your convenience; children should work this investigation with minimal guidance.)

Let's explore union and intersection of sets.

Investigating the Ideas

Make a loop out of a 30-cm piece of string. The figure shows another loop with 3 letters inside and 1 letter outside.

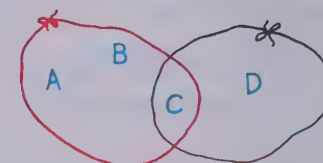


See Investigation.

How many ways can you place your loop of string on the figure so that **each letter is inside one of the loops** **and** **at least one letter is inside both loops?**	Draw pictures to show the ways you placed your loop.

Discussing the Ideas

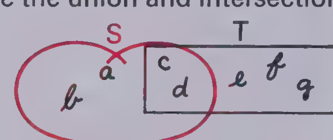
1. Each loop in your Investigation contained a set of letters. The **union** of the two sets is all the letters that are in one set or the other or in both sets. What is the union of the two sets in the figure? {A, B, C, D}



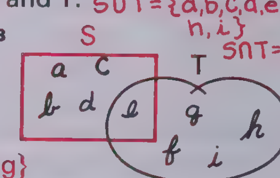
2. A The **intersection** of the two sets is the set of letters that are in **both** sets. What is the intersection of the sets in the figure? {C}
   B What letters were in the intersections of the sets you formed in the Investigation? See Investigation.

3. Give the union and intersection of sets S and T. S∪T = {a, b, c, d, e, f, g, h, i} S∩T = {e}

A



B



Answer: Union: S ∪ T = {a, b, c, d, e, f, g}
Intersection: S ∩ T = {c, d}

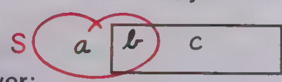
Discussion

Exercises 1 and 2 introduce the terms *union* and *intersection* through a discussion of the investigation. It would be helpful to exhibit the chart illustrated at the left, in the investigation section. Since the union includes letters that are in one set or in the other set or in both sets, the union of each of the seven pairs of sets is the same, namely, $\{A, B, C, D\}$. The intersection, however, contains only those letters which are in both of the two sets; thus, the intersection of the two sets is different for each different pair of sets.

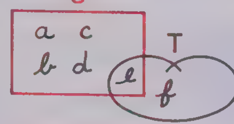
Exercise 3 introduces the symbols for union and intersection. You might help children distinguish between ∪ (union) and ∩ (intersection), by associating the letter "u" of union with the "u"-shaped symbol, ∪. It would be helpful to use these symbols with sets from the investigation. Observe with the children that members of a set are enclosed within brackets. For example, if we let $R = \{D, A\}$ and $S = \{A, B, C\}$, then $R \cup S = \{A, B, C, D\}$ and $R \cap S = \{A\}$ or, more simply, $\{D, A\} \cup \{A, B, C\} = \{A, B, C, D\}$ and $\{D, A\} \cap \{A, B, C\} = \{A\}$.

Using the Ideas

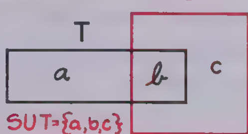
1. For each exercise, give the union and the intersection of the two sets. (Note: If there are no letters in the intersection of the sets, just write $S \cap T = \{\ \ \}$.)

A 

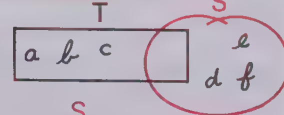
Answer:
$S \cup T = \{a, b, c,\}$
$S \cap T = \{b\}$

B 

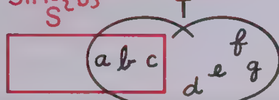
$S \cup T = \{a,b,c,d,e,f\}$
$S \cap T = \{e\}$

C 

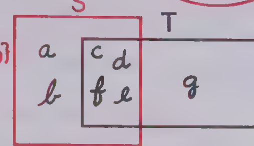
$S \cup T = \{a,b,c\}$
$S \cap T = \{b\}$

D 

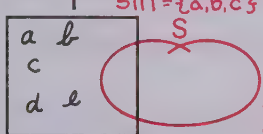
$S \cup T = \{a,b,c,d,e,f\}$
$S \cap T = \{\ \}$

E 

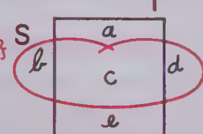
$S \cup T = \{a,b,c,d,e,f,g\}$
$S \cap T = \{a,b,c\}$

F 

$S \cup T = \{a,b,c,d,e,f,g\}$
$S \cap T = \{c,d,e,f\}$

G 

$S \cup T = \{a,b,c,d,e\}$
$S \cap T = \{\ \}$

H 

$S \cup T = \{a,b,c,d,e\}$
$S \cap T = \{c\}$

2. Study the example. Then give the union and the intersection for each exercise.

Example: $S = \{1, 2, 3, 4\}$ $T = \{3, 4, 5, 6, 7\}$ — $S \cup T = \{1, 2, 3, 4, 5, 6, 7\}$ $S \cap T = \{3, 4\}$

A $S = \{1, 2, 3\}$ $S \cup T = \{1,2,3,4\}$
$T = \{3, 4\}$ $S \cap T = \{3\}$

B $S = \{0, 1, 2, 3, 4, 5\}$ $S \cup T = \{0,1,2,3,4,5,6\}$
$T = \{3, 4, 5, 6\}$ $S \cap T = \{3,4,5\}$

C $S = \{0, 1, 2\}$ $S \cup T = \{0,1,2,3,4,5,6\}$
$T = \{1, 2, 3, 4, 5, 6\}$ $S \cap T = \{1,2\}$

D $S = \{0, 1, 2, 3\}$ $S \cup T = \{0,1,2,3,4,5,6\}$
$T = \{4, 5, 6\}$ $S \cap T = \{\ \}$

E $S = \{0, 1, 2, 3, 4, 5, 6\}$ $S \cup T = \{0,1,2,3,4,5,6\}$
$T = \{2, 3, 4\}$ $S \cap T = \{2,3,4\}$

F $S = \{7, 8, 9\}$ $S \cup T = \{1,2,3,7,8,9\}$
$T = \{1, 2, 3\}$ $S \cap T = \{\ \}$

G $S = \{5, 6, 7\}$ $S \cup T = \{2,3,4,5,6,7,8\}$
$T = \{2, 3, 4, 5, 6, 7, 8\}$ $S \cap T = \{5,6,7\}$

H $S = \{9, 10, 11\}$ $S \cup T = \{7,8,9,10,11,12,13\}$
$T = \{7, 8, 9, 11, 12, 13\}$ $S \cap T = \{9,11\}$

More practice, page A-15, Set 29

Resources for Active Learning

Discovery, Section II, Units 19/2, 3; 20/1–4, Encyclopaedia Britannica Educational Corp.

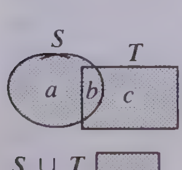
Nuffield Project; *Problems*—Green Set, No. 37; Red Set, No. 1, Wiley.

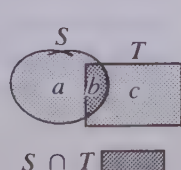
SMSG: *Probability for Intermediate Grades*, "Both, And; Either, Or," Lesson 9, Stanford University.

Duplicator Masters, page 39
Workbook, page 55
Skill Masters, page 39

Using the Exercises

Assign the exercises on page 177 as independent work. When the children finish, you might use colored chalk to demonstrate answers to some of the exercises. For example, draw the picture in exercise 1A twice; color one drawing to illustrate the union of the two sets and the other to show the intersection of the two sets.





Assignments (page 177)
Minimum: 1, 2A–D.
Average: 1–2. Maximum: 1–2.

Objective

Given a pair of composite numbers, the child will be able to find their greatest common factor.

Preparation

To prepare for this lesson, review the intersection of sets discussed in the previous lesson. For example, list the following pairs of sets on the chalkboard and ask children to give the members of the sets which are in the intersection.

$\{1, 2, 4, 6, 8, 10, 12\} \cap \{1, 3, 6, 9, 12\}$

$\{1, 2, 4, 6, 8, 12\} \cap \{1, 4, 8, 12, 16, 20\}$

$\{1, 3, 4, 6, 12, 24\} \cap \{1, 2, 3, 6, 9, 12, 36\}$

$\{1, 3, 5, 15\} \cap \{1, 2, 5, 10\}$

$\{1, 3, 7, 21\} \cap \{1, 5, 7, 35\}$

$\{1, 3, 6, 7, 21, 42\} \cap \{1, 3, 4, 6, 8, 12, 24\}$

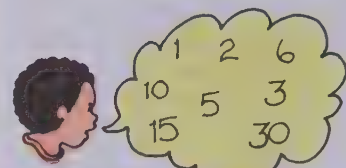
Investigation

If the children worked well with the suggestion in the preparation, you might suggest that they first find the intersection of Don's set and Linda's set. The intersection, of course, would consist of the numbers given by both children, that is, of the numbers which are factors of both 30 and 48. They must then proceed to find the largest of these numbers (6). Encourage the children to work independently. This investigation is short, so they should not need very much time.

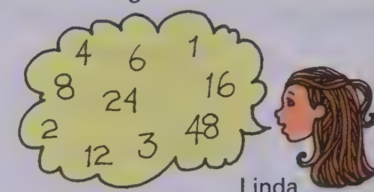
What is the greatest common factor of two numbers?

Investigating the Ideas

Don gave the factors of 30.



Linda gave the factors of 48.



See Investigation.

? Can you find the intersection of Don's and Linda's sets? 1,2,3,6

Record the largest number in the intersection of the sets. 6

Discussing the Ideas

1. The largest number in the intersection of the children's sets is the **greatest common factor** of 30 and 48. What is it? 6

2. Study each table and give the greatest common factor of the two numbers.

$S = \{1, 2, 3, 4, 6, 12\}$ ←	The factors of 12
$T = \{1, 2, 4, 5, 10, 20\}$ ←	The factors of 20
$S \cap T = \{1, 2, 4\}$ ←	The common factors of 12 and 20
4 is the greatest common factor of 12 and 20.	

$R = \{1, 2, 3, 6, 9, 18\}$ ←	The factors of 18
$Q = \{1, 3, 9, 27\}$ ←	The factors of 27
$R \cap Q = \{1, 3, 9\}$ ←	The common factors of 18 and 27
9 is the greatest common factor of 18 and 27.	

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Discussion

Before discussing exercise 1, point out that the numbers 1, 2, 3, and 6 were given by both children, so they are *common factors* of 30 and 48. Then discuss the fact that since 6 is the greatest of these numbers, it is called the *greatest common factor.*

As you discuss exercise 2, point out that first the factors of each number are listed; then all the common factors are identified; and finally the greatest of these common factors is noted. To give children practice in finding the greatest common factor, use several examples such as those suggested in the preparation. Also use examples in which the children must first find the factors of pairs of numbers and then find the greatest common factor. For example, ask children to find the greatest common factor of these pairs: 12, 18; 12, 15; 16, 20; 12, 24; 9, 16. Observe with the children that sometimes the greatest common factor of two numbers is one of the numbers: the greatest common factor of 12 and 24 is 12. Also, sometimes it is the number one: the greatest common factor of 9 and 16 is 1.

Using the Ideas

1. A List the factors of 12. 1,2,3,4,6,12 B List the factors of 18. 1,2,3,6,9,18
   C List the common factors of 12 and 18. 1,2,3,6
   D What is the greatest common factor of 12 and 18? 6

2. A List the factors of 18. 1,2,3,6,9,18 B List the factors of 20. 1,2,4,5,10,20
   C List the common factors of 18 and 20. 1,2
   D What is the greatest common factor of 18 and 20? 2

3. For each pair of numbers below:
   [A] List the factors of each number.
   [B] List the common factors of the two numbers.
   [C] Give the greatest common factor of the two numbers.

A 12, 4	C 8, 12	E 10, 14	G 5, 15	I 7, 13
B 8, 20	D 9, 27	F 40, 50	H 25, 20	J 16, 24

See Answers, T.E. page 179.

4. Give the greatest common factor of each pair of numbers.

A 15, 25 5	D 24, 18 6	G 9, 10 1	J 18, 30 6
B 10, 30 10	E 3, 21 3	H 18, 15 3	K 12, 9 3
C 18, 8 2	F 8, 7 1	I 50, 20 10	L 15, 8 1

5. What number is a common factor of any pair of numbers? 1

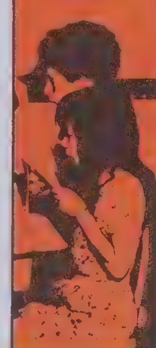
★6. If both of the numbers are prime, what can you say about the greatest common factor of the two numbers? It is 1.

★7. If one of two numbers is prime, what can you say about the greatest common factor of the two numbers? It is either 1 or the prime number.

think

We each would like to be
A composite or a prime.
But when we try to join,
We are turned down
every time.

More practice, page A-16, Set 30



Using the Exercises

Notice that, in exercises 1 through 3 on page 179, children are guided through the procedure of first listing the factors of each number, then the common factors, and finally the greatest common factor. But by the time they get to exercise 4, more able children might be able to give the greatest common factor of some pairs simply by inspection. We hope that children will develop this type of skill for their later work with fractions. However, do not expect them to do it efficiently at this time.

Assignments (page 179)
Minimum: 1–4. Average 1–5.
Maximum: 1–7.

Answers, exercise 3, page 179

A 1, 2, 3, 4, 6, 12
1, 2, 4
Common factors: 1, 2, 4
GCF: 4

B 1, 2, 4, 8
1, 2, 4, 5, 10, 20
Common factors: 1, 2, 4
GCF: 4

C 1, 2, 4, 8
1, 2, 3, 4, 6, 12
Common factors: 1, 2, 4
GCF: 4

D 1, 3, 9
1, 3, 9, 27
Common factors: 1, 3, 9
GCF: 9

E 1, 2, 5, 10
1, 2, 7, 14
Common factors: 1, 2
GCF: 2

F 1, 2, 4, 5, 8, 10, 20, 40
1, 2, 5, 10, 25, 50
Common factors: 1, 2, 5, 10
GCF: 10

G 1, 5
1, 3, 5, 15
Common factors: 1, 5
GCF: 5

H 1, 5, 25
1, 2, 4, 5, 10, 20
Common factors: 1, 5
GCF: 5

I 1, 7
1, 13
Common factors: 1
GCF: 1

J 1, 2, 4, 8, 16
1, 2, 3, 4, 6, 8, 12, 24
Common factors: 1, 2, 4, 8
GCF: 8

Duplicator Masters, page 40
Workbook, page 56
Skill Masters, page 40

PAGES 180–181

Objective

Given two numbers, the child will be able to find the least common multiple of the two numbers.

Preparation

Materials

colored strips; centimetre ruler

The nature of this investigation makes it suitable to omit any specific preparation. Have the children begin immediately with the investigation.

Investigation

Suggest to the children that they first use their 2-strip and 3-strip as illustrated in the text. When they are working with the 6-strip and the 8-strip, you might challenge some children to find out some other numbers these strips will match. You might also suggest that they do the same investigation with another pair of strips, such as the 5-strip and the 3-strip, or 2 and 4, or 6 and 9.

● *What is the least common multiple of two numbers?*

Investigating the Ideas

When your 2-strips and 3-strips "match" at 0, they "match" at 6, 12 . . . .

? Can you find where your 6-strips and 8-strips first match after 0? 24 See Investigation.

Discussing the Ideas

You were finding multiples and common multiples in the Investigation. The common multiples of 2 and 3 are

{0, 6, 12, 18, . . .}

The first time the strips match after 0 is called the **least common multiple** of the two numbers.

1. What is the least common multiple of 2 and 3? 6

2. What is the least common multiple of 4 and 6? 12

Discussion

Use the text explanation to relate the investigation to the set of common multiples of 2 and 3. Help children see why 6 is the least common multiple of 2 and 3; note that it is *not* because 6 happens also to be the product of 2 and 3.

Have a volunteer list the numbers where his 8-strip and 6-strip match. Have someone circle the number where these strips first match (after zero). Help children to see that this number, 24, is the least common multiple of 8 and 6. Note that although 0 is a common multiple of every number, we do not consider it as the least common multiple of two numbers. More precisely, we might call this the "least common nonzero multiple" of two numbers, but we shorten it to "least common multiple."

Use other examples of finding the least common multiple, such as those suggested in the investigation. Make sure children realize that the multiple of a number is simply a product of the given number and another. Thus, to find the least common multiple of 6 and 9, they should list the first five or six multiples of 6, then the first five or six multiples of 9, compare the lists, and find the smallest number which appears in both lists.

Using the Ideas

1. A List the multiples (to 36) of 3. {0,3,6,9,12,15,18,21,24,27,30,33,36}
   B List the multiples (to 36) of 4. {0,4,8,12,16,20,24,28,32,36}
   C List the common multiples (to 36) of 3 and 4. {0,12,24,36}
2. A List the multiples (to 24) of 2. {0,2,4,6,8,10,12,14,16,18,20,22,24}
   B List the multiples (to 24) of 4. {0,4,8,12,16,20,24}
   C List the common multiples (to 24) of 2 and 4. {0,4,8,12,16,20,24}
3. A List the multiples (to 48) of 6. {0,6,12,18,24,30,36,42,48}
   B List the multiples (to 48) of 8. {0,8,16,24,32,40,48}
   C List the common multiples (to 48) of 6 and 8. {0,24,48}
4. A List the multiples (to 42) of 3. {0,3,6,9,12,15,18,21,24,27,30,33,36,39,42}
   B List the multiples (to 42) of 6. {0,6,12,18,24,30,36,42}
   C List the common multiples (to 42) of 3 and 6. {0,6,12,18,24,30,36,42}
5. A List the multiples (to 60) of 10. {0,10,20,30,40,50,60}
   B List the multiples (to 60) of 4. {0,4,8,12,16,20,24,28,32,36,40,44,48,52,56,60}
   C List the common multiples (to 60) of 10 and 4. {0,20,40,60}
6. A List the multiples (to 60) of 5. {0,5,10,15,20,25,30,35,40,45,50,55,60}
   B List the multiples (to 60) of 6. {0,6,12,18,24,30,36,42,48,54,60}
   C List the common multiples (to 60) of 5 and 6. {0,30,60}
7. A List the multiples (to 80) of 8. {0,8,16,24,32,40,48,56,64,72,80}
   B List the multiples (to 80) of 10. {0,10,20,30,40,50,60,70,80}
   C List the common multiples (to 80) of 8 and 10. {0,40,80}
8. A List the multiples (to 70) of 5. {0,5,10,15,20,25,30,35,40,45,50,55,60,65,70}
   B List the multiples (to 70) of 7. {0,7,14,21,28,35,42,49,56,63,70}
   C List the common multiples (to 70) of 5 and 7. {0,35,70}
9. What number is a common multiple for every pair of numbers? 0
10. Give the least common multiple for:

A 3 and 4 12	G 8 and 10 40
B 2 and 4 4	H 5 and 7 35
C 6 and 8 24	I 5 and 2 10
D 3 and 6 6	J 9 and 4 36
E 10 and 4 20	K 5 and 4 20
F 5 and 6 30	L 3 and 5 15

think

I'm an odd composite.
The smallest one you'll see
That has two different primes
Appearing on its tree.

WHO AM I? 15

More practice, page A-16, Set 31

Using the Exercises

After children complete the exercises on page 181, point out certain related ideas. For example, although we could list all the factors of a number, we can never list all the multiples of a number. Also, the LCM (a common abbreviation for least common multiple) of two numbers will sometimes be the product of those numbers, as with 3 and 4. Also, the LCM may sometimes be one of the numbers, as with 2 and 4, or 2 and 8.

Assignments (page 181)
Minimum: 1–4, 10. Average: Even-numbered problems. Maximum: All.

Duplicator Masters, page 41
Workbook, page 57
Skill Masters, page 41

PAGES 182–183

Objective

The child will demonstrate his ability to work with the concepts presented in this chapter.

Preparation

Review with the children any of the chapter topics with which they had particular difficulty. Some beneficial preparation activities would be drawing factor trees, listing all the factors of several numbers, and listing some of the multiples of several small numbers. Making these lists of factors and multiples is nearly always a worthwhile activity for the children, and should be done frequently to reinforce and maintain their ability to work with factors and multiples.

Reviewing the Ideas

1. Copy each factor tree and give the missing factors.

A 7 × 3 × 3
7 × 9
63

B 3 × 3 × 2 × 3
9 × 6
54

C 2 × 5 × 3 × 3
10 × 9
90

2. Draw a factor tree for each number. See Answers, T.E. page 183.

A 14 B 10 C 28 D 45 E 56

3. Tell whether or not each number is prime. Show a factor tree for each number that is not prime.

A 21 3×7 No
B 29 Yes
C 33 3×11 No
D 67 Yes
E 81 3×3×3×3, 9×9 No
F 51 3×17 No
G 47 Yes

4. List all the factors of each number.

A 14 1,2,7,14
B 12 1,2,3,4,6,12
C 16 1,2,4,8,16
D 18 1,2,3,6,9,18
E 20 1,2,4,5,10,20
F 22 1,2,11,22
G 40 1,2,4,5,8,10,20,40

5. Give S ∪ T and S ∩ T for each pair of sets.

A S = {2, 4, 6, 8, 10} S∪T={0,1,2,3,4,5,6,8,10}
T = {0, 1, 2, 3, 4, 5} S∩T={2,4}

B S = {6, 7, 8, 9, 10, 11} S∪T={0,1,2,3,4,5,6,7,8,9,10,11}
T = {0, 1, 2, 3, 4, 5, 6} S∩T={6}

C S = {0, 2, 4, 6, 8, 10} S∪T={0,1,2,3,4,5,6,7,8,9,10,11}
T = {1, 3, 5, 7, 9, 11} S∩T={ }

D S = { } S∪T={1,2,3}
T = {1, 2, 3} S∩T={ }

E S = {5, 7, 9} S∪T={4,5,6,7,8,9,10}
T = {4, 5, 6, 8, 9, 10} S∩T={5,9}

F S = {7, 8, 9, 10} S∪T={7,8,9,10}
T = {7, 8, 9, 10} S∩T={7,8,9,10}

6. A List all the factors of 30. 1,2,3,5,6,10,15,30
B List all the factors of 18. 1,2,3,6,9,18
C List the common factors of 18 and 30. 1,2,3,6
D What is the greatest common factor of 18 and 30? 6

7. A List the first 10 multiples of 6. {0,6,12,18,24,30,36,42,48,54}
B List the first 10 multiples of 8. {0,8,16,24,32,40,48,56,64,72}
C List some common multiples of 6 and 8. 24, 48
D What is the least common multiple of 6 and 8? 24

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Discussion

If you assign the exercises on these two pages as independent work, the results should enable you to make a fairly accurate evaluation of the children's understanding and skill with prime numbers, factorization, set union and intersection, greatest common factors, and least common multiples. Check the work yourself, but return the papers and allow the children to ask questions. Plan to reteach areas of weakness by letting the children work at the board or in small groups on common problem areas.

Although the complete solution to the *Think* problem may not be accessible to any but the most able children, most children will benefit from a discussion of it once the correct answer is given. Be sure to give all children an opportunity to try the problem and, later, to discuss it. A table of primes should help less capable children.

8. Give the greatest common factor for each pair of numbers.
   - A 4 and 10 2
   - B 15 and 25 5
   - C 18 and 48 6
   - D 24 and 40 8

9. Give the least common multiple for each pair of numbers.
   - A 4 and 10 20
   - B 6 and 15 30
   - C 8 and 5 40
   - D 12 and 4 12

10. If the numbers 1, 2, 3, and 6 are the common factors of two numbers, what is the greatest common factor of the numbers? 6

think

No one has yet found an even number greater than 2 that is not the sum of two primes. Here are some examples:

$4 = 2 + 2$	$10 = 5 + 5$
$6 = 3 + 3$	$12 = 5 + 7$
$8 = 5 + 3$	$14 = 7 + 7$

You try this for the rest of the even numbers to 50.
See Solution, T.E. page 183.

11. If the first four common multiples of two numbers are 0, 12, 24, and 36, what is the least common multiple? 12

12. Give the missing word or number.
   - A No prime number greater than 7 ends with one of the digits 0, 2, 4, 6, 8, or ||||. 5
   - B If a number is prime, then it has exactly two __?__. factors
   - C If a number is the product of two smaller numbers, then it is __?__. composite
   - D The numbers less than 10 that have exactly three factors are 4 and ||||. 9
   - E |||| is the smallest 2-digit number that has exactly three factors. 25
   - F If 7 and 3 are both factors of a number, then |||| is a factor of the number. 21
   - G If 10 is a factor of a number, then 2 and |||| are factors of the number. 5
   - H If 6 and 5 are factors of a number, then 30, 15, and |||| are also factors of the number. 2, 3, or 10

Solution, Think, page 183

Several answers are possible for each of the even numbers in the *Think* problem. The following are sample answers. Your students may suggest other acceptable answers as well.

$16 = 3 + 13$	$34 = 17 + 17$
$18 = 7 + 11$	$36 = 19 + 17$
$20 = 7 + 13$	$38 = 19 + 19$
$22 = 3 + 19$	$40 = 23 + 17$
$24 = 5 + 19$	$42 = 23 + 19$
$26 = 7 + 19$	$44 = 31 + 13$
$28 = 11 + 17$	$46 = 17 + 29$
$30 = 13 + 17$	$48 = 29 + 19$
$32 = 13 + 19$	$50 = 31 + 19$

Answers, exercise 2, page 182
(These are sample answers.)

A 2×7
14

B 2×5
10

C $2 \times 2 \times 7$
4×7
28

D $5 \times 3 \times 3$
5×9
45

E $7 \times 2 \times 2 \times 2$
$7 \times 4 \times 2$
7×8
56

Resources for Active Learning

Modern Math Games . . ., "Prime Factorization," pp. 49-50, Fearon. [Puzzles] (Available from Clarke, Irwin)

Workbook, page 58

PAGES 184–185

Objective

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

Review with the children any topics covered so far in the text which they have found difficult. If no apparent difficulty comes to mind, it will probably be most helpful to spend this time reviewing the division algorithm. For example, have children work through the following:

$88\overline{)2024}$ (quotient: 23)

or $26\overline{)2867}$ (quotient: 110 R7)

If you prefer, you might have children practice rounding numbers, or you might discuss area and perimeter before the children do the problems. In any case, keep the preparation short and lively.

Keeping in Touch with Addition, Subtraction, Multiplication, Division, Measurement, Place value

1. Solve the equations.

A $9 \times n = 36$ 4   C $n = 23 - 9$ 14   E $46 = (n \times 10) + 6$ 4

B $70 \div 10 = n$ 7   D $35 = n + 28$ 7   F $63 - (n \times 7) = 0$ 9

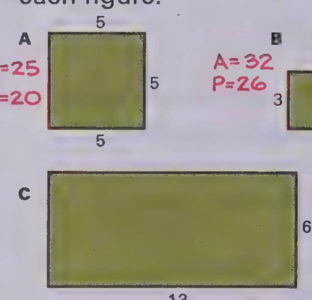
2. Compute the sum, product, difference, or quotient.

A 576 + 985 = 1561	B 489 × 4 = 1956	C 482 − 167 = 315	D 86 × 34 = 2924	E 804 − 457 = 347
F 583 × 26 = 15 158	G 999 + 888 = 1887	H 900 − 398 = 502	I 467 × 231 = 107 877	J 4002 − 879 = 3123
K 839 × 207 = 173 673	L 7034 − 2769 = 4265	M $5\overline{)460}$ 92	N $7\overline{)1867}$ 266R5	
O $8\overline{)46\,592}$ 5824	P $32\overline{)256}$ 8	Q $59\overline{)1416}$ 24	R $75\overline{)3075}$ 41	

3. A bottle of perfume holds a little more than 29 millilitres of liquid. **Estimate** the number of millilitres of perfume in 8 bottles.

240 cm³

4. Find the **area** and **perimeter** of each figure.

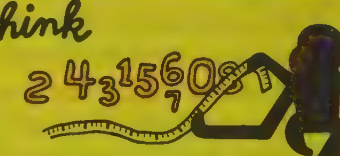


A=25 P=20

A=32 P=26

A=78 P=38

think



I'm the smallest number
Ever to make this claim.
I have 4 prime factors,
With none of them the same.
$a \times b \times c \times d = ?$

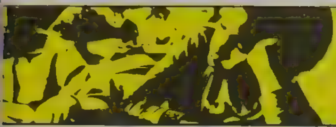
WHO AM I? 210

(See Discussion.)

Discussion

Have the children do the exercises. When they have finished, allow time for checking papers and for discussion of the exercises. It may prove helpful to have several parts of exercise 2 presented on the chalkboard and explained to the class. It may also be helpful to discuss in some detail the parts of exercise 4. Illustrate for the children why we multiply to find the area (pages 128–129). Most of them should be able to see quickly why we add in order to find perimeter.

The given equation should help those who want to attempt the *Think* problem. The key ideas are *smallest number* with *four unlike* prime factors. Replacing $a, b, c,$ and d with the four smallest primes gives $2 \times 3 \times 5 \times 7$, or 210. Urge the solvers to share their methods with the rest of the class.



Solving Story Problems

FIND THE NUMBER

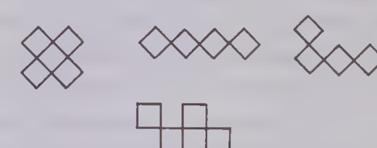
1. Find the number that is 78 more than 296. 374
2. What number must be added to 346 to get 501 for the sum? 155
3. Find the number that is 59 less than the product of 26 and 59. 1475
4. What number must you multiply by 36 to get 1512 for the product? 42
5. Give the number that is 269 less than the sum of 3268, 4297, and 6598. 13 894
6. Find the number that is three times the difference of 2003 and 867. 3408
7. Give the number that must be added to the product of 46 and 18 to get 985. 157
8. What number is six times the sum of 386, 265, 19, and 1268? 11 628
9. Find the number that is twice the product of 62 and 7864. 975 136
10. Give the number that is 75 less than the sum of 687, 346, 928, 467, 159, 847, and 698. 4057
11. What number is 268 more than the product of 34 and 6925? 235 718
12. What number is eight times twice the sum of 6289 and 75 668? 1 311 312
13. Find the number that is 67 more than the quotient of 1728 and 27. 131
14. Give the number that is 64 times the sum of 326, 547, 832, and 964. 170 816

You are invited to explore

ACTIVITY CARD 8
Page 337

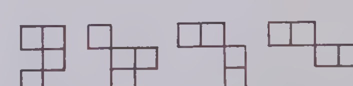
Using the Exercises
During the discussion of page 185, emphasize the key words that help us decide which operation to use for certain situations. For example, "more than" quite often indicates addition; "less than," subtraction; etc. Give the children an opportunity to explain how they decided upon a particular operation for a given problem.

Follow-up
To improve understanding of the differences between perimeter and area, give each child four 5-cm squares cut from colored poster board. Illustrate on the chalkboard how the four regions can be arranged to form a figure with a perimeter of 8 units. (See the figures below.) Challenge the children to try to arrange the four squares in such a way as to get perimeters of 10, 12, 14, and then 16 units. Suggest that they sketch each of their solutions on paper to share later with their classmates. Encourage interested children to find as many different arrangements for each given perimeter as possible. Some of them may even try to find perimeters of 9, 11, 13, and 15 units. As solutions are shared, emphasize that the total area of the four square regions remains constant, while the various arrangements change the number of edges exposed. Some possible solutions are shown below.



16 units

14 units



12 units

10 units

8 units

General Objectives

To provide a formal introduction to fraction concepts

To provide experience with equivalent fractions

To stress working with sets of equivalent fractions

To provide background material for a smooth transition to fractional-number concepts

To develop skills in finding lowest-terms fractions

A fraction is a written symbol for a number pair. However, it is often convenient to use the word "fraction" in place of "number pair," and you will notice that we will occasionally use "fraction" when, actually, we mean "number pair." It should be clear at all times from the context of the material whether we are speaking of the number pair or the symbol. Later, of course, we will use fractions as symbols for fractional numbers. Here again, it should always be clear from the context whether we are speaking of the fraction, the number pair, or the fractional number.

Following the introduction of fractions and number pairs, the lessons of this chapter develop further concepts of fractions, which culminate in the idea of equivalent fractions. Considerable stress is placed upon building sets of equivalent fractions in order to lead the children toward the idea that exactly one fractional number is associated with each set of equivalent fractions, which is the central idea treated in Chapter 10. The terms numerator, denominator, and improper fraction are introduced. Next, a test is provided to determine whether or not two fractions are equivalent. Then, toward the end of the chapter, the lowest-terms fractions are introduced; and, finally, children are presented with materials which provide the techniques and practice they need in order to gain skill in changing fractions to lowest terms.

Mathematics

This chapter is concerned primarily with developing concepts of fractions in preparation for the introduction of fractional-number concepts in the next chapter. The study of fractions is conceptually different from the study of the numbers represented by fractions (fractional numbers). You will observe that, except for the references to number pairs early in the chapter, essentially nothing is said about numbers throughout Chapter 9. Fractions are considered to be symbols for number pairs in the study of this chapter, and in the next chapter they will be considered to be symbols for fractional numbers.

One of the clearest explanations we can give for the concepts presented in this chapter appears in the tables on page 187. These tables show that a number pair is associated with a given set or region and that a fraction is used to communicate this number pair. We might say, therefore, that this chapter is a study of number pairs used in a certain way and that fractions are the symbols we write to represent these number pairs.

The work with equivalent fractions is an important part of Chapter 9, since it lays the groundwork for the development of fractional-number concepts. We do not give a formal definition of equivalent fractions because such a definition would be too abstract for the children. What is intended is that the children recognize two fractions as being equivalent by picturing them in terms of sets or parts of a region.

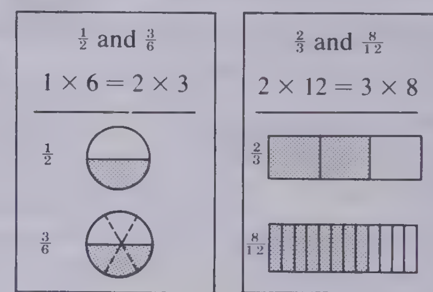
A formal definition of equivalent fractions follows.

Two fractions

$$\frac{a}{b} \text{ and } \frac{c}{d}$$

are equivalent to each other if and only if $a \times d = b \times c$.

The diagrams below each example illustrate the equivalence relation with regard to part of an object.



Having provided the children with experiences in working with equivalent fractions, we lead them to build sets of equivalent fractions, starting with lowest-terms fractions such as $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$, etc. The following sets are built from such fractions:

$$A = \{\tfrac{1}{2}, \tfrac{2}{4}, \tfrac{3}{6}, \tfrac{4}{8}, \tfrac{5}{10}, \tfrac{6}{12}, \tfrac{7}{14}, \tfrac{8}{16}, \ldots\}$$
$$B = \{\tfrac{2}{3}, \tfrac{4}{6}, \tfrac{6}{9}, \tfrac{8}{12}, \ldots\}$$
$$C = \{\tfrac{3}{4}, \tfrac{6}{8}, \tfrac{9}{12}, \tfrac{12}{16}, \ldots\}$$

Clearly, if we continue the established pattern indefinitely, each set contains an unlimited number of fractions. It is important to note that any fraction that is equivalent to one half is in set A (assuming an unlimited continuation of the obvious pattern) and that a fraction that is not equivalent to one half is not in set A. Of course, similar statements are true for sets B and C, as well as any other similarly constructed set of fractions.

We list below points concerning equivalent fractions and sets of equivalent fractions; these points are essential to the general objectives of Chapter 9.

(a) The definition of equivalent fractions partitions the set of all fractions into classes.

(b) Any two fractions in one class are equivalent.
(c) A fraction from one class and a fraction from a different class are not equivalent.
(d) In every class, there is an unlimited number of fractions.
(e) Every fraction is in exactly one class.

Since this chapter is concerned with fractions as symbols and not with fractional numbers, do not write equalities such as $\frac{1}{2}=\frac{2}{4}$. Eventually, we will give meaning to statements of this type, but only when $\frac{1}{2}$ and $\frac{2}{4}$ represent fractional numbers. Clearly, $\frac{1}{2}$ and $\frac{2}{4}$ are different fractions, but they are equivalent in that they represent the same number. We want the children to understand that pairs of fractions, such as $\frac{1}{2}$ and $\frac{2}{4}$, are, in a sense, the same; we use the word "equivalent" to denote this.

Teaching the Chapter

Materials

Colored strips
Crayons
Flannelboard
Objects for set demonstrations (plastic miniatures such as ships, planes, autos, dinosaurs, pencils, checkers, and so on)
Objects to demonstrate fractional parts (apples, candy bars, modeling clay, paper strips, cutouts, and so on)
Overhead projector (if available) and transparencies
Scissors
Tracing paper

Vocabulary

denominator	lower terms
equivalent fractions	lowest terms
fraction	number pair
higher terms	numerator
improper fraction	

Although most of the children's previous experiences with fractions have consisted of work with parts of sets or parts of regions, most children will need further experiences of this type in the initial work for this chapter.

In the early stages, have children work with concrete materials, and then, as they are able to think more abstractly about fractions, reduce the use of these materials. However, if most of your children still need concrete materials you should continue to provide them. Have available materials such as the colored strips; counters for work with sets and fractional parts of sets; construction paper for work with parts of regions; rulers; volume containers which show fractional parts; number lines and charts for work with equivalent fractions.

Most of the words in the vocabulary list are familiar to the children. We minimize use of the expression *improper fraction* in this development because of the misleading connotation of the word *improper.* Expose the children to the term *improper fraction,* but when possible, just use the word *fraction* for the improper fraction.

Lesson Schedule

Plan to cover the material in this chapter in about three to three and one half weeks. Of course, you will want to adjust this time schedule to the needs and abilities of your children. Some children will require more manipulative experiences and closer guidance than others.

Evaluation of Progress

It is not a simple matter to evaluate the children's achievement for a chapter such as this. While several important skills are developed, you must not underrate the importance of a child's acquiring the ability to think of a single quantitative idea for a given set of equivalent fractions. This is one of the key concepts of this chapter, and, certainly, it is difficult to test. Therefore, we suggest that you base a considerable part of your evaluation of the children's achievement on their day-to-day experiences rather than on skills such as changing fractions to lowest terms or constructing sets of equivalent fractions in accordance with a rule that they have learned. If you can successfully build the children's comprehension of a quantitative sameness between pairs of equivalent fractions, they will be ready for the more abstract concepts presented in the next chapter.

Chapter and cumulative reviews are provided on pages 204–207. Use them either for evaluation of progress or strictly for review, as you prefer.

Resources for Active Learning

General Activities

A Cloudburst, Vol. 3, Nos. 3115–3145, Midwest Publications [Experiments using fractions]
Franklin Series: *Patterns and Puzzles,* "Fraction Fun," pp. 64–67, Lyons and Carnahan [Games] (Available from McGraw-Hill Ryerson)
Mathex: Numeration No. 7, "The Meaning of Fractions," pp. 34-40 (pupil pages 27-30), Encyclopaedia Britannica Publications Ltd.
Notes on Mathematics in Primary Schools, "Fractions," pp. 190-193, Cambridge University Press (Available from Macmillan of Canada)
SMSG: *Probability for Intermediate Grades,* "Using Numbers to Express Probability," Lesson 6, Stanford University [Fractions and chance]

Manipulative Devices

Capacity Measures (Childcraft; Educational Teaching Aids)
Cuisenaire Rods (Cuisenaire Co.)
Fraction Chart (Ideal; school supplier)
Fraction Line Set (Ideal; school supplier)
Geoboards (Addison-Wesley)
"Invicta" Math Balance (Math Media; Selective Educational Equipment)
Modulated Cut-outs (Encyclopaedia Britannica Publications Ltd.)
Unifix Fractions Kit (Educational Teaching Aids)

Commercial Games

Fraction Dominoes (Responsive Environments Corp.; Selective Educational Equipment)

PAGES 186–187

Objective

Given fractional parts of a whole, the child will be able to recognize related number pairs and use them in writing fractions.

Preparation

Materials

colored strips

Since the lesson introduces a new topic, proceed immediately with the investigation, explaining how the children may think of partitioning the strips as mentioned below.

Investigation

Before children begin the investigation, point out the unit marks in the illustrations of the purple and brown strips. Help the children realize that each strip may be thought of as partitioned into such units, the 3-strip into three units, the 6-strip into six units, and so on. (Of course, children can verify this by turning the strip to its uncolored side.) Also, point out that, although in the illustration of the parts of strips being covered the parts are not completely covered, the children should think of those parts as being fully covered. This idea will become clear to them as they actually do the covering with their own strips.

Encourage children to work independently on this investigation. You might distribute duplicated copies of the table, or simply have the children copy the headings of the columns from the text. As you move around the room, ask children to read some of their fractions to you as a review. Some of the fractions which the children might use are:

red, purple $\frac{2}{4}$
yellow, brown $\frac{5}{8}$
black, brown $\frac{7}{8}$
white, light green $\frac{1}{3}$
purple, brown $\frac{4}{8}$
dark green, blue $\frac{6}{9}$
blue, orange $\frac{9}{10}$
light green, purple $\frac{3}{4}$
yellow, orange $\frac{5}{10}$
red, light green $\frac{2}{3}$

9 Fractions

Let's explore number pairs and fractions.

Investigating the Ideas

2 of the [illegible] parts are covered.

$\frac{2}{4}$ of the purple strip is covered.

5 of the [illegible] parts are covered.

$\frac{5}{8}$ of the brown strip is covered.

? Can you use your strips and complete at least 10 more lines of a table like this? See Investigation.

Strips	Number Pair		Fraction of the strip covered
	Parts covered	Parts in all	
red purple	2	[illegible]	$\frac{2}{4}$
yellow brown	5	[illegible]	$\frac{5}{8}$

Discussing the Ideas

1. Cover parts of the orange strip with other strips. In this way, how many fractions can you show? $\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \cdots \frac{9}{10}$ (See Discussion)

2. Cover parts of the black strip with other strips. Now what fractions can you name? $\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \cdots \frac{6}{7}$

3. What strips would you use to show each of these?
A $\frac{7}{8}$ $\frac{\text{black}}{\text{brown}}$ **B** $\frac{5}{7}$ $\frac{\text{yellow}}{\text{black}}$ **C** $\frac{3}{5}$ $\frac{\text{lt.green}}{\text{yellow}}$ **D** $\frac{6}{9}$ $\frac{\text{dk.green}}{\text{blue}}$ **E** $\frac{5}{6}$ $\frac{\text{yellow}}{\text{dk.green}}$ **F** $\frac{2}{3}$ $\frac{\text{red}}{\text{lt.green}}$ **G** $\frac{8}{8}$ $\frac{\text{brown}}{\text{brown}}$

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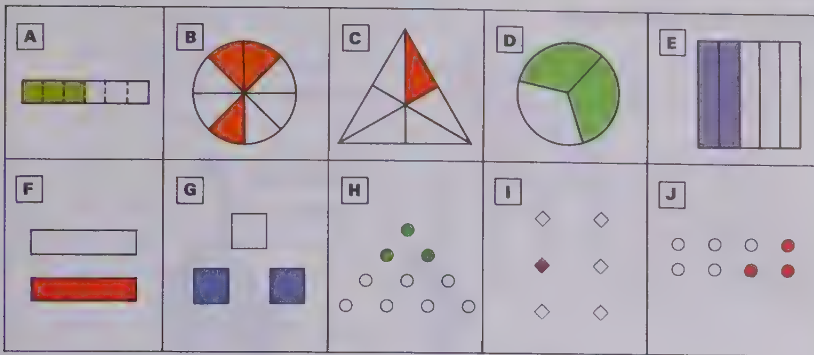
Discussion

Encourage children to move through the discussion of exercise 1 at a lively pace. For example, they might say, the red strip over the orange strip suggests 2 over 10, or $\frac{2}{10}$ (read: two tenths); the yellow strip over the orange strip suggests $\frac{5}{10}$ (if a child calls this one half, accept his answer but elaboration on this point would be untimely). Work through exercise 2 similarly, pointing out how a number pair can be expressed as a fraction.

Have children use their strips to show each part in exercise 3. You might also show a few of these fractions with demonstration materials or with transparencies for the overhead projector, using shapes similar to those at the top of page 187.

Using the Ideas



Each row of the table refers to one of the pictures (A through J) above. Give the missing pictures, numbers, or fractions.

	Picture	Number Pair: Number colored	Number Pair: Total number	Fraction of figure or objects that are colored
1.	A	A 3	6	$\frac{3}{6}$ of the strip is colored.
2.	G	A 2	B 3	$\frac{2}{3}$ of the objects are blue.
3.	B	A 3	B 8	$\frac{3}{8}$ of the circle is colored.
4.	F	A 1	B 2	$\frac{1}{2}$ C of the objects are red.
5.	A D	2	3	$\frac{2}{3}$ B of the figure is colored.
6.	H	A 3	B 10	$\frac{3}{10}$ C of the objects are green.
7.	C	A 1	B 6	$\frac{1}{6}$ C of the triangle is red.
8.	A E	2	B 5	$\frac{2}{5}$ of the figure is colored.
9.	I	A 1	B 6	$\frac{1}{6}$ C of the objects are purple.
10.	J	A 3	B 8	$\frac{3}{8}$ C of the objects are colored.

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Follow-up
Direct children to draw illustrations and write a fraction for each of the following conditions.

4 objects colored, 7 objects in all
3 objects colored, 5 objects in all
8 objects colored, 12 objects in all
6 objects colored, 10 objects in all

This type of activity might also be done by using actual counters and yarn or string. For example:

12 counters, 3 circled with yarn
10 counters, 7 circled with yarn

Resources for Active Learning
Mathematics in Modules, F1, Addison-Wesley.

Workbook, page 59

Using the Exercises
Encourage children to work the exercises on page 187 on their own. However, you may ask some children to name the fraction they think of for a few specific examples. When the children finish, carefully check and discuss each exercise.

Assignments (page 187)
Minimum: 1–10, oral.
Average: 1-5, oral; 6–10.
Maximum: All.

PAGES 188–189

Objectives

Given fractional parts of a set, the child will be able (1) to use a fraction to express this part and (2) to identify the numerator *and* denominator.

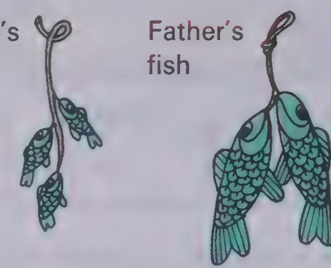
Preparation

To prepare for this lesson, you might review material studied in the previous lesson. For example, exhibit figures or sets of figures with some parts shaded, similar to those on page 187; then have the children give the various number pairs and fractions associated with them.

Let's find out more about fractions.

Discussing the Ideas

1. A Did Billy catch $\frac{3}{5}$ of the number of fish caught? Yes
Explain your answer. See Discussion.



B Did Gary eat $\frac{3}{5}$ of the pizza? No
Explain your answer. See Discussion.

2. Draw five small circles.
Color three of them red.

$$\frac{3}{5} \quad \begin{array}{l}\leftarrow \text{NUMERATOR} \\ \leftarrow \text{DENOMINATOR}\end{array}$$

A What fraction of the circles are red? $\frac{3}{5}$
B What does the numerator of the fraction tell? The number of red circles in the set
C What does the denominator of the fraction tell?
The total number of circles in the set

3. What fraction of the children in your room are girls? Answers will vary from class to class.
What denominator did you use? What numerator?

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Discussion

One of the important points to emphasize in this lesson is that, when we consider fractional parts of a region, we are thinking about the region as divided into parts of the same size; on the other hand, when we consider fractional parts of a set we are merely considering a particular subset which has a given numerical property, but the objects in the set need not all be the same. For example in exercise 1A, each of the five fish is $\frac{1}{5}$ of a set of five items. But Billy's $\frac{3}{5}$ of the fish probably weigh less than the other $\frac{2}{5}$ of the fish. Thus, when we consider a fraction of a set of objects, we are concerned only with the number of objects, not with the size of the objects. However, in exercise 1B, although Gary ate 3 pieces and David ate 2, Gary's share is less than David's. Help the children realize that when we consider a fraction of a *region* or of a *single object*, we are concerned with the *size of each part* and with the *number of parts*. If we speak of an object as being divided into fractional parts, such as fifths, we must have all parts of the same size.

In exercises 2 and 3, stress that the numerator represents the number of *parts* of a region or of a set we have; the denominator represents the *number of equal-size parts* into which the region has been partitioned, or the *number of items* in the set being considered.

Using the Ideas

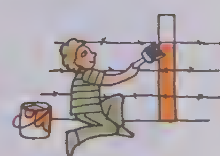
1. A What fraction of the children in this group are girls? $\frac{3}{5}$
   B What is the numerator of this fraction? 3

2. A What fraction of the children in the group wear glasses? $\frac{1}{5}$
   B What is the denominator of this fraction? 5

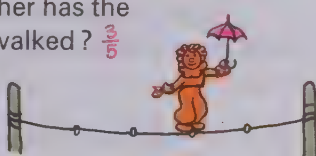
3. A What fraction of the pencils in this set are red? $\frac{4}{7}$

   B What fraction of the pencils in this set are green? $\frac{3}{7}$

4. A What fraction of the post is painted? $\frac{3}{4}$
   B What fraction of the post is not painted? $\frac{1}{4}$
   C Give the numerator and denominator of the fraction in exercise 4B.
   Numerator – 1; denominator – 4



5. What fraction of the way from one post to the other has the tightrope walker walked? $\frac{3}{5}$



6. What fraction of the window must be replaced? $\frac{1}{6}$

7. If 2 more window panes had been broken, what would be the numerator of the fraction in exercise 6? 3

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Using the Exercises
Have the children do the exercises on page 189. When they have finished, allow time for checking papers and discussion.

Note that exercises 1, 2, and 3 have to do with sets of distinct objects; exercises 4 and 5 have to do with whole objects considered individually. Exercise 4 can be thought of as work with measurement or with a region, and exercise 5 can be thought of merely as work with measurement. Exercises 6 and 7, concerning panes of glass, can be considered both as region problems and as set problems. The children can consider this as one window which is divided into sixths (hence, the region concept) or as six separate window panes (hence, the set concept). Of course, the answers will be the same either way inasmuch as each of the six window panes is the same size.

Assignments (page 189)
Minimum: 1, oral; 2–4.
Average: 1–7. Maximum: 1–7.

Mathematics
Note that *numerator* and *denominator* are defined as numbers, whereas our basic definition of *fraction* is as a symbol. Note that the numerator is the number, the whole number, represented by the numeral above the line; and the denominator is the whole number represented by the numeral below the line. It is important that we define *numerator* and *denominator* as numbers because later, in our work with computation involving fractional numbers, we will speak about multiplying numerator times denominator or numerator times numerator, etc.

Follow-up
Provide children with set material or felt cutouts and a flannelboard and let them handle materials to describe fractions you list on the chalkboard such as $\frac{2}{3}$; $\frac{4}{5}$; $\frac{8}{9}$; $\frac{1}{2}$; $\frac{4}{8}$; and so on. Emphasize that within a single object or region, the parts must be the same size. Ask the children to choose subsets of a given set or parts of a region or object to represent fractions that you have written. Assignment cards similar to the following would also provide beneficial experiences with fractional concepts:

> Cut out a set of 24 small triangles.
> Color $\frac{1}{8}$ of them red.
> Color $\frac{1}{3}$ of them blue.
> Color $\frac{1}{2}$ of them yellow.
> Do any remain uncolored? If so, how many?

Duplicator Masters, page 42
Workbook, page 60
Skill Masters, page 42

PAGES 190–191

Objective

Given fractional parts of an object or of a set, the child will be able to give a pair of equivalent fractions to describe them.

Preparation

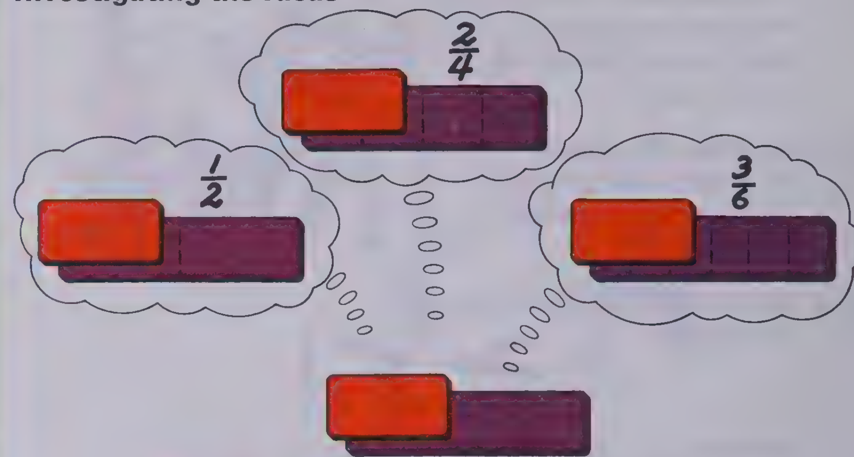
Review with the children the way the strips were used in the first lesson of this chapter. For example, remind the children of the way the orange strip when covered by the purple strip matched the fraction $\frac{4}{10}$. Or, ask them what strips match the fractions $\frac{5}{8}$, $\frac{3}{5}$, $\frac{1}{4}$, and so on. Then explain that in this investigation the purple strip will be thought of not only as partitioned into four units but into other numbers of units as well.

Investigation

Study with the children the investigation illustration of the three ways of thinking of the red and purple strip. Help the children realize that each illustration represents a different way of partitioning the purple strip; and that there are still other ways of thinking about the partitioning. Since the fraction they are working with, $\frac{1}{2}$, is the most familiar to them, they should be able to find many fractions that tell what part of the purple strip is covered. However, some children may need to be stimulated by such questions from you as: "What fraction would you use if you thought of the purple strip as 100? as 50? as 40? as 8? as 10? as 400?" and so on. Note that, since children are asked to *imagine* these different divisions of the purple strip, manipulation of the strip itself is not necessary; the illustration in the text should be sufficient guidance for most children.

● *What are equivalent fractions?*

Investigating the Ideas



Imagine the purple strip divided equally in other ways.

? How many more fractions that tell what part of the purple strip is covered can you find?

$\frac{4}{8}, \frac{5}{10}, \frac{6}{12}$

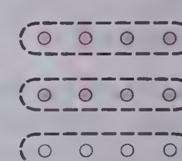
See Investigation.

Discussing the Ideas

A pair of fractions that suggest the same number of objects in a set or the same part of an object are called equivalent fractions

1. Use the idea in the Investigation and give some fractions equivalent to $\frac{1}{3}$. $\frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots$

2. **A** Explain what you might be thinking if you said, "$\frac{8}{12}$ of the dots are pink."
   B Explain what you might be thinking if you said, "$\frac{2}{3}$ of the dots are pink."
   C Explain why $\frac{2}{3}$ is equivalent to $\frac{8}{12}$.
   See Discussion.



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Discussion

One of the principal aims of this lesson is for children to recall that two different fractions may suggest the same number of objects in a set or in the same part of an object and that such fractions are called *equivalent fractions*. Discuss the fractions children found in the investigation, pointing out how, for each, the denominator is twice the numerator.

For discussion exercise 1, suggest that children think of the white strip (1-strip) covering the light green strip (3-strip). Again, they should think of dividing the light green strip into other partitions, such as 6, 9, 12, 24, or 30. For both fractions, $\frac{1}{2}$ and $\frac{1}{3}$, stress that the same amount of the larger strip is covered, no matter which pair of numbers describes it.

In exercise 2, children are asked to consider a set of objects rather than a region that is covered. The illustration suggests that the dots may be thought of as single dots, of which 8 of 12 are colored; or as groups of 4 dots, of which 2 groups of 3 are colored. Stress that, no matter which fraction is used, $\frac{8}{12}$ or $\frac{2}{3}$, the same number of dots are colored.

Using the Ideas

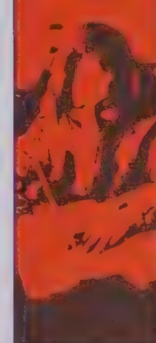
Study examples **A** and **B** in the first two rows of the table.
Then copy the statement and give the missing fractions.

A		$\frac{1}{2}$ is equivalent to $\frac{2}{4}$.	
B		$\frac{2}{3}$ is equivalent to $\frac{6}{9}$.	
1.		$\frac{3}{4}$ ▮ is equivalent to $\frac{6}{8}$.	
2.		$\frac{1}{4}$ is equivalent to ▮. $\frac{3}{12}$	
3.		$\frac{1}{3}$ ▮ is equivalent to ▮. $\frac{2}{6}$	
4.		$\frac{1}{3}$ ▮ is equivalent to ▮. $\frac{5}{15}$	
5.		$\frac{2}{6}$ ▮ is equivalent to ▮. $\frac{4}{12}$	
6.		$\frac{1}{8}$ ▮ is equivalent to ▮. $\frac{2}{16}$	
7.		$\frac{5}{6}$ ▮ is equivalent to ▮. $\frac{10}{12}$	
8.		$\frac{4}{5}$ ▮ is equivalent to ▮. $\frac{12}{15}$	

think

$\frac{1}{2}$ of a brick plus 6 kilograms weighs the same as the brick.
How much does the brick weigh?

More practice, page A-17, Set 32



Using the Exercises

Assign the exercises on page 191 as independent work. When the children finish, check their work and discuss several exercises carefully. Stress with the children that in problems concerning regions, such as example A, although a different partition is indicated, the same amount of region is shaded. Similarly, in problems concerning sets, such as example B, although different groupings are thought of, the same number of items are colored.

All children would benefit from a discussion of the solution to the *Think* problem.

Assignments (page 191)
Minimum: 1–5. Average: 1–8.
Maximum: 1–8.

Mathematics

The standard definition for equivalent fractions follows.

The fraction $\frac{a}{b}$ is equivalent to the fraction $\frac{c}{d}$ if and only if

$$a \times d = b \times c.$$

The fact that this definition expresses the same general idea as does the intuitive definition given in the children's text requires further consideration. Certainly, using either the intuitive definition or the standard definition, we can easily observe that a fraction is equivalent to itself. Consider the fraction $\frac{2}{3}$. Two thirds is equivalent to two thirds because $2 \times 3 = 3 \times 2$. Now, consider the two fractions $\frac{2}{3}$ and $\frac{4}{6}$. If we write the numerator and denominator of $\frac{4}{6}$ in factored form, we are saying that $\frac{2}{3}$ is equivalent to the fraction

$$\frac{2 \times 2}{2 \times 3}.$$

That is, $\frac{4}{6}$ differs from $\frac{2}{3}$ only in having a factor of two in both the numerator and the denominator. Hence, $2 \times (2 \times 3) = 3 \times (2 \times 2)$. Note that the difference between this and $2 \times 3 = 3 \times 2$ is simply a factor of 2 on each side.

It is easy to illustrate the equivalence of $\frac{2}{3}$ and $\frac{4}{6}$ with diagrams such as those in exercises 1F and 1G, page 193, which show the quantitative sameness of these fractions. The importance of the idea of equivalent fractions lies in the fact that it allows us to separate fractions into large classes from which we can abstract the concept of fractional number. For example, we will associate just one number with the two fractions, $\frac{2}{3}$ and $\frac{4}{6}$, and we will also want to observe that there are other fractions for this same number, such as $\frac{6}{9}$, $\frac{8}{12}$, $\frac{10}{15}$.

Duplicator Masters, page 43
Workbook, page 61
Skill Masters, page 43

PAGES 192–193

Objective

Given a partially shaded region, the child will be able to give several members from the set of equivalent fractions suggested by the shaded portion of the region.

Preparation

To prepare for this lesson, review some of the fractions studied previously. For example, display two lists like those below and ask the children to match equivalent fractions.

$\frac{2}{4}$	$\frac{6}{10}$
$\frac{3}{5}$	$\frac{4}{8}$
$\frac{1}{2}$	$\frac{2}{3}$
$\frac{3}{4}$	$\frac{3}{6}$
$\frac{6}{9}$	$\frac{6}{8}$

Or, name a familiar fraction and ask children to respond with an equivalent fraction.

Investigation

In this investigation, children are asked to study a shaded strip being divided into increasingly numerous parts. Point out how the first two fractions listed in the set, $\frac{3}{4}$ and $\frac{6}{8}$, relate to the first two strips illustrated in the chart. Encourage children to work independently on the investigation question, but allow the sharing of ideas among those who find it helpful.

Let's look at sets of equivalent fractions.

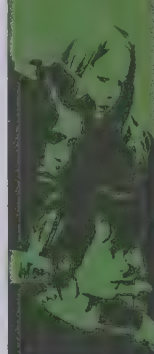
Investigating the Ideas

The same amount of each strip is shaded, but different fractions can be used to describe it.

Imagine that the chart goes on and on without end.

? If you think about the chart, can you give eight more fractions in this set of fractions equivalent to $\frac{3}{4}$? (Any equivalents of $\frac{3}{4}$ are acceptable.)

$\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}, \frac{18}{24}, \frac{21}{28}, \frac{24}{32}, \frac{27}{36}, \frac{30}{40}, \ldots\}$



Discussing the Ideas

This chart suggests a set of equivalent fractions. $\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{5}{15}, \cdots\}$
Tell as much as you can about the chart and the set of fractions.

See Discussion.

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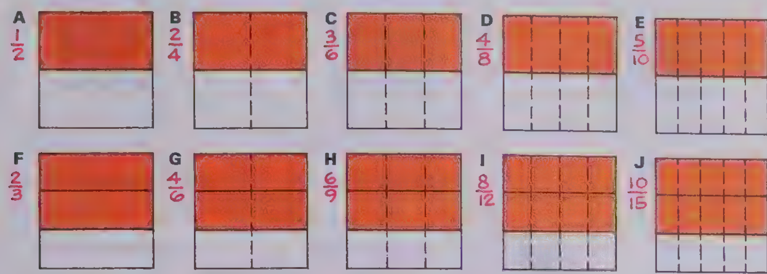
Discussion

Use both the chart in the investigation and in this discussion section to stress the idea that the same amount of each strip is shaded (although different fractional parts are used) and that different, but equivalent, fractions are used to represent this. Point out that the second strip has twice as many shaded parts as the first, and twice as many parts in all; in the third strip, there are three times as many shaded parts and three times as many parts in all; and so on. Referring to the chart at the bottom of the page, which depicts one third and some equivalents of one third, ask children to give the name of the chart, the fractions which match the strips, and several fractions beyond those shown. Note that the appearance of the chart being torn at the bottom is intended to suggest that the chart continues indefinitely. Throughout this discussion, it is essential that you stress that all the fractions for a single chart represent the same amount; that is, each set of equivalent fractions represents a single quantitative idea.

Using the Ideas

1. Give the fraction suggested by each figure.



2. Give the next three fractions in each set.

A $\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \ldots\}$ $\frac{6}{12}, \frac{7}{14}, \frac{8}{16}$ B $\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \ldots\}$ $\frac{12}{18}, \frac{14}{21}, \frac{16}{24}$

3. Study the chart and give the missing fractions.

4. Study the chart and give the missing fractions.

5. Give the next three fractions in each set.

A $\{\frac{4}{5}, \frac{8}{10}, \frac{12}{15}, \frac{16}{20}, \frac{20}{25}, \ldots\}$ $\frac{24}{30}, \frac{28}{35}, \frac{32}{40}$ B $\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \frac{5}{20}, \ldots\}$ $\frac{6}{24}, \frac{7}{28}, \frac{8}{32}$

More practice, page A-18, Set 33

Using the Exercises

Have the children do the exercises on page 193. When they have finished, allow time for discussion and checking papers. Lead the children to see how they build sets of equivalent fractions by thinking about different division marks for a given figure and a given shading of the figure, as in exercises 1A–1E and 1F–1J.

Treat exercises 3 and 4 primarily as discovery exercises. Give the children an opportunity to name the correct fractions without much additional help.

Assignments (page 193)
Minimum: 1–3. Average: 1–4. Maximum: 1–5.

Mathematics

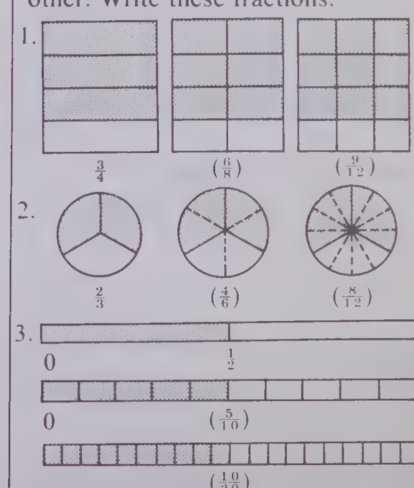
Having established the concept of the equivalence of two fractions, we can consider the set of all fractions as divided into equivalence classes. That is, any two fractions within the same class of fractions are equivalent. Of course, having divided the fractions into these classes, we can see that if two fractions are not equivalent they are in different classes. When we speak of sets of equivalent fractions, we are simply calling attention to the fact that the equivalence relation between pairs of fractions actually divides the set of all fractions into equivalence classes.

It is important also to note that there is an unlimited number of fractions in each of the equivalence classes. For example, in the equivalence class containing $\frac{1}{3}$, there are the fractions $\frac{2}{6}$, $\frac{3}{9}$, $\frac{4}{12}$, $\frac{5}{15}$, and so on endlessly. This idea of dividing the fractions into equivalence classes leads us to the concept of fractional number in Chapter 10.

Follow-up

Provide children with additional practice with sets of equivalent fractions by duplicating worksheets with exercises like these:

In each row, the fractions suggested by the figures are equivalent to each other. Write these fractions.



Resources for Active Learning

Discovery, Section II, Unit 13/5, Encyclopaedia Britannica Educational Corp.

Workbook, page 62

PAGES 194–195

Objective

Given a few fractions from a set of equivalent fractions, the child will be able to give several other fractions in that set.

Preparation

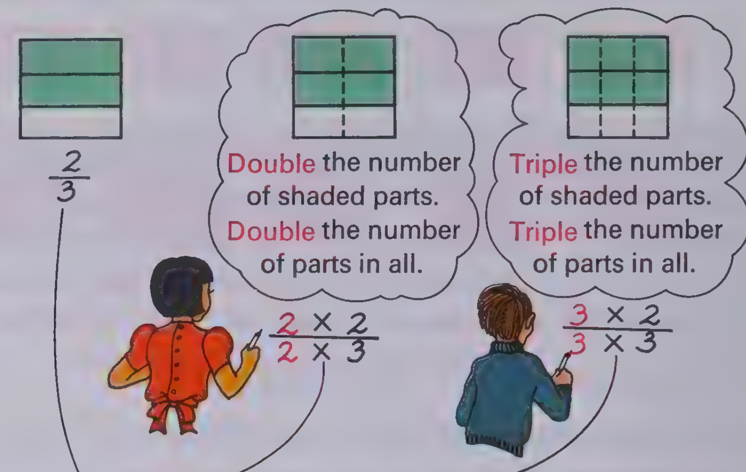
To prepare for this lesson, you might have the children turn back to page 192 and again point out how in both illustrations the second strip shows twice as many shaded parts and twice as many parts in all; and the third strip shows three times as many shaded parts and three times as many parts in all. This idea is the basis for the content of this lesson.

Investigation

For this investigation, you might want to divide the class into small groups and encourage the children to study the given set of equivalent fractions and explain to each other how the fractions relate to the illustrations. Then they should work together to build the set for $\frac{4}{5}$. If you prefer, study the illustrations and their related fractions with the children, pointing out the factor 2 in the numerator and denominator of $\frac{2 \times 2}{2 \times 3}$ and the factor 3 in $\frac{3 \times 2}{3 \times 3}$. You might suggest to some children that they build a chart for $\frac{4}{5}$, using strips like those shown on page 192. However, it is intended in this lesson that children develop the skill of finding sets of equivalent fractions arithmetically, by using factors, rather than by depending on illustrations.

● *How can you build a set of equivalent fractions?*

Investigating the Ideas



$\left\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{\blacksquare}{\blacksquare}, \frac{\blacksquare}{\blacksquare}, \frac{\blacksquare}{\blacksquare}, \frac{\blacksquare}{\blacksquare}, \frac{\blacksquare}{\blacksquare}, \ldots\right\}$ $\frac{8}{12}, \frac{10}{15}, \frac{12}{18}, \frac{14}{21}, \frac{16}{24}$

? Can you use this idea to give 5 more fractions for this set and to build a set of equivalent fractions starting with $\frac{4}{5}$? See Investigation and Discussion.

Discussing the Ideas

1. What set of equivalent fractions does this set of pictures suggest? $\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \cdots\}$

2. Can you explain how to find some more fractions for this set of equivalent fractions? See Discussion.

$\{\frac{3}{10}, \frac{6}{20}, \frac{9}{30}, \ . \ . \ . \}$

Discussion

In this lesson, children would benefit from a demonstration of building a set of equivalent fractions. First, have volunteers explain how they built a set of equivalent fractions for $\frac{4}{5}$. Write these examples on the chalkboard:

$$\frac{1 \times 4}{1 \times 5} \quad \frac{2 \times 4}{2 \times 5} \quad \frac{3 \times 4}{3 \times 5} \quad \frac{4 \times 4}{4 \times 5} \quad \frac{5 \times 4}{5 \times 5}$$

$$\frac{4}{5} \quad \frac{8}{10} \quad \frac{12}{15} \quad \frac{16}{20} \quad \frac{20}{25}$$

Develop other sets similarly, using the discussion exercises as examples. Such examples should help children realize that these sets of equivalent fractions are built by multiplying both the numerator and denominator of a given fraction by the same number, increasing the number by one for each successive equivalent.

Using the Ideas

Give the missing fractions for exercises 1, 2, and 3.

1. $\frac{1\times 1}{1\times 5}$, $\frac{2\times 1}{2\times 5}$, $\frac{3\times 1}{3\times 5}$, . . .

$\frac{1}{5}$ $\frac{2}{10}$ **A** $\frac{3}{15}$ **B** $\frac{4}{20}$ **C** $\frac{5}{25}$ **D** $\frac{6}{30}$ **E** $\frac{7}{35}$ **F** $\frac{8}{40}$

2. $\frac{1\times 3}{1\times 8}$, $\frac{2\times 3}{2\times 8}$, $\frac{3\times 3}{3\times 8}$, . . .

$\frac{3}{8}$ $\frac{6}{16}$ **A** $\frac{9}{24}$ **B** $\frac{12}{32}$ **C** $\frac{15}{40}$ **D** $\frac{18}{48}$ **E** $\frac{21}{56}$ **F** $\frac{24}{64}$

3. $\frac{1\times 3}{1\times 4}$, $\frac{2\times 3}{2\times 4}$, $\frac{3\times 3}{3\times 4}$, . . .

$\frac{3}{4}$ $\frac{6}{8}$ **A** $\frac{9}{12}$ **B** $\frac{12}{16}$ **C** $\frac{15}{20}$ **D** $\frac{18}{24}$ **E** $\frac{21}{28}$ **F** $\frac{24}{32}$

4. Give the next three fractions for each set.

A $\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \ldots\}$ $\frac{15}{20}, \frac{18}{24}, \frac{21}{28}$

B $\{\frac{1}{6}, \frac{2}{12}, \frac{3}{18}, \frac{4}{24}, \ldots\}$ $\frac{5}{30}, \frac{6}{36}, \frac{7}{42}$

C $\{\frac{5}{8}, \frac{10}{16}, \frac{15}{24}, \frac{20}{32}, \ldots\}$ $\frac{25}{40}, \frac{30}{48}, \frac{35}{56}$

D $\{\frac{3}{5}, \frac{6}{10}, \frac{9}{15}, \frac{12}{20}, \ldots\}$ $\frac{15}{25}, \frac{18}{30}, \frac{21}{35}$

E $\{\frac{7}{10}, \frac{14}{20}, \frac{21}{30}, \frac{28}{40}, \ldots\}$ $\frac{35}{50}, \frac{42}{60}, \frac{49}{70}$

F $\{\frac{4}{7}, \frac{8}{14}, \frac{12}{21}, \frac{16}{28}, \ldots\}$ $\frac{20}{35}, \frac{24}{42}, \frac{28}{49}$

5. Give the missing numerators and denominators to form sets of equivalent fractions.

$\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{5\ \mathbf{A}}{15}, \frac{6}{18\ \mathbf{B}}, \frac{7\ \mathbf{C}}{21}, \frac{8\ \mathbf{D}}{24}, \frac{9}{27\ \mathbf{E}}, \ldots\}$

$\{\frac{4}{5}, \frac{8}{10}, \frac{12}{15}, \frac{16}{20}, \frac{20\ \mathbf{F}}{25}, \frac{24}{30\ \mathbf{G}}, \frac{28\ \mathbf{H}}{35}, \frac{32}{40\ \mathbf{I}}, \frac{36\ \mathbf{J}}{45}, \ldots\}$

$\{\frac{1}{8}, \frac{2}{16}, \frac{3}{24}, \frac{4}{32}, \frac{5\ \mathbf{K}}{40}, \frac{6\ \mathbf{L}}{48}, \frac{7}{56\ \mathbf{M}}, \frac{8}{64\ \mathbf{N}}, \frac{9\ \mathbf{O}}{72}, \ldots\}$

$\{\frac{5}{6}, \frac{10}{12}, \frac{15}{18}, \frac{20}{24}, \frac{25}{30\ \mathbf{P}}, \frac{30\ \mathbf{Q}}{36}, \frac{35\ \mathbf{R}}{42}, \frac{40}{48\ \mathbf{S}}, \frac{45\ \mathbf{T}}{54}, \ldots\}$

★ **6.** Give the missing numerator or denominator so that the fraction will belong to the set. See Using the Exercises.

A $\frac{3}{10}$ | $\frac{6}{20}$ | $\frac{9}{30}$ . . . $\frac{300}{1000}$

B $\frac{3}{4}$ | $\frac{6}{8}$ | $\frac{9}{12}$ . . . $\frac{75}{100}$

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Using the Exercises

Have the children do the exercises on page 195 independently. Note that children are not required to show the pattern that they use but simply the new resulting fraction. However, some children might find it helpful to write the patterned development (as shown in exercises 1, 2, and 3) for all exercises. For those who want to try starred exercise 6, explain that the illustrations represent a chart with a middle section missing; the ragged edges represent not only missing fractions but also a set which continues indefinitely.

Assignments (page 195)

Minimum: 1–4. Average: 1–5. Maximum: 1–6.

Mathematics

The method of building a set of equivalent fractions from a given fraction is a direct consequence of the definition of equivalent fractions stated in the mathematics section for pages 190–191 of the student text. This consequence can be stated formally as follows.

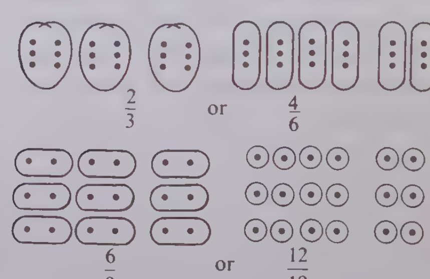
If $\frac{a}{b}$ is any fraction and k is a nonzero whole number, then $\frac{a}{b}$ is equivalent to $\frac{a\times k}{b\times k}$.

To show this, we need only apply the definition of equivalent fractions and observe that $a \times (b \times k) = b \times (a \times k)$ for all whole numbers a, b, and k where neither b nor k is zero.

Note that in applying this theorem we do *not* multiply the fractional number $\frac{a}{b}$ by 1. In fact, at this point, multiplication for fractional numbers has not been discussed or defined. Thus, we simply apply the theorem which is made plausible to the children by the examples in their book. Later, when multiplication of fractional numbers has been defined, we can show that multiplying the numerator and denominator of a fraction by the same nonzero whole number is equivalent to multiplying the fractional number by 1.

Follow-up

Suggest that the children choose a set of counters such as 12, 18, 20, 24, 30, 36 and show them as a set of equivalent fractions. For example, with 18 counters, a grouping such as the following may be described as:



$\frac{2}{3}$ or $\frac{4}{6}$

$\frac{6}{9}$ or $\frac{12}{18}$

Workbook, page 63

PAGES 196–197

Objective

Given a pair of fractions, the child will be able to determine whether or not they are equivalent.

Preparation

It would be suitable to begin immediately with the investigation, but, if you prefer, you might try to help children realize the importance of being able to recognize whether two fractions are equivalent. For this purpose, write the following group of fractions on the chalkboard:

$$\left\{\frac{1}{2}, \frac{2}{4}, \frac{1}{3}, \frac{4}{8}, \frac{5}{10}, \frac{6}{9}, \frac{7}{14}, \frac{8}{12}\right\}$$

Ask the children to identify any fractions which do not belong in the set. Since $\frac{1}{2}$ is familiar to them, most should see that $\frac{1}{3}$, $\frac{6}{9}$, and $\frac{8}{12}$ do not fit in the set for $\frac{1}{2}$. Then point out that it is not always so easy to see whether two fractions are or are not equivalent (for example, $\frac{42}{50}$ and $\frac{7}{8}$, or $\frac{5}{6}$ and $\frac{25}{30}$). The lesson should help them discover a check for equivalent fractions.

Investigation

Encourage the children to work independently on this investigation. However, make sure they understand which numbers they are to multiply. Do not let the children be satisfied that they *cannot* find two fractions whose cross products are different until they have multiplied every fraction at least once by another. The children might not anticipate that the answer to the investigation question is negative; but, in working through this question, they should discover that the products of the numbers in the rings are equal when the two fractions are equivalent. As they discover the pattern, stress that they are working with pairs of *equivalent* fractions.

When are two fractions equivalent?

Investigating the Ideas

Pick **any two** fractions from a set of equivalent fractions.

$$\left\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{5}{15}, \frac{6}{18}, \frac{7}{21}, \frac{8}{24}, \ldots\right\}$$

Find the product of the numbers in each ring.

$$\frac{2}{6} \quad \frac{7}{21}$$

Can you pick two fractions from the set above so that the products of the numbers in the rings are different? No. See Investigation.

Discussing the Ideas

Explain each part of this poster by using another pair of equivalent fractions. See Discussion.

Two fractions (like $\frac{2}{6}$ and $\frac{7}{21}$) **are equivalent when they:**

1 **Show the same part of an object.**

2 **Can be "built" from the same fraction.** $\frac{2 \times 1}{2 \times 3} \rightarrow \frac{2}{6}$ $\quad \frac{7 \times 1}{7 \times 3} \rightarrow \frac{7}{21}$

3 **Have the same cross products.** $\frac{2 \times 1}{2 \times 3} \quad \frac{7 \times 1}{7 \times 3}$ $\quad \frac{2}{6} \quad \frac{7}{21}$

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Discussion

The three points made in the chart shown in the discussion section relate building a set of equivalent fractions to the check for equivalent fractions. In part 2, stress that

$$\frac{2 \times 1}{2 \times 3} \text{ and } \frac{7 \times 1}{7 \times 3}$$

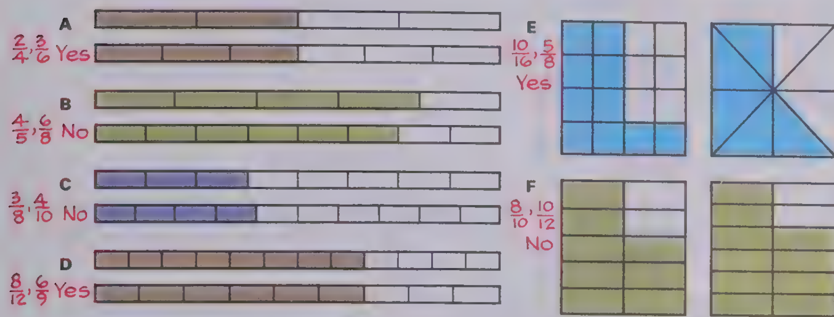
both are built from the fraction $\frac{1}{3}$. Then use part 3 to help children observe that the numbers in the numerator of one fraction and the denominator of the other fraction are the same as the numbers in the denominator of the first fraction and the numerator of the second fraction. In the given example, the numbers inside the blue ring are, in order, 2, 1, 7, and 3 and those inside the red ring are, in order, 2, 3, 7, and 1. Explain that the product of the numbers in the red ring and the product of the numbers in the blue ring are called *cross products*. Use several examples to "prove" that, given two fractions which are equivalent, their cross products are equal. For example, show

$$\frac{2}{4} \quad \frac{5}{10} \quad \text{or} \quad \frac{2 \times 1}{2 \times 2} \quad \frac{5 \times 1}{5 \times 2}$$

and ask: "What is the product of the numbers ringed by the dashed lines? of those ringed by the solid lines? Are the products the same?"

Using the Ideas

1. For each exercise, write the two fractions that are suggested by the shaded parts of the two regions. Then, by looking at the pictures, tell whether or not the two fractions are equivalent.



2. Find the product of the numbers in each ring.

A 15 × 4 = 60, 20 × 3 = 60 B 9 × 28 = 252, 12 × 21 = 252 C 12 × 32 = 384, 16 × 24 = 384 D 18 × 8 = 144, 24 × 6 = 144

A	15 3 / 20 4	60	60
B	9 21 / 12 28	252	252
C	12 24 / 16 32	384	384
D	18 6 / 24 8	144	144

Is each pair of fractions from this set of equivalent fractions? Yes

$$\left\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}, \frac{18}{24}, \frac{21}{28}, \frac{24}{32}, \ldots\right\}$$

3. Which pairs are equivalent?

A $\frac{1}{2}, \frac{5}{10}$ E $\frac{5}{7}, \frac{10}{14}$ I $\frac{9}{9}, \frac{27}{27}$

B $\frac{3}{8}, \frac{9}{24}$ F $\frac{12}{5}, \frac{20}{8}$ J $\frac{6}{2}, \frac{24}{8}$

C $\frac{7}{5}, \frac{3}{2}$ G $\frac{8}{3}, \frac{6}{2}$ K $\frac{6}{15}, \frac{8}{20}$

D $\frac{3}{4}, \frac{2}{5}$ H $\frac{6}{9}, \frac{2}{3}$ L $\frac{8}{6}, \frac{28}{21}$

Circled parts are equivalent.

★ **4.** Find the number for n so that the fractions will be equivalent.

A $\frac{3}{4}, \frac{n}{8}$ 6 C $\frac{5}{10}, \frac{n}{2}$ 1 E $\frac{10}{30}, \frac{n}{3}$ 1

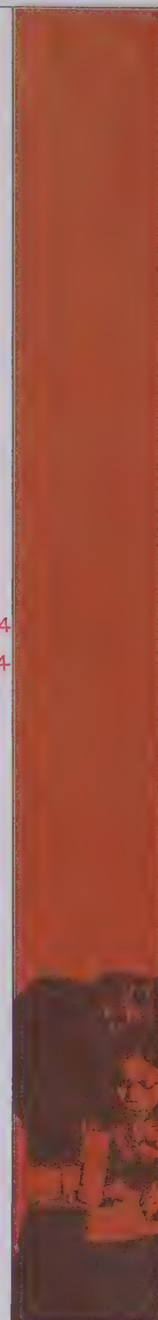
B $\frac{1}{2}, \frac{n}{6}$ 3 D $\frac{12}{8}, \frac{3}{n}$ 2 F $\frac{3}{6}, \frac{n}{42}$ 21

think

The figure below shows a house floor plan. Draw a figure like this. Draw a path to show where you will be if you start inside and use each door exactly once. Outside

Is there an even or odd number of doors? Where would you end if the number of doors were even? Inside

More practice, page A-18, Set 34



Follow-up

Most children will benefit from a worksheet which provides practice in checking for the equivalence of fractions. One such as the following would be suitable:

> Use the cross-product method to decide whether or not the two fractions are equivalent:
>
> 1. $\frac{50}{100}, \frac{1}{2}$ 5. $\frac{20}{30}, \frac{3}{4}$
> 2. $\frac{3}{4}, \frac{75}{100}$ 6. $\frac{7}{8}, \frac{9}{10}$
> 3. $\frac{24}{60}, \frac{18}{40}$ 7. $\frac{30}{20}, \frac{2}{3}$
> 4. $\frac{10}{16}, \frac{25}{40}$ 8. $\frac{5}{6}, \frac{11}{12}$

Duplicator Masters, page 44
Workbook, page 64
Skill Masters, page 44

Using the Exercises

The exercises on page 197 emphasize the fact that, given two equivalent fractions, the cross products are equal and, given that the cross products of two fractions are equal, the two fractions are equivalent. It is important for children to realize that if the two cross products are the same, then the fractions are equivalent; if the two cross products are not the same, the fractions are not equivalent.

If you imagine the figure in the *Think* problem to be a rectangular polygon, it is an application of the same idea as that suggested by the *Think* problem on page 71.

Assignments (page 197)

Minimum: 1, 2, 3A–H.
Average: 1–3. Maximum: 1–4.

PAGES 198–199

Objective

The child will be able to use both proper and improper fractions to express lengths of objects in terms of various units.

Preparation

Materials

colored strips

No specific preparation is necessary for this lesson. However, you might want to review the meaning of the terms *numerator* and *denominator;* familiarity with this language is important in the present discussion of improper fractions.

Investigation

This investigation is similar to that on page 186. However, notice with the children that here they are not *covering* one strip with another; rather they are *comparing* one strip to another. Thus, a longer strip may be *compared* to a shorter strip, whereas it would be awkward, if not meaningless, to ask how much a shorter strip is "covered by" a longer strip. Making comparisons, however, serves as an effective means of introducing improper fractions.

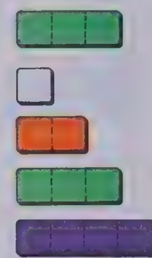
Encourage children to study the investigation section and then try to find and record the fractions that compare the other strips to the yellow strip.

white to yellow: $\frac{1}{5}$
red to yellow: $\frac{2}{5}$
light green to yellow: $\frac{3}{5}$
purple to yellow: $\frac{4}{5}$
yellow to yellow: $\frac{5}{5}$
dark green to yellow: $\frac{6}{5}$
black to yellow: $\frac{7}{5}$
brown to yellow: $\frac{8}{5}$
blue to yellow: $\frac{9}{5}$
orange to yellow: $\frac{10}{5}$

● *Can fractions be used for comparison?*

Investigating the Ideas

Fractions can be used to compare the lengths of strips.
What is the missing fraction below? What are some fractions that compare the other strips to the light green strip?



The unit

$\frac{1}{3}$ as long as the unit

$\frac{2}{3}$ as long as the unit

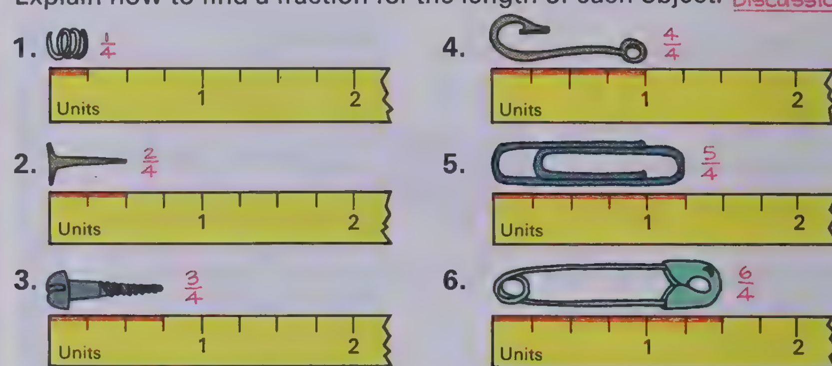
$\frac{3}{3}$ as long as the unit

$\frac{\square}{\square}$ as long as the unit

? Can you use your strips to find fractions that compare each strip to the yellow strip? See Investigation.	Record the fractions that you find.

Discussing the Ideas

The spring is $\frac{1}{4}$ as long as the unit. Its length is $\frac{1}{4}$ unit.
Explain how to find a fraction for the length of each object. See Discussion.



▶ Fractions that have the numerator equal to or greater than the denominator are sometimes called **improper fractions.**

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Discussion

Before discussing the illustrations in the discussion section, have children give some fractions which they wrote in the investigation. Point out in particular how a comparison may be expressed by a fraction in which the numerator is the larger number. This idea is progressively developed in the discussion section. Children must be able to understand why it is possible to have $\frac{5}{4}$ or $\frac{6}{4}$ of a unit.

Work through the illustrated exercises carefully. We are using fractions here to compare lengths; for example, in order to compare the spring with the unit, we use the fraction $\frac{1}{4}$. (We would say that the spring has a length of one-fourth unit.) Stress with the children the sequence of fractions ($1/4$, $2/4$, $3/4$, $4/4$, $5/4$, and $6/4$) which results from comparing the various objects with the unit.

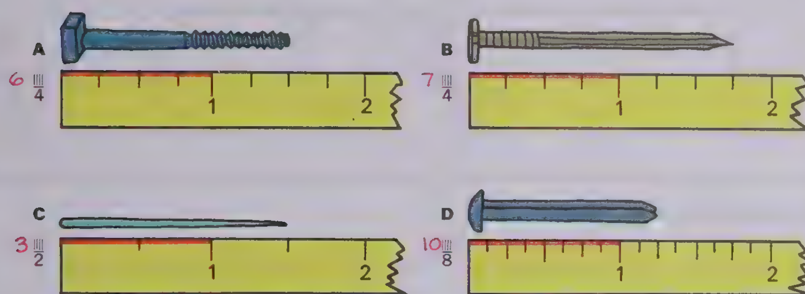
Using the Ideas

1. Give the fraction for each exercise.

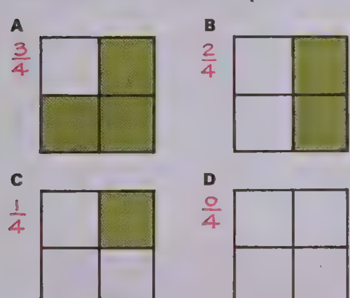
   A The purple strip is ▥ as $\frac{4}{5}$ long as the yellow strip.

   B The blue strip is ▥ as long as $\frac{9}{2}$ the red strip.

2. Give an improper fraction that compares each object with the unit. The denominator of each fraction is given.



3. Give a fraction that compares the number of shaded parts to the total number of parts.



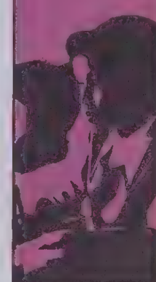
think

Guess which number below is greatest and which is smallest.

1. Number of seconds in an hour. 3600
2. Number of hours in a year. 8760
3. Number of minutes in a week. 10 080

Now find each number to check your guesses.

See Using the Exercises.



Using the Exercises

Assign the exercises on page 199 as independent work. Exercises 1 and 2 are similar to those the children worked with on page 198. Exercise 3 uses the children's understanding of fractions to introduce the idea of zero as a numerator. In exercise 3D, zero of the shaded parts are compared to the total four parts, so zero is the numerator and the fraction is $\frac{0}{4}$.

Most children will be able to guess that part 3 of the *Think* problem gives the greatest number of the three. However, by setting the problem up as follows, they can compare the factors and see without computing that part 3, the number of minutes in a week, is the greatest number of the three.

1. $60 \times 60 = ?$
2. $24 \times 365 = ?$
3. $60 \times 24 \times 7 = ?$
   or $(60 \times 7) \times 24 = ?$
   so $420 \times 24 = ?$

Assignments (page 199)
Minimum: 1–2. Average: 1–3.
Maximum: 1–3.

Workbook, page 65

Objective

The child will be able to give the lowest-terms fraction for a given fraction that is not in lowest terms.

Preparation

To prepare for this lesson, you might review building sets of equivalent fractions. For example, write $\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \ldots$ and ask the children to name the next three or four fractions in the set. Point out that $\frac{2}{8}$ is equivalent to $\frac{2 \times 1}{2 \times 4}$ and $\frac{3}{12}$ is equivalent to $\frac{3 \times 1}{3 \times 4}$, and write the fractions the children give you in a similar manner. However, do not introduce the relationship of lowest-terms fraction to the factors of the numerator and denominator; this will be explored in the investigation. The purpose of your review should be to reinforce the concept of building sets of equivalent fractions; it is intended that children explore on their own how to find lowest-terms fractions.

Investigation

For this investigation, it would be appropriate to have the children work in groups of two or three. Direct them to think of building the set of equivalent fractions for each fraction given. If any children have difficulty with the first three fractions, you might suggest that they think of finding the missing factor. To help individuals with the last three, you might point out that, in the previous fractions, factors common to numerator and denominator were used. A common factor of 4 and 8 is 4, so thinking of $\frac{4 \times ?}{4 \times ?}$ might be a help in finding the lowest-terms fraction. For those who finish quickly, suggest other fractions, such as $\frac{4}{12}, \frac{3}{15}, \frac{20}{25}, \frac{6}{18}$.

● *How do you find lowest-terms fractions?*

Investigating the Ideas

When you build a set of equivalent fractions, you start with a **lowest-terms fraction**.

LOWEST-TERMS FRACTION

$\frac{2 \times 3}{2 \times 4}$ $\frac{3 \times 3}{3 \times 4}$ $\frac{4 \times 3}{4 \times 4}$ $\frac{5 \times 3}{5 \times 4}$

$$\left\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}, \ldots\right\}$$

? Can you find the lowest-terms fraction that was used to "build" each of these fractions?

$\frac{4 \times ?}{4 \times ?}$	$\frac{2 \times ?}{2 \times ?}$	$\frac{5 \times ?}{5 \times ?}$	$\frac{\|\|\|\|\| \times ?}{\|\|\|\|\| \times ?}$	$\frac{\|\|\|\|\| \times ?}{\|\|\|\|\| \times ?}$	$\frac{\|\|\|\|\| \times ?}{\|\|\|\|\| \times ?}$
$\frac{8}{12}$ $\frac{2}{3}$	$\frac{14}{16}$ $\frac{7}{8}$	$\frac{25}{35}$ $\frac{5}{7}$	$\frac{4}{8}$ $\frac{1}{2}$	$\frac{6}{9}$ $\frac{2}{3}$	$\frac{20}{24}$ $\frac{5}{6}$

Discussing the Ideas

1. Explain the method shown here for finding the lowest-terms fraction for $\frac{9}{15}$. See Discussion.

$$\boxed{\frac{9}{15}} \rightarrow \frac{9 \div 3}{15 \div 3} \rightarrow \boxed{\frac{3}{5}}$$

2. Sometimes you may need to use more than one step to find the lowest-terms fraction. Explain each step shown in finding the lowest-terms fraction for $\frac{90}{120}$. See Discussion.

$$\boxed{\frac{90}{120}} \rightarrow \frac{90 \div 10}{120 \div 10} \rightarrow \frac{9}{12} \rightarrow \frac{9 \div 3}{12 \div 3} \rightarrow \boxed{\frac{3}{4}}$$

3. Use some lowest-terms fractions and some fractions not in lowest terms to explain the following.

If the greatest common factor of the numerator and denominator is 1, the fraction is in lowest terms.

Sample answers: $\frac{5}{8} = \frac{1 \times 5}{2 \times 4} = \frac{5}{8}$ (No common factor greater than 1)

$\frac{8}{12} = \frac{2 \times 4}{3 \times 4} = \frac{2}{3}$ (4 is a common factor, so $\frac{8}{12}$ is not in lowest terms.)

Discussion

Have children explain how they found the lowest-terms fractions in the investigation. In particular, discuss the fractions $\frac{4}{8}$, $\frac{6}{9}$, and $\frac{20}{24}$, asking children to explain how they found a common factor and how they used it.

Work through the example in discussion exercise 1 and show this relationship:

$$\frac{9}{15} \rightarrow \frac{3 \times 3}{3 \times 5} \rightarrow \frac{3}{5}$$

$$\frac{9}{15} \rightarrow \frac{9 \div 3}{15 \div 3} \rightarrow \frac{3}{5}$$

Have the children observe that to find the lowest-terms fraction for $\frac{9}{15}$ we divide the numerator and denominator by the greatest common factor of the two numbers. Stress this same concept as you work through exercise 2. Here, note that, although 10 is not the *greatest* common factor of 90 and 120, dividing by it simplifies the fraction to a lower-terms fraction which can be further simplified.

Use other examples to show how factoring can be helpful in finding lowest-terms fractions. For example, write $\frac{8}{12}$, $\frac{9}{15}$, and $\frac{20}{21}$, and ask children to identify the factors and the greatest common factor of each. Then they should try to find equivalent fractions in lower terms until

Using the Ideas

1. Give the lowest-terms fraction for each fraction. The sets of equivalent fractions may help.

A $\frac{12}{30}$ $\frac{2}{5}$ B $\frac{35}{50}$ $\frac{7}{10}$ C $\frac{14}{20}$ $\frac{7}{10}$ D $\frac{7}{21}$ $\frac{1}{3}$

E $\frac{4}{6}$ $\frac{2}{3}$ F $\frac{49}{70}$ $\frac{7}{10}$ G $\frac{9}{24}$ $\frac{3}{8}$ H $\frac{6}{15}$ $\frac{2}{5}$

I $\frac{56}{80}$ $\frac{7}{10}$ J $\frac{8}{24}$ $\frac{1}{3}$ K $\frac{5}{15}$ $\frac{1}{3}$ L $\frac{8}{12}$ $\frac{2}{3}$

M $\frac{42}{60}$ $\frac{7}{10}$ N $\frac{10}{25}$ $\frac{2}{5}$ O $\frac{15}{40}$ $\frac{3}{8}$ P $\frac{6}{18}$ $\frac{1}{3}$

Q $\frac{14}{21}$ $\frac{2}{3}$ R $\frac{21}{30}$ $\frac{7}{10}$ S $\frac{16}{40}$ $\frac{2}{5}$ T $\frac{28}{40}$ $\frac{7}{10}$

$\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{5}{15}, \frac{6}{18}, \frac{7}{21}, \frac{8}{24}, \ldots\}$

$\{\frac{3}{8}, \frac{6}{16}, \frac{9}{24}, \frac{12}{32}, \frac{15}{40}, \frac{18}{48}, \frac{21}{56}, \frac{24}{64}, \ldots\}$

$\{\frac{2}{5}, \frac{4}{10}, \frac{6}{15}, \frac{8}{20}, \frac{10}{25}, \frac{12}{30}, \frac{14}{35}, \frac{16}{40}, \ldots\}$

$\{\frac{7}{10}, \frac{14}{20}, \frac{21}{30}, \frac{28}{40}, \frac{35}{50}, \frac{42}{60}, \frac{49}{70}, \frac{56}{80}, \ldots\}$

$\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \frac{12}{18}, \frac{14}{21}, \frac{16}{24}, \ldots\}$

2. Tell whether or not the fraction is in lowest terms. All the factors of the numerator and denominator are given in red.

A No $\frac{12\{1, 2, 3, 4, 6, 12\}}{15\{1, 3, 5, 15\}}$ D Yes $\frac{11\{1, 11\}}{18\{1, 2, 3, 6, 9, 18\}}$

B Yes $\frac{10\{1, 2, 5, 10\}}{9\{1, 3, 9\}}$ E Yes $\frac{15\{1, 3, 5, 15\}}{16\{1, 2, 4, 8, 16\}}$

C Yes $\frac{6\{1, 2, 3, 6\}}{35\{1, 5, 7, 35\}}$ F No $\frac{24\{1, 2, 3, 4, 6, 8, 12, 24\}}{27\{1, 3, 9, 27\}}$

3. Give the lowest-terms fraction for each of the following.

A $\frac{4}{8}$ $\frac{1}{2}$ B $\frac{9}{30}$ $\frac{3}{10}$ C $\frac{15}{20}$ $\frac{3}{4}$ D $\frac{21}{10}$ $\frac{21}{10}$ E $\frac{6}{15}$ $\frac{2}{5}$ F $\frac{14}{5}$ $\frac{14}{5}$ G $\frac{27}{18}$ $\frac{3}{2}$ H $\frac{25}{35}$ $\frac{5}{7}$

I $\frac{6}{12}$ $\frac{1}{2}$ J $\frac{5}{10}$ $\frac{1}{2}$ K $\frac{1}{10}$ $\frac{1}{10}$ L $\frac{5}{15}$ $\frac{1}{3}$ M $\frac{2}{3}$ $\frac{2}{3}$ N $\frac{4}{6}$ $\frac{2}{3}$ O $\frac{4}{7}$ $\frac{4}{7}$ P $\frac{14}{7}$ $\frac{2}{1}$

Q $\frac{3}{2}$ $\frac{3}{2}$ R $\frac{8}{5}$ $\frac{8}{5}$ S $\frac{6}{4}$ $\frac{3}{2}$ T $\frac{7}{14}$ $\frac{1}{2}$ U $\frac{6}{8}$ $\frac{3}{4}$ V $\frac{25}{30}$ $\frac{5}{6}$ W $\frac{7}{20}$ $\frac{7}{20}$ X $\frac{26}{39}$ $\frac{2}{3}$

4. Find the greatest common factor of the numerator and denominator; then give the lowest-terms fraction.

A $\frac{7}{14}$ 7, $\frac{1}{2}$ B $\frac{15}{10}$ 5, $\frac{3}{2}$ C $\frac{3}{15}$ 3, $\frac{1}{5}$ D $\frac{6}{8}$ 2, $\frac{3}{4}$

E $\frac{20}{15}$ 5, $\frac{4}{3}$ F $\frac{50}{60}$ 10, $\frac{5}{6}$ G $\frac{6}{15}$ 3, $\frac{2}{5}$ H $\frac{75}{100}$ 25, $\frac{3}{4}$

I $\frac{55}{20}$ 5, $\frac{11}{4}$ J $\frac{50}{100}$ 50, $\frac{1}{2}$ K $\frac{20}{50}$ 10, $\frac{2}{5}$ L $\frac{56}{80}$ 8, $\frac{7}{10}$

M $\frac{44}{28}$ 4, $\frac{11}{7}$ N $\frac{5}{20}$ 5, $\frac{1}{4}$ O $\frac{0}{7}$ 7, $\frac{0}{1}$ P $\frac{6}{6}$ 6, $\frac{1}{1}$

think

Find a fraction that is equivalent to $\frac{1}{2}$ and has a denominator that is 5 less than 3 times the numerator. $\frac{5}{10}$

More practice, page A-19, Set 35

the only common factor is one, as in

$$\frac{20}{21} \rightarrow \frac{2 \times 2 \times 5 \times 1}{3 \times 7 \times 1}.$$

Encourage children who volunteer a rule for finding lowest terms when the greatest common factor of the numerator and denominator is not one, but do not require use of a rule by all the children.

Using the Exercises

On page 201, note with the children that in exercise 2 we list the factors for the numerator and the denominator and that all they need do is to check to see whether the greatest common factor is one. Discuss the exercises when the children have finished.

If the children examine the set of fractions equivalent to $\frac{1}{2}$, they can find the solution to the *Think* problem through trial and error.

Assignments (page 201)

Minimum: 1–3 A–H.
Average: 1–3.
Maximum: 1–4.

Follow-up

Worksheets similar to the following will provide children with further practice with lowest-terms fractions.

Give the lowest-terms fraction for each set.

1. $\frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}, \frac{7}{14}$
2. $\frac{2}{12}, \frac{3}{18}, \frac{4}{24}, \frac{5}{30}, \frac{6}{36}, \frac{7}{42}$
3. $\frac{2}{14}, \frac{3}{21}, \frac{4}{28}, \frac{5}{35}, \frac{6}{42}, \frac{7}{49}$
4. $\frac{14}{16}, \frac{21}{24}, \frac{28}{32}, \frac{35}{40}, \frac{42}{48}, \frac{49}{56}$
5. $\frac{2}{20}, \frac{3}{30}, \frac{4}{40}, \frac{5}{50}, \frac{6}{60}, \frac{7}{70}$
6. $\frac{6}{12}, \frac{3}{6}, \frac{8}{16}, \frac{10}{20}, \frac{4}{8}, \frac{9}{18}$
7. $\frac{5}{15}, \frac{9}{27}, \frac{2}{6}, \frac{4}{12}, \frac{10}{30}, \frac{11}{33}$
8. $\frac{7}{70}, \frac{9}{90}, \frac{2}{20}, \frac{10}{100}, \frac{6}{60}, \frac{5}{50}$
9. $\frac{5}{5}, \frac{9}{9}, \frac{10}{10}, \frac{14}{14}, \frac{2}{2}, \frac{6}{6}, \frac{7}{7}$
10. $\frac{0}{8}, \frac{0}{6}, \frac{0}{15}, \frac{0}{18}, \frac{0}{7}, \frac{0}{2}, \frac{0}{9}$

More capable children may be ready to determine that a fraction is in lowest terms by inspection, simply by observing that the numerator and denominator do not share any common factors. They can be expected to handle the following types of exercises:

Circle the lowest-terms fractions.

1. A $\frac{6}{12}$ B $\frac{5}{8}$ . . . 2. A $\frac{6}{4}$ B $\frac{8}{5}$ . . .

Build a set of six equivalent fractions.

1. $\frac{3}{5} \rightarrow \{\frac{3}{5}, \frac{6}{10}, \ldots\}$
2. $\frac{7}{4} \rightarrow \{\frac{7}{4}, \ldots\}$

Duplicator Masters, page 45
Workbook, page 66
Skill Masters, page 45

PAGES 202–203

Objective

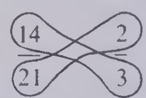
Given word problems, the child will be able to use his understanding of fractions to solve the problems.

Preparation

It would be helpful to review both checking equivalent fractions, and expressing fractions in lowest terms. For example, ask children to give the lowest-terms fraction for each of the following:

$\frac{14}{21}, \frac{8}{16}, \frac{9}{15}, \frac{6}{18}$

Then ask children to use the cross-product method to see whether the fractions are equivalent.



$3 \times 14 = 2 \times 21$

Fraction Short Stories

1 10 children. 8 girls.

A What fraction of the children are girls? $\frac{8}{10}$

B Give a different fraction to tell what part of the children are girls. $\frac{4}{5}$

2 2 home runs. 8 runs in all. Give two fractions to tell what part are home runs. $\frac{2}{8}, \frac{1}{4}$

3 Expos, 7. White Sox, 5. What fraction of the runs did the White Sox score? $\frac{5}{12}$

4 Pie cut into eighths. 6 pieces eaten. Give two fractions for the part that was eaten. $\frac{6}{8}, \frac{3}{4}$

5 10-metre rope.

A Climb up $\frac{6}{10}$ of the way. How far? 6 m

B Climb up $\frac{9}{10}$ of the way. How far? 9 m

C Climb up $\frac{1}{2}$ of the way. How far? 5 m

D Climb up $\frac{2}{5}$ of the way. How far? 4 m

6 3 out of each 4 are blue. 5 groups of 4. How many are blue? 15

7 20 children. $\frac{3}{4}$ of these children are boys. How many are boys? 15

8 2 out of each 5 are missing. 7 groups of 5. How many are missing? 14

9 $\frac{2}{5}$ of the apples are rotten. 35 apples. How many are rotten? 14

10 $\frac{2}{3}$ of the children wear boots. Less than 6 children. How many wear boots? 2

★ **11** 24 blocks to school. Run 2 blocks and then walk 1 block all the way to school. Give two fractions to tell what part of the way is covered by running. $\frac{2}{3}, \frac{16}{24}$

202

Discussion

Have the children do the problems on page 202. When they have finished, allow time for discussion and checking papers. You should take particular care to help the children think about the sets involved rather than about any specific operation. For example, multiplication of rational numbers should not be mentioned as exercise 7 is discussed. Rather, you should elicit from the children the fact that knowing that $\frac{3}{4}$ of the children are boys allows them to conclude that 3 out of each group of 4 are boys. Hence, they should observe that the 20 children can be thought of as 5 groups of 4, and 3 of each group are boys; three times five is 15, so 15 of the 20 children are boys.

Assignments (page 202)

Minimum: 1–5. Average: 1–8.
Maximum: 1–11.

Fraction Puzzlers

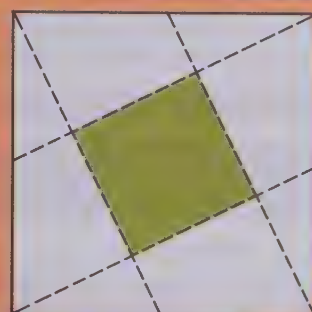
See Solutions, T.E. page 203.

1st Puzzler

This figure is $\frac{1}{4}$ of a square. Can you cut out 4 pieces exactly this shape and size and put them together to form the square?

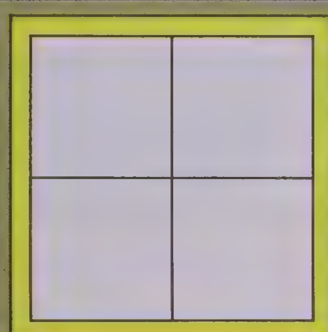
2nd Puzzler

The shaded square region is $\frac{1}{5}$ the size of the large square region. Trace the large square and cut on the dotted lines. Can you arrange the pieces to form 5 small squares?



3rd Puzzler

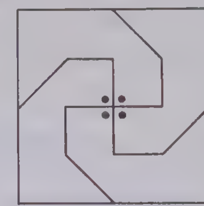
Can you trace this picture and color exactly half of this square window so that the uncolored half is still a perfect square?



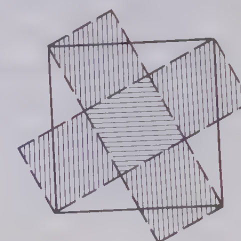
203

Solutions, Fraction Puzzlers, page 203

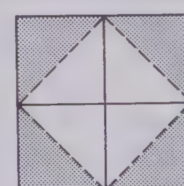
1.



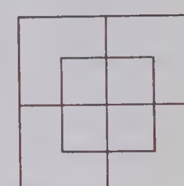
2. The four small triangles can be placed on the four trapezoids to form four additional squares as shown below.



3.



Correct solution



Incorrect solution

Note that, although other solutions are possible, building a square from the midpoints of each of the interior segments does *not* yield a correct solution.

Resources for Active Learning
Developmental Math Cards, H²19, Addison-Wesley.

Workbook, page 67

Using the Exercises

The puzzles on page 203 should be treated with a light touch. For the first puzzle, a hint may be given by explaining that if a red dot is drawn in the lower right-hand corner of each figure, the completed figure will show 4 red dots at the centre. If the third puzzle seems too difficult, suggest that a "stained glass" window could be made by drawing the diagonals and by connecting the points where the horizontal and vertical perpendicular lines hit the edge.

Assignments (page 203)
Minimum: Any one puzzle.
Average: Any two puzzles.
Maximum: All puzzles.

PAGES 204–205

Objective

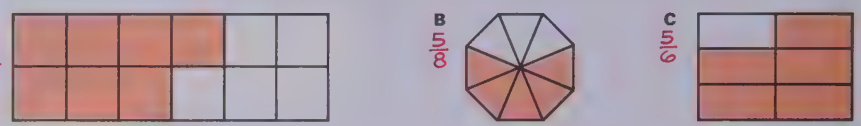
The child will demonstrate his ability to work with the concepts presented in this chapter.

Preparation

Review with the children any of the chapter topics with which they have had particular difficulty. Since the most important topics in the chapter concern equivalent fractions, you should make a special effort to review the concept of equivalent fractions and the building of sets of equivalent fractions. You will want to provide additional practice on finding lowest-terms fractions if your children are weak in this skill.

Reviewing the Ideas

1. Give the fraction suggested by the shaded part of each region.

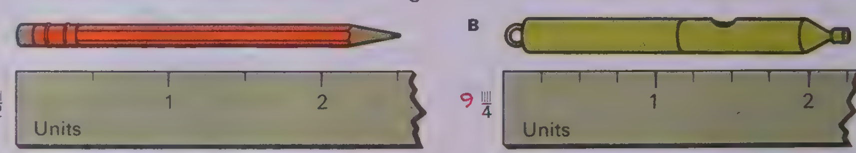


A $\frac{7}{12}$ B $\frac{5}{8}$ C $\frac{5}{6}$

2. Give the fraction suggested by the shaded part of each set.

A $\frac{7}{10}$ B $\frac{7}{9}$ C $\frac{3}{11}$

3. Give an improper fraction that compares each object with the unit. The denominator of each fraction is given.



A $\frac{5}{2}$ B $\frac{9}{4}$

4. Each of these figures suggests two fractions. What are they?

A $\frac{3}{6}, \frac{1}{2}$ B $\frac{6}{8}, \frac{3}{4}$

5. Give the fraction suggested by the shaded divisions of each rod.

A $\frac{2}{3}$
B $\frac{4}{6}$
C $\frac{6}{9}$
D $\frac{8}{12}$
E $\frac{10}{15}$

6. What can you say about the five fractions you wrote for exercise 5?
They are equivalent.

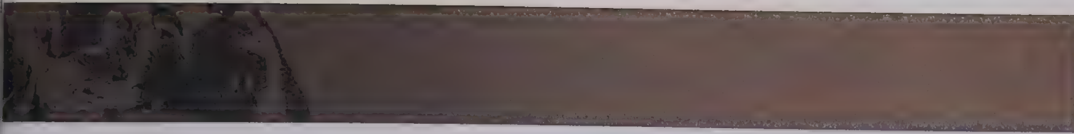
7. If a first fraction is equivalent to a second and the second is equivalent to a third, what can you say about the first and third fractions?
They are equivalent.

204

Discussion

Have the children do the exercises either for your use in evaluation or have them work through the exercises with two or three classmates as a review. Stress particularly those exercises which treat the building of sets of equivalent fractions; this is important background for the development of the concept of fractional numbers explored in the next chapter.

Encourage those children who finish the exercises quickly to try the *Think* problem. You might suggest that they make a chart to show the heights which the ball reaches after each bounce. Children might also use rubber balls or tennis balls and actually measure and record the heights reached by the ball as it bounces after being dropped from various heights.



8. Give the missing fractions for each set.

$\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, A, \frac{10}{15}, \frac{12}{18}, B, \frac{16}{24}, \ldots\}$ A $\frac{8}{12}$ B $\frac{14}{21}$

$\{\frac{1}{8}, C, \frac{3}{24}, \frac{4}{32}, D, \frac{6}{48}, \frac{7}{56}, E, \ldots\}$ C $\frac{2}{16}$ D $\frac{5}{40}$ E $\frac{8}{64}$

9. Tell whether or not the two fractions are equivalent.

A $\frac{5}{10}, \frac{48}{100}$ No B $\frac{15}{20}, \frac{3}{4}$ Yes C $\frac{6}{9}, \frac{8}{10}$ No D $\frac{8}{20}, \frac{12}{25}$ No E $\frac{36}{100}, \frac{18}{50}$ Yes

10. Build a set of equivalent fractions (about 8) for each of the following lowest-terms fractions. See Answers, T.E. page 205.

A $\frac{1}{2}$ B $\frac{3}{4}$ C $\frac{1}{5}$ D $\frac{3}{10}$ E $\frac{7}{2}$ F $\frac{4}{7}$

11. Give the lowest-terms fraction for each of the following fractions. Some are in lowest terms already.

A $\frac{18}{20}$ $\frac{9}{10}$ B $\frac{5}{20}$ $\frac{1}{4}$ C $\frac{4}{10}$ $\frac{2}{5}$ D $\frac{10}{40}$ $\frac{1}{4}$ E $\frac{8}{9}$ $\frac{8}{9}$ F $\frac{12}{30}$ $\frac{2}{5}$ G $\frac{11}{100}$ $\frac{11}{100}$ H $\frac{12}{5}$ $\frac{12}{5}$

I $\frac{4}{7}$ $\frac{4}{7}$ J $\frac{8}{50}$ $\frac{4}{25}$ K $\frac{32}{20}$ $\frac{8}{5}$ L $\frac{16}{60}$ $\frac{4}{15}$ M $\frac{12}{10}$ $\frac{6}{5}$ N $\frac{65}{100}$ $\frac{13}{20}$ O $\frac{35}{50}$ $\frac{7}{10}$ P $\frac{10}{15}$ $\frac{2}{3}$

12. Jim said, "Exactly $\frac{3}{4}$ of the children in my class are girls."

A If there are only 4 children in Jim's class, how many of them are girls? 3

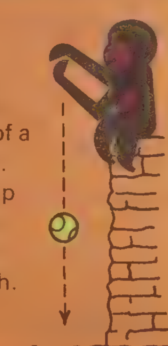
B How many are girls if there are just 8 children in the class? 6

C How many are boys if there are 12 children in the class? 3

D Could there be 10 children in Jim's class? No

E Could there be 10 girls in the class? No

think

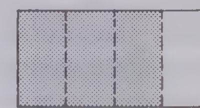


A special rubber ball is dropped from the top of a wall that is 16 m high. Each time it bounces up $\frac{1}{2}$ as high as it fell. The ball is caught when its bounce is 1 metre high. Find the total distance the ball travels. 45 m

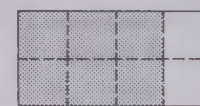
Follow-up

To stress sets of equivalent fractions, you might ask children to build and display charts which show sets of equivalent fractions. For example, they might use a rectangle with dimensions 10 by 15 centimetres and show the set $^3/_4$.

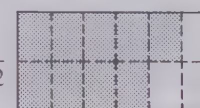
$\frac{3}{4}$



$\frac{6}{8}$



$\frac{9}{12}$



Some might even try to find a way of showing that the amount shaded in the $\frac{9}{12}$ illustration is the same as that in the $\frac{3}{4}$ illustration. (One method is to cut out the $\frac{9}{12}$ and put them on the $\frac{3}{4}$ piece. The ninth twelfth will have to be cut in half to show that $\frac{9}{12}$ can exactly cover $\frac{3}{4}$.

Answers, exercise 10, page 205

A $\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}, \frac{7}{14}, \frac{8}{16}\}$

B $\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}, \frac{18}{24}, \frac{21}{28}, \frac{24}{32}\}$

C $\{\frac{1}{5}, \frac{2}{10}, \frac{3}{15}, \frac{4}{20}, \frac{5}{25}, \frac{6}{30}, \frac{7}{35}, \frac{8}{40}\}$

D $\{\frac{3}{10}, \frac{6}{20}, \frac{9}{30}, \frac{12}{40}, \frac{15}{50}, \frac{18}{60}, \frac{21}{70}, \frac{24}{80}\}$

E $\{\frac{7}{2}, \frac{14}{4}, \frac{21}{6}, \frac{28}{8}, \frac{35}{10}, \frac{42}{12}, \frac{49}{14}, \frac{56}{16}\}$

F $\{\frac{4}{7}, \frac{8}{14}, \frac{12}{21}, \frac{16}{28}, \frac{20}{35}, \frac{24}{42}, \frac{28}{49}, \frac{32}{56}\}$

Workbook, page 68

PAGES 206–207

Objective

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

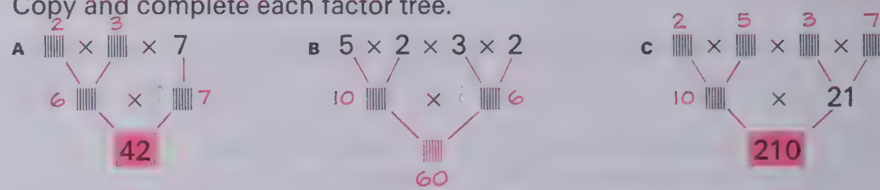
Review with the children any topics with which they have had special difficulty thus far in the text. Since the children have just finished working with material in which number theory is particularly important, you may find it helpful to place special stress on writing a composite number as the product of primes (using factor trees, perhaps), finding the least common multiple of two numbers, and finding the greatest common factor of two numbers.

Keeping in Touch with Computing, Measurement, Geometry, Number theory

1. Find the sums, products, differences, and quotients.

A	B	C	D	E	F
697 + 867 = 1564	642 × 9 = 5778	8)376 = 47	57 × 62 = 3534	19)114 = 6	6003 − 2764 = 3239

2. Copy and complete each factor tree.

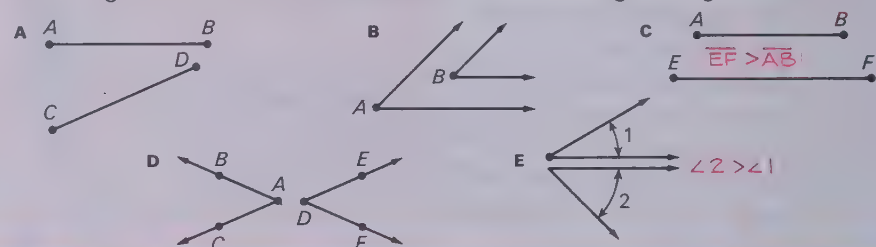


3. Write the numbers at the bottom of each tree in exercise 2 as the product of prime factors. 42=2×3×7 60=5×2×3×2 210=2×5×3×7

4. A List the multiples of 12 (up to 60). {0,12,24,36,48,60}
   B List the multiples of 9 (up to 60). {0,9,18,27,36,45,54}
   C List the common multiples of 12 and 9 (up to 60). 0,36
   D What is the least common multiple of 12 and 9? 36

5. What is the least common multiple of 4 and 6? 12

6. Give the letters of the following exercises in which figures are congruent. In the other exercises, tell which figure is greater. A,B,D



You are invited to explore **ACTIVITY CARD 9** Page 337

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Discussion

The exercises on page 206 should be assigned as independent work. You might have volunteers put parts of exercise 1 on the chalkboard, to help children review work with algorithms.

For page 207, you will probably want children to work together. For example, after children write a story problem for exercise 1, they should share their problem with four or five others and each of the group should then work out each others' problems. Give all of the children a chance to try to solve some of the more imaginative and well expressed problems that have been written. You should stress that a variety of problems are acceptable, although more capable children may find some of them easy. For exercise 1, the obvious problem would be: How long does it take a bicycle to travel 104 kilometres at the rate of 13 kilometres per hour? Another child, observing that the bicycle is almost half the way from City A to City B, might make up this problem: The bicycle has gone 52 kilometres. How much farther does it have to go? Such a problem is acceptable though you should point out to the child that his problem does not make full use of the data given about the speed. (It

Writing Problems

Each picture below suggests a problem. Study the picture carefully; then write and solve your own problem for the picture.

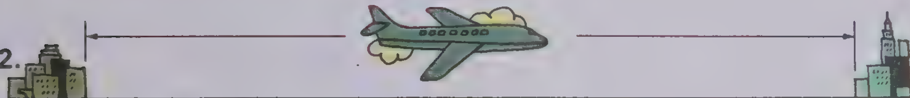
Sample problems are given below. Numerous variations may be acceptable.

1. 

13 kilometres per hour →

City A ← 104 kilometres → City B

How long does the bicycle trip take? 104 ÷ 13 = 8 h

2. 

5 hours

Vancouver ← 3340 kilometres → Toronto

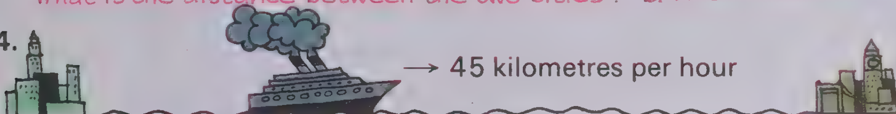
At what speed does the airplane travel? 3340 ÷ 5 = 668 km/h

3. 

← 88 kilometres per hour

City E ← 37 hours → City F

What is the distance between the two cities? 37 × 88 = 3256 km

4. 

→ 45 kilometres per hour

New York ← 5850 kilometres → London

How long does the voyage take? 5850 ÷ 45 = 130 h

5. 

220 kilometres   310 kilometres

City C ← 800 kilometres → City D

How far apart are the car and the bus? 800 − (220 + 310) = 270 km

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is not always necessary to use all the data!)

Follow-up

Suggest that each group of children make up at least 3 problems or task cards related to each story problem in the text. Remind them that the answers should be recorded but kept separate from the problems themselves.

CHAPTER 10 | Fractional Numbers

Pages 208–225

General Objectives

To introduce fractional numbers

To provide a transition from sets of equivalent fractions to fractional numbers

To introduce the fractional-number line

To establish the fact that each whole number is also a fractional number (that the set of whole numbers is a proper subset of the set of fractional numbers)

To introduce equality and inequality for fractional numbers

The opening pages of this chapter are devoted to teaching the following concept: associated with each set of equivalent fractions there is exactly one fractional number and one point on the number line. Since length is a number, the concept of many names for one fractional number is introduced in the first lesson through a study of length, for a chosen unit of measure. The second lesson extends this concept by showing that a set of equivalent fractions not only represents one fractional number but also that this number may be associated with a single point on the number line.

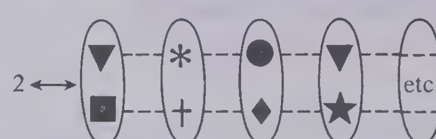
Following this introduction of the fractional-number concept, the idea that fractional numbers may be named by any one of the fractions from the set of equivalent fractions for that fractional number is presented. Next, equality and inequality for fractional numbers is introduced, and material is provided to give practice in working with these ideas. Then, fractional numbers greater than one are introduced, followed by word problems, ratio and scale drawing, and their relationships to fractional numbers.

Note that at no point in this chapter do we state exactly what a fractional number is. Basically, we treat a fractional number as an undefined concept. Thus, for fractional numbers, we associate one number with each set of equivalent fractions, and then use any of these fractions to name the number.

Mathematics

Building the fractional-number concept from fractions (number pairs) is very much like building the cardinal-number concept from sets. In the earlier *Investigating School Mathematics* texts, we began to develop the concept of cardinal numbers by establishing an understanding of equivalent sets. Next, children were led to think about classes of equivalent sets and then about the cardinal number associated with each class. For example, they learned to associate cardinal number two with the class of sets that are equivalent to:

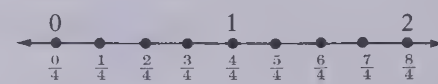
The equivalence of the sets in this class is illustrated as follows:



Just as we abstracted cardinal number two from the class of sets illustrated above, so can we abstract fractional number one half from this class of equivalent fractions:

$$\{\tfrac{1}{2}, \tfrac{2}{4}, \tfrac{3}{6}, \tfrac{4}{8}, \tfrac{5}{10}, \tfrac{6}{12}, \tfrac{7}{14}, \ldots\}.$$

As you saw in the last chapter, we can start with any lowest-terms fraction and imagine an infinite set of equivalent fractions. With each of the equivalent sets, we associate a different fractional number. The number line is very effective for illustrating these ideas. For example, the number-line charts on page 210 of the children's text demonstrate that for each set of equivalent fractions there is one and only one fractional number. As illustrated in the following figure, extending the number line and labelling fourths help give meaning to improper fractions and corresponding fractional numbers.



The number line also helps illustrate that each whole number can be considered a fractional number. That is, the set of whole numbers is a subset of the set of fractional numbers.

Although we think of exactly one number for a given point on the number line (and exactly one point for a given fractional number), we may label the point with any fraction from the set $\{\tfrac{1}{2}, \tfrac{2}{4}, \tfrac{3}{6}, \tfrac{4}{8}, \tfrac{5}{10}, \tfrac{6}{12}, \ldots\}$.

The fact that each fractional number has many different names (any fraction in the set) leads us to consider the concept of equality for fractional numbers. When we write

$$\tfrac{1}{2} = \tfrac{4}{8}$$

we are indicating that $\tfrac{1}{2}$ and $\tfrac{4}{8}$ represent the same fractional number. Obviously, this would indicate also that the fractions are equivalent. Keep in mind, however, that

$$\tfrac{1}{2} = \tfrac{4}{8}$$

is to be considered as a statement about numbers, not about fractions. In general, we say that

$$\frac{a}{b} = \frac{c}{d}$$

means that $\frac{a}{b}$ and $\frac{c}{d}$ name the same fractional number.

Again we observe that:

Two fractions name the same fractional number if and only if they are equivalent.

Therefore,

$\frac{a}{b} = \frac{c}{d}$ if and only if $a \times d = b \times c$.

If we wish to compare two fractional numbers such as $\frac{3}{8}$ and $\frac{5}{8}$, we could easily arrive at $\frac{3}{8} < \frac{5}{8}$ by considering the physical interpretations from which the fractions $\frac{3}{8}$ and $\frac{5}{8}$ arise. Several examples will make the following definition seem plausible.

$\frac{a}{b} > \frac{c}{d}$ if and only if $a \times d > b \times c$.

Consider these examples.

Example 1

$\frac{3}{8} > \frac{5}{14}$ since $3 \times 14 > 8 \times 5$

Example 2

$\frac{9}{15} < \frac{4}{6}$ since $9 \times 6 < 15 \times 4$

Using the number line or some other physical representation, we could make the preceding examples seem obvious. Also, we could find fractions with common denominators for the two fractional numbers. The inequality becomes obvious when we do this. Consider example 2. Since $\frac{9}{15} = \frac{18}{30}$ and $\frac{4}{6} = \frac{20}{30}$, it is clear that $\frac{9}{15} < \frac{4}{6}$ because $\frac{18}{30} < \frac{20}{30}$.

Although it may be inconvenient to use physical objects or the number line to consider the inequality in the preceding example, such interpretations for certain simple pairs of fractions will aid the children. For example, to show that $1/5 < 1/4$, we could use a number line or some physical object. Keep in mind that such demonstrations should facilitate understanding of the definition with respect to the generally accepted notion of "less than" and "greater than."

Teaching the Chapter

Materials

Centimetre ruler
Colored strips
Figures, such as squares and circles, showing fractional parts
Maps, particularly highway and road maps
Number lines, for demonstration of fractional numbers

Vocabulary

fractional number
ratio
scale drawing

Just as with the words number and numeral, correct usage of the terms *fraction* and *fractional number* may sometimes be awkward. The term fraction normally refers to one name, or one way of expressing a fractional number, whereas the term fractional number is the undefined mathematical idea associated with a set of equivalent fractions and corresponding to one point on the number line. The use of the terms in the text should help children develop correct usage themselves, but precision of usage should not be overemphasized. In many cases, it will be more meaningful to children if you say fraction when you really mean fractional number. The development of the concepts involved should lead to intuitive understanding of the terms from their context.

Encourage children to continue using concrete materials or pictorial representations for as long as they want. Children will usually put these materials aside naturally when the abstractions become a part of themselves.

Lesson Schedule

Plan to cover this chapter in about three-and-a-half to four weeks. Naturally, your schedule should be adjusted to the needs and abilities of your children.

Evaluation of Progress

Evaluating children's achievement for this chapter may be more difficult than for any other chapter in the text. We do not expect the children to aquire great efficiency in particular skills in this chapter. The primary subjects of evaluation should be the children's understanding of the general concept of fractional numbers, their relation to whole numbers, and their location on the number line. Of course, associated with this is the concept of equality and inequality for fractional numbers. The best method for such evaluation is daily observation of the children's responses and participation in discussions.

Keep in mind that it is not expected that all children will completely master fractional-number concepts during this exposure. We do, however, expect most children to gain a feeling for a number concept associated with a set of equivalent fractions. Do not be overly concerned if some children have difficulty grasping this idea. These children can continue to study fractional numbers without fully abstracting the idea of equivalent fractions.

Pages 222–223 can be used for evaluation or for review and diagnostic purposes. Pages 224–225 review concepts taught in previous chapters.

Resources for Active Learning

General Activities

Mathex: Numeration No. 7, "Equivalent Fractions," pp. 41–45 (pupil pages 31–41), Encyclopaedia Britannica Publications Ltd.

Manipulative Devices

Cuisenaire Rods (Cuisenaire Co.)
Fraction Chart (Ideal; school supplier)
Fraction Line Set (Ideal; school supplier)
"Invicta" Math Balance (Math Media; Selective Educational Equipment)
Map Measurer and compass (Edmund Scientific; Math Media)
Modulated Cut-outs (Encyclopaedia Britannica Publications Ltd.)
Unifix Fractions Kit (Educational Teaching Aids)

Commercial Games

Fraction Dominoes (Responsive Environments Corp.; Selective Educational Equipment)

PAGES 208–209

Objective

Given one of the colored strips as the unit of length, the child will be able to name several equivalent fractions for the length of another strip.

Preparation

Materials

colored strips

Since the strips are clearly illustrated in the text, most children will not need to use them in the investigation. However, many may want to use them as you discuss the other examples suggested in the discussion section.

Unless you prefer to review building sets of equivalent fractions, have the children begin immediately with the investigation.

Investigation

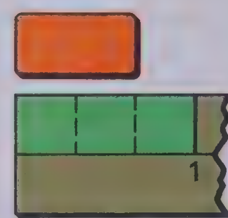
It would be appropriate for children to work through this investigation in groups of two or three, reading and studying the text material together. Stimulate their exchange of ideas with questions such as, "Does the length of the red strip change in each picture?" (No) "When we ask 'How long' an object is, how do we express its length?" (By a number) "What do you notice about the three ways the text shows for expressing the length of the red strip?" (They are all from the same set of equivalent fractions.) "How would you give the length of the red strip if you thought of the unit green strip as having 12 parts?" (8) "15 parts?" (10) Questions such as these should help the child not only to write five other equivalent fractions, but also to attain a firmer grasp of the concept that all of these equivalent fractions represent the same number.

10 Fractional Numbers

Can fractions be used to represent length?

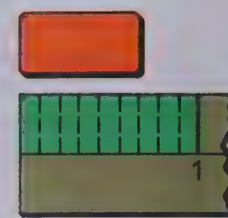
Investigating the Ideas

After a unit is chosen, **the length of the red strip does not change, but different fractions can be used to represent it.**



The length of the red strip is $\frac{2}{3}$

The length of the red strip is $\frac{4}{6}$



The length of the red strip is $\frac{6}{9}$

? Can you write five more fractions that represent the length of the red strip? $\frac{8}{12}, \frac{10}{15}, \frac{12}{18}, \frac{14}{21}, \frac{16}{24}$

See Investigation.

Discussing the Ideas

1. What can you say about the set of fractions that represent the length of the red strip in the Investigation?
   They all belong to one set of equivalent fractions.
2. After a unit is chosen, the length of a strip does not change. That length is a certain number.
   - A If the white strip is the unit, what might we write to represent the number that is the length of the yellow strip? $\frac{5}{1}$
   - B If the purple strip is the unit, what might we write to represent the number that is the length of the dark green strip? $\frac{6}{4}$ or $\frac{3}{2}$

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Discussion

This lesson develops one of the most important ideas of the chapter. Children are familiar with the idea that, once a unit of length is chosen, one number matches that length, no matter how it may be expressed. In this investigation, children use many equivalent fractions to tell how long the red strip is when the green strip is the unit. Thus, they are building the concept that all of these equivalent fractions represent one number idea.

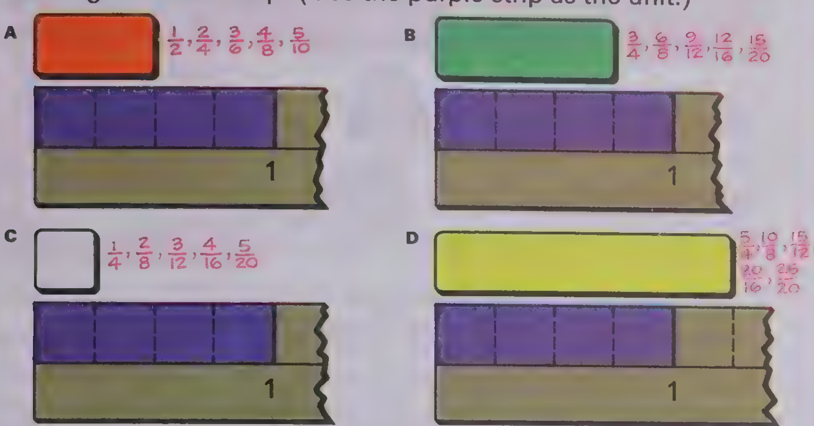
In discussion exercise 1, stress the fact that any fraction which expresses the length of the red strip when the light green strip is the unit must belong to one and only one set of equivalent fractions and that any fraction from that set expresses the same number idea.

In exercise 2, stress the idea that all fractions which belong to one set of equivalent fractions represent one *fractional number*.

Note that exercises 2A and 2B deal with the improper fractions $\{\frac{5}{1}, \frac{10}{2}, \frac{15}{3}, \ldots\}$ and $\{\frac{3}{2}, \frac{6}{4}, \frac{9}{6}, \frac{12}{8}, \ldots\}$. Before discussing these exercises, however, it would be helpful to use other examples of proper fractions, such as "If the yellow strip is the unit, what might we write to express the length of the light green strip?" ($\{\frac{3}{5}, \frac{6}{10}, \frac{9}{15}, \ldots\}$)

Using the Ideas

1. Give five equivalent fractions that can be used to represent the length of each strip. (Use the purple strip as the unit.)



2. Suppose the orange strip is the unit. Name the strip whose length is represented by the fractions in each set.

A $\{\frac{1}{10}, \frac{2}{20}, \frac{3}{30}, \frac{4}{40} \ldots\}$ White
C $\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8} \ldots\}$ Yellow
B $\{\frac{2}{5}, \frac{4}{10}, \frac{6}{15}, \frac{8}{20} \ldots\}$ Purple
D $\{\frac{4}{5}, \frac{8}{10}, \frac{12}{15}, \frac{16}{20} \ldots\}$ Brown

think

$\frac{a}{b}$	1	2	3	4	5	6	7
1	$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$
2	$\frac{2}{1}$	$\frac{2}{2}$	$\frac{2}{3}$	$\frac{2}{4}$	$\frac{2}{5}$	$\frac{2}{6}$	
3	$\frac{3}{1}$	$\frac{3}{2}$	$\frac{3}{3}$	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{3}{6}$	
4	$\frac{4}{1}$	$\frac{4}{2}$	$\frac{4}{3}$	$\frac{4}{4}$	$\frac{4}{5}$		
5	$\frac{5}{1}$	$\frac{5}{2}$	$\frac{5}{3}$				
6	$\frac{6}{1}$	$\frac{6}{2}$					
7	$\frac{7}{1}$						

Think of the table as going on without end.

1. What fraction is in:
   (A) the 3rd row and the 5th column? $\frac{3}{5}$
   (B) the 165th row and the 348th column? $\frac{165}{348}$
2. Do you think every fraction (except those like $\frac{0}{7}$) is somewhere in this table? Yes

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Mathematics

The use of the strips to represent length in this lesson involves a very subtle transition from the idea of fraction to that of fractional number. We take the point of view that length is a number; if we were to say that the fraction $\frac{2}{3}$ compares the red strip with the unit (the green strip), we would be working with the fraction concept. When we say, "The length of the red strip is two thirds," we are moving from the concept of two-out-of-three parts to the concept of a number which indicates how long the red strip is when the green strip is 1. Then the concept of many names for this number is developed: this length of the red strip can be represented by a complete set of equivalent fractions generated from imagining the green strip partitioned in different ways.

Throughout this chapter the term *fractional number* is used repeatedly. Although each fractional number is also a rational number, the term fractional number is preferred at this level. Rational numbers include the set of all fractional numbers as well as the negatives of all the fractional numbers. The term fractional number refers only to the set of positive rational numbers and zero.

Follow-up

Exercises such as the following should reinforce the concepts treated in this lesson.

1. Name six fractions which express the length of the yellow strip when the black strip is the unit.
2. Name six fractions which express the length of the light green strip when the dark green strip is the unit.

Resources for Active Learning

Developmental Math Cards, H-2, Addison-Wesley. [Fractions and weight]

Using the Exercises

Depending on the needs and abilities of the children, assign the exercises on page 209 as independent work or use them as the basis for further discussion.

When you discuss the *Think* problem, help the children see that all fractions are in this table and we can locate any fraction by matching the row number with the numerator and the column number with the denominator. Be sure that all children who attempt this problem can distinguish between rows and columns.

Assignments (page 209)

Minimum: 1, oral.
Average: 1, oral; 2.
Maximum: 1–2.

PAGES 210–211

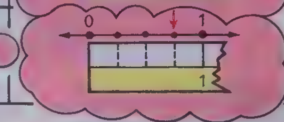
Objective

Given a set of equivalent fractions, the child will be able to think of one fractional number and find the point on the number line for the number.

Preparation

To prepare for this lesson, review the building of sets of equivalent fractions beginning with the lowest-terms fraction of a set. For example, give the children the fraction $\frac{3}{4}$ and ask them to list five more members in the set. Note that a list such as $\{\frac{9}{12}, \frac{30}{40}, \frac{6}{8}, \frac{75}{100}, \frac{300}{400}, \ldots\}$ is as correct as $\{\frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}, \frac{18}{24}, \ldots\}$. Remember, also, to keep the preparation brief and lively.

Discussing the Ideas

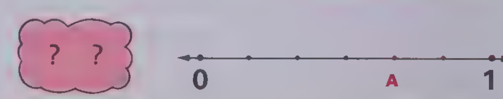
For each set of equivalent fractions	we think of **one** fractional number	and **one** point on the number line.
$\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15} \ldots\}$		
$\{\frac{3}{4}, \frac{6}{6}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20} \ldots\}$		

1. Pat is thinking of a fractional number. Explain how she might have found the point on the number line for her number.
She might have thought of dividing the unit into 3 equal parts and then counting off 2 of the 3 parts.
2. Jim is thinking of a fractional number. Explain how he might have found the point on the number line for his number.
He might have thought of dividing the unit into 4 equal parts and then counting off 3 of the 4 parts.
3. How can you find which set of equivalent fractions goes with point A on the number line? See Discussion.

$\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots\}$

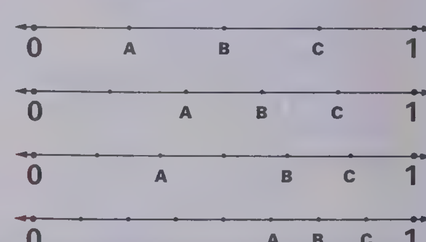
$\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \ldots\}$

$\{\frac{4}{7}, \frac{8}{14}, \frac{12}{21}, \frac{16}{28}, \ldots\}$



For each set of equivalent fractions there is one fractional number. Give the point (A, B, or C) for the given fractional number in questions 4 through 7.

4. $\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \ldots\}$ A
5. $\{\frac{3}{5}, \frac{6}{10}, \frac{9}{15}, \frac{12}{20}, \ldots\}$ B
6. $\{\frac{5}{6}, \frac{10}{12}, \frac{15}{18}, \frac{20}{24}, \ldots\}$ C
7. $\{\frac{7}{8}, \frac{14}{16}, \frac{21}{24}, \frac{28}{32}, \ldots\}$ C



210

Discussion

This discussion section might be thought of as an extension of the previous lesson. The point of emphasis is stated by the headings in the chart: "For each set of equivalent fractions, we think of one fractional number and one point on the number line." As you work through the discussion exercises, help children realize that the partitioning of the unit relates to the set of equivalent fractions. For example, in exercise 3, children should realize that the unit has been partitioned into sixths and that point A corresponds to $\frac{4}{6}$. If some children ask what a fractional number is, point out that it is the number idea we think about for a given set of equivalent fractions. Just as we do not define whole number, neither do we define fractional number.

Using the Ideas

Give the correct point for the fractional number that is indicated by the set of equivalent fractions.

1. 0 A B C 1

$\{\frac{4}{5}, \frac{8}{10}, \frac{12}{15}, \frac{16}{20}, \ldots\}$ C

2. 0 A B C 1

$\{\frac{1}{6}, \frac{2}{12}, \frac{3}{18}, \frac{4}{24}, \ldots\}$ A

3. 0 A B C 1

$\{\frac{2}{7}, \frac{4}{14}, \frac{6}{21}, \frac{8}{28}, \ldots\}$ B

4. 0 A B C 1

$\{\frac{5}{8}, \frac{10}{16}, \frac{15}{24}, \frac{20}{32}, \ldots\}$ C

5. 0 A B C 1

$\{\frac{1}{5}, \frac{2}{10}, \frac{3}{15}, \frac{4}{20}, \ldots\}$ A

6. 0 A B C 1

$\{\frac{7}{10}, \frac{14}{20}, \frac{21}{30}, \frac{28}{40}, \ldots\}$ C

7. 0 A B C 1

$\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots\}$ B

8. 0 A B C 1

$\{\frac{2}{5}, \frac{4}{10}, \frac{6}{15}, \frac{8}{20}, \ldots\}$ B

9. 0 A B C 1

$\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \ldots\}$ C

10. 0 A B C 1

$\{\frac{5}{6}, \frac{10}{12}, \frac{15}{18}, \frac{20}{24}, \ldots\}$ C

think

Jill and June are twins.
Jim and John are twins.
The girls are 2 years older than the boys. The sum of all their ages is 40.
How old are they?
Girls, 11; boys, 9

More practice, page A-19, Set 36

Mathematics

This lesson continues to stress the mathematical development of the key fractional-number concepts. The chart at the top of page 210 illustrates the ideas. Notice that the left column shows a set of equivalent fractions, in which the three dots indicate that we are considering the set of all fractions equivalent to the first, or lowest-terms, fraction in the set. The second column shows a child thinking about exactly one number for each set of equivalent fractions. The third column indicates that for this one number there is exactly one point on the number line. Thus, there are three distinct things to be considered: the set of equivalent fractions, the fractional number (which is undefined), and the point on the number line for that fractional number.

As we have mentioned before in discussing concepts of fractional numbers, it is not unusual to leave undefined the idea of number. It is important that the children understand that there is a certain quantitative sameness associated with all the fractions in a given set of equivalent fractions, but we do not attempt to say exactly what a fractional number is.

We do not need to know what whole numbers are or what fractional numbers are in order to work them. It is necessary to have a general feeling for the ideas, symbols to represent them, an intuition about the quantitative size of numbers (locations on the number line), and some rules (basic principles) to guide us in computing with them.

Using the Exercises

When children finish the exercises on page 211, ask them to explain how they reasoned to arrive at the correct point for the fractional number for each set of equivalent fractions. Although, in most instances, children will have used the lowest-terms fractions to locate the correct point, they should be aware that the *entire set* of equivalent fractions is associated with that point.

Discuss any solutions to the *Think* problem for the benefit of all interested children. If no child solves it, you might display the following table on the chalkboard or overhead projector so that children can organize their guesses.

Guess	5
Boys: B + B	$5 + 5 = 10$
Girls: $(B + 2) \times 2$	$(5 + 2) \times 2$ $7 \times 2 = 14$
Boys + Girls (Sum = 40)	$10 + 14 \neq 40$ (Try again!)

Assignments (page 211)

Minimum: 1–6. Average: 1–8.
Maximum: 1–10.

Resources for Active Learning

Developmental Math Cards, J-19, Addison-Wesley.

Workbook, page 69

PAGES 212–213

Objective

The child will be able to name the fractional number for a point on the number line by giving one of the fractions from the set of equivalent fractions representing the point.

Preparation

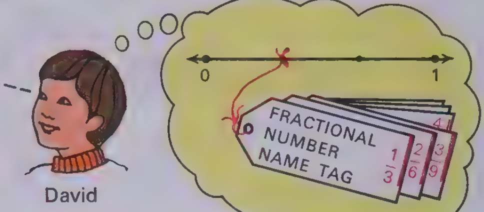
To prepare for this lesson, you might briefly review the check for equivalent fractions. For example, write pairs of fractions on the chalkboard and ask children to tell whether or not they are equivalent and to explain the reasons for their answer.

$\frac{2}{3}, \frac{14}{21}$	Equivalent because $3 \times 14 = 2 \times 21$
$\frac{4}{5}, \frac{15}{25}$	Not equivalent because $5 \times 15 \neq 4 \times 25$
$\frac{1}{3}, \frac{8}{24}$	Equivalent because $3 \times 8 = 1 \times 24$

Discussing the Ideas

You have learned that **for each set of equivalent fractions there is exactly one fractional number and one point on the number line.**

$\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{5}{15}, \ldots\}$



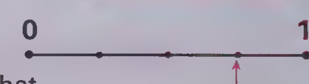
In order to talk and write about fractional numbers, we need names (symbols) for them. We agree that:

> Any fraction from a set of equivalent fractions can be used to name the fractional number for that set.

1. As you can see, **each fractional number has many names.** To show this, give six fractions for this sentence: "▮" names the fractional number that David is thinking about. $\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{5}{15}, \frac{6}{18}$

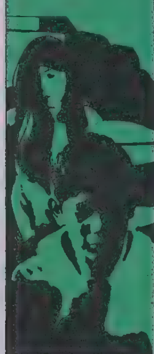
2. To show that two fractions name the same fractional number, we write an equation:
   $\frac{3}{9}$ names the fractional number for this set.
   $\frac{5}{15}$ names the fractional number for this set. → $\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{5}{15}, \ldots\}$

   We write: $\frac{3}{9} = \frac{5}{15}$

   Write some equations using fractions that name the fractional number for the point above the red arrow.
   $\frac{3}{4} = \frac{6}{8}$, $\frac{9}{12} = \frac{12}{16}$, etc.

3. The lowest-terms fraction is the most commonly used name for a fractional number. Choose some fractional numbers and give the lowest-terms fraction that names each.
   Answers will vary.



Discussion

This lesson develops not only the idea that we can use any fraction from a set of equivalent fractions to name the fractional number for that set but also that when we write a statement that uses equality between two fraction symbols, we are stating that these two fractions name the same fractional number. Another point to stress during this discussion is the customary use of the lowest-terms fraction to name a fractional number, as noted in exercise 3.

Also, during this discussion, emphasize for the children that, when two fractions are equivalent to one another, they name the same fractional number. Of course, this is true since we associate with a set of equivalent fractions exactly one fractional number. Hence, if two fractions are equivalent, they belong to the same set of fractions. It is for this reason that the children should see that in order to decide whether or not two fractional numbers are equal, they can use the cross-products check for equivalence of fractions.

Using the Ideas

1. Give fractions to name the fractional numbers for the points over the red arrows. The denominators are indicated.

Fourths → (0 to 1): $\frac{0}{4}$, $\frac{1}{4}$, $\frac{2}{4}$, A $\frac{3}{4}$, $\frac{4}{4}$

Eighths → (0 to 1): $\frac{0}{8}$, B $\frac{1}{8}$, $\frac{2}{8}$, $\frac{3}{8}$, C $\frac{4}{8}$, $\frac{5}{8}$, D $\frac{6}{8}$, $\frac{7}{8}$, E $\frac{8}{8}$

Sixteenths → (0 to 1): F $\frac{0}{16}$, $\frac{1}{16}$, $\frac{2}{16}$, $\frac{3}{16}$, G $\frac{4}{16}$, $\frac{5}{16}$, H $\frac{6}{16}$, $\frac{7}{16}$, $\frac{8}{16}$, I $\frac{9}{16}$, $\frac{10}{16}$, $\frac{11}{16}$, $\frac{12}{16}$, $\frac{13}{16}$, J $\frac{14}{16}$, $\frac{15}{16}$, $\frac{16}{16}$

Thirds → (0 to 2): $\frac{0}{3}$, K $\frac{2}{3}$, $\frac{3}{3}$, $\frac{4}{3}$, $\frac{5}{3}$, L $\frac{6}{3}$

Sixths → (0 to 2): M $\frac{0}{6}$, $\frac{2}{6}$, $\frac{4}{6}$, N $\frac{6}{6}$, O $\frac{8}{6}$, $\frac{10}{6}$, $\frac{12}{6}$

Twelfths → (0 to 2): $\frac{0}{12}$, $\frac{2}{12}$, P $\frac{4}{12}$, $\frac{6}{12}$, Q $\frac{8}{12}$, R $\frac{10}{12}$, $\frac{12}{12}$, $\frac{14}{12}$, S $\frac{16}{12}$, $\frac{18}{12}$, T $\frac{20}{12}$, $\frac{22}{12}$, $\frac{24}{12}$

2. Give three other names for each fractional number.

A $\frac{1}{5}$ B $\frac{3}{4}$ C $\frac{2}{8}$ D $\frac{6}{10}$ E $\frac{3}{7}$

$\frac{2}{10}, \frac{3}{15}, \frac{4}{20}$; $\frac{6}{8}, \frac{9}{12}, \frac{12}{16}$; $\frac{1}{4}, \frac{3}{12}, \frac{4}{16}$; $\frac{3}{5}, \frac{9}{15}, \frac{12}{20}$; $\frac{6}{14}, \frac{9}{21}, \frac{12}{28}$

3. Which fractions name the same fractional number? Write three equations.

$\frac{1}{6}$, $\frac{2}{8}$, $\frac{3}{18}$, $\frac{2}{16}$, $\frac{1}{4}$, $\frac{1}{8}$

$\frac{1}{6} = \frac{3}{18}$; $\frac{2}{8} = \frac{1}{4}$; $\frac{2}{16} = \frac{1}{8}$

4. Give the missing numerator or denominator.

A $\frac{1}{2} = \frac{2}{4}$ C $\frac{2}{3} = \frac{4}{6}$

B $\frac{1}{10} = \frac{10}{100}$ D $\frac{3}{7} = \frac{9}{21}$

think

If today is Tuesday, what day of the week is:

A 10 days from now? Fri

B 50 days from now? Wed

C 100 days from now? Thur

More practice, page A-20, Set 37

Using the Exercises

You might choose to use exercise 1 on page 213 as a basis for discussion. Make sure that the children see the relation between the number lines for the fourths, eighths, and sixteenths and that they realize that the same number is named by three different fractions. For example, point A ($\frac{3}{4}$), point D ($\frac{6}{8}$), and the point for $\frac{12}{16}$ all correspond to one fractional number. Also, note with the children that the whole numbers 0, 1, and 2 may all be thought of as fractional numbers.

Stress the importance of understanding the method of checking equivalent fractions when you check exercises 3 and 4. It is intended that children work these exercises with understanding, not by mechanically following a rule.

The children will need to think about multiples of 7 in order to solve this *Think* problem, since every 7 days it will be Tuesday again. Thus, since $10 = 7 + 3$, answer A is 3 days from Tuesday, or Friday.

$$50 = (7 \times 7) + 1,$$

so answer B is 1 day from Tuesday, or Wednesday, and so on.

Assignments (page 213)

Minimum: 1–2. Average: 1–3. Maximum: 1–4.

Workbook, page 70

PAGES 214–215

Objective

Given two fractional numbers, the child will be able to determine which is greater.

Preparation

Materials

colored strips

It would be appropriate to begin immediately with the investigation, omitting specific preparation. However, if you prefer, you might review the terms *numerator* and *denominator* and the fact that the lengths of strips depend on a chosen unit. It would also be helpful to review the meaning of the inequality signs: > (greater than) and < (less than).

Investigation

Have children work on this investigation in groups of two or three. However, each child should keep his own record of inequalities and strips. To help them do this, duplicate or exhibit for copying a table similar to the following:

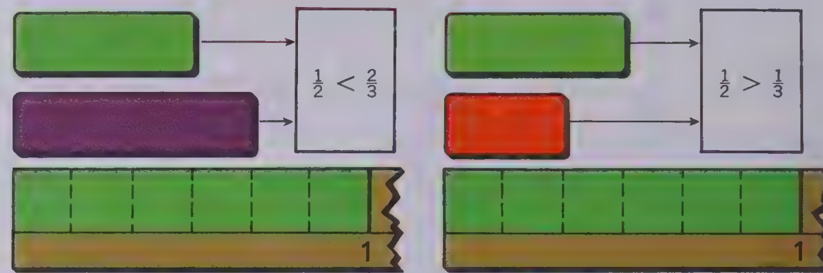
Top Strip	Bottom Strip	Inequality
yellow	red	$\frac{5}{6} > \frac{2}{6}$ or $\frac{5}{6} > \frac{1}{3}$
white	purple	$\frac{1}{6} < \frac{4}{6}$ or $\frac{1}{6} < \frac{2}{3}$
black	light green	$\frac{7}{6} > \frac{3}{6}$ or $\frac{7}{6} > \frac{1}{2}$

It might be necessary to remind children that each strip must be seen in relation to the unit, and its length expressed as a fraction, before arriving at an inequality statement.

Which of two fractional numbers is greater?

Investigating the Ideas

Here are two inequality statements

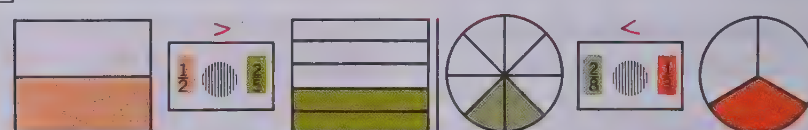


? Can you use the unit shown and other strips to write some more inequality statements? List the strips used. See Investigation.

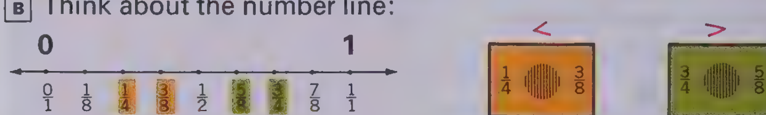
Discussing the Ideas

Here are three more ways to tell which of two fractional numbers is greater. Choose the correct symbol (>, <) and explain how you made your choice. See Discussion.

A Think about parts of a region:



B Think about the number line:



C Think about fractions with the same denominator.

$\frac{3}{5}$ ◯ $\frac{1}{2}$ because $\frac{6}{10} > \frac{5}{10}$. >

$\frac{1}{4}$ ◯ $\frac{1}{3}$ because $\frac{3}{12} < \frac{4}{12}$. <

214

Discussion

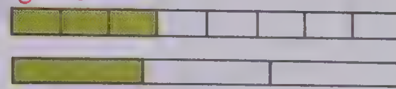
Use the table suggested in the investigation section as a basis for discussing some of the inequalities the children found. Make sure children are reading the inequality signs correctly. For each of the exercises, ask children to give reasons for their choice. For example, when we consider the regions in part A, we see that more of the first region is shaded than of the second, so we think $\frac{1}{2}$ must be greater than $\frac{2}{5}$. In part B, we can compare the distances of the points from zero: since $\frac{1}{4}$ is not as far from zero as $\frac{3}{8}$, then $\frac{1}{4}$ must be less than $\frac{3}{8}$.

Finally, observe with the children that in part C, if the fractions have the same denominators, they can simply examine the numerators and determine the correct inequality. For example, by inspection they know that $\frac{6}{10}$ is greater than $\frac{5}{10}$; hence, they can say that $\frac{3}{5}$ is greater than $\frac{1}{2}$ because $\frac{3}{5}$ is the same fractional number as $\frac{6}{10}$, and $\frac{1}{2}$ is the same fractional number as $\frac{5}{10}$.

1. Write an inequality for each exercise.

A $\frac{3}{4} > \frac{5}{8}$

B $\frac{3}{8} > \frac{1}{3}$



2. Use the number line to give the correct sign (> or <) for each ●.

$0 \quad \frac{1}{16} \quad \frac{1}{8} \quad \frac{3}{16} \quad \frac{1}{4} \quad \frac{5}{16} \quad \frac{3}{8} \quad \frac{7}{16} \quad \frac{1}{2} \quad \frac{9}{16} \quad \frac{5}{8} \quad \frac{11}{16} \quad \frac{3}{4} \quad \frac{13}{16} \quad \frac{7}{8} \quad \frac{15}{16} \quad 1$

A $\frac{5}{16}$ ● $\frac{3}{8}$ <
B $\frac{9}{16}$ ● $\frac{1}{2}$ >
C $\frac{5}{8}$ ● $\frac{7}{16}$ >
D $\frac{1}{2}$ ● $\frac{3}{16}$ >
E $\frac{15}{16}$ ● $\frac{7}{8}$ >

3. Give the numbers for ***a*** and ***b***. Then give the correct sign (>, <) for each ●.

A $\frac{2}{3} = \frac{a}{6}$ 4, $\frac{1}{2} = \frac{b}{6}$ 3 → $\frac{2}{3}$ ● $\frac{1}{2}$ >
B $\frac{3}{4} = \frac{a}{20}$ 15, $\frac{4}{5} = \frac{b}{20}$ 16 → $\frac{3}{4}$ ● $\frac{4}{5}$ <
C $\frac{1}{6} = \frac{a}{30}$ 5, $\frac{1}{5} = \frac{b}{30}$ 6 → $\frac{1}{6}$ ● $\frac{1}{5}$ <
D $\frac{4}{7} = \frac{a}{21}$ 12, $\frac{2}{3} = \frac{b}{21}$ 14 → $\frac{4}{7}$ ● $\frac{2}{3}$ <
E $\frac{5}{8} = \frac{a}{56}$ 35, $\frac{4}{7} = \frac{b}{56}$ 32 → $\frac{5}{8}$ ● $\frac{4}{7}$ >
F $\frac{2}{5} = \frac{a}{45}$ 18, $\frac{4}{9} = \frac{b}{45}$ 20 → $\frac{2}{5}$ ● $\frac{4}{9}$ <
G $\frac{1}{2} = \frac{a}{6}$ 3, $\frac{1}{3} = \frac{b}{6}$ 2 → $\frac{1}{2}$ ● $\frac{1}{3}$ >
H $\frac{2}{3} = \frac{a}{12}$ 8, $\frac{3}{4} = \frac{b}{12}$ 9 → $\frac{2}{3}$ ● $\frac{3}{4}$ <
I $\frac{5}{6} = \frac{a}{12}$ 10, $\frac{11}{12} = \frac{b}{12}$ 11 → $\frac{5}{6}$ ● $\frac{11}{12}$ <
J $\frac{3}{8} = \frac{a}{8}$ 3, $\frac{1}{2} = \frac{b}{8}$ 4 → $\frac{3}{8}$ ● $\frac{1}{2}$ <
K $\frac{6}{7} = \frac{a}{21}$ 18, $\frac{2}{3} = \frac{b}{21}$ 14 → $\frac{6}{7}$ ● $\frac{2}{3}$ >
L $\frac{7}{8} = \frac{a}{24}$ 21, $\frac{5}{6} = \frac{b}{24}$ 20 → $\frac{7}{8}$ ● $\frac{5}{6}$ >

4. Give the correct sign (>, =, or <) for each ●.

A $\frac{5}{6}$ ● $\frac{11}{12}$ <
B $\frac{2}{3}$ ● $\frac{3}{6}$ >
C $\frac{1}{12}$ ● $\frac{0}{12}$ >
D $\frac{1}{2}$ ● $\frac{6}{12}$ =
E $\frac{7}{12}$ ● $\frac{1}{2}$ >
F $\frac{7}{12}$ ● $\frac{2}{3}$ <
G $\frac{5}{12}$ ● $\frac{1}{2}$ <
H $\frac{3}{4}$ ● $\frac{5}{6}$ <
I $\frac{1}{2}$ ● $\frac{1}{3}$ >
J $\frac{1}{4}$ ● $\frac{1}{6}$ >
K $\frac{1}{4}$ ● $\frac{1}{3}$ <
L $\frac{1}{3}$ ● $\frac{4}{12}$ =

think

$\frac{5}{8} \quad \frac{21}{32} \quad \frac{22}{32} \quad \frac{23}{32} \quad \frac{3}{4}$

Find three fractions with denominator 32 so that the numbers they name are between $\frac{5}{8}$ and $\frac{3}{4}$.

More practice, page A-20, Set 38

Using the Exercises

Have the children do the exercises on page 215 on their own or with one or two classmates. Make sure they realize that the inequalities presented in exercise 2 may all be worked by referring to the number line. The inequalities in exercise 3 should be thought about in relation to fractions with the same denominator. The children must realize that by selecting fractions having the same denominator for the two numbers involved, they can compare the two fractional numbers simply by comparing the numerators and without using a physical model.

The children will need to convert the fractions $\frac{5}{8}$ and $\frac{3}{4}$ to thirty-seconds in order to solve the *Think* problem. After they make the conversions, the solution is relatively simple inasmuch as there are only three whole-number numerators between $\frac{20}{32}$ and $\frac{24}{32}$.

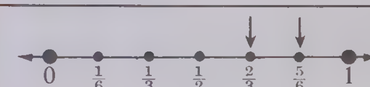
Assignments (page 215)
Minimum: 1–2, 3A–H.
Average: 1–3, 4A–F.
Maximum: 1–4.

Mathematics

In this lesson, we present informally, through the use of pictures of various objects, the general concept of inequalities for fractional numbers. Such a concept involves a precise definition (which is not presented to the children at this time). Fractional numbers represented by fractions with equal denominators are easily compared:

$$\frac{a}{b} > \frac{c}{b} \text{ if and only if } a > c.$$

If the denominators are unequal, we can choose fractions having equal denominators that are equivalent to our given fractions. We illustrate the use of the definition by comparing fractional numbers with unequal denominators in the following diagram.



$\frac{2}{3} < \frac{5}{6}$, and $\frac{5}{6} > \frac{2}{3}$

Comparing Rational Numbers on the Number Line

$\frac{3}{4} = \frac{15}{20}$, and $\frac{4}{5} = \frac{16}{20}$

Since $\frac{15}{20} < \frac{16}{20}$, then $\frac{3}{4} < \frac{4}{5}$.

Comparing Rational Numbers by Finding a Common Denominator

Notice that we can compare $\frac{7}{5}$ and $\frac{17}{12}$, for example, by using the facts that

$$\frac{7}{5} = \frac{7 \times 12}{5 \times 12} \text{ and } \frac{17}{12} = \frac{17 \times 5}{12 \times 5}.$$

Since $7 \times 12 < 17 \times 5$, we see that $\frac{7}{5} < \frac{17}{12}$. In general, we note that $\frac{a}{b} > \frac{c}{d}$ if and only if $a \times d > b \times c$ (where $\frac{a}{b}$ and $\frac{c}{d}$ are positive fractional numbers).

Resources for Active Learning

Mathematics in Modules, F1, Addison-Wesley.

Duplicator Masters, page 46
Workbook, page 71
Skill Masters, page 46

PAGES 216–217

Objective

Given short story problems, the child will be able to solve them by applying his understanding of inequalities of fractional numbers.

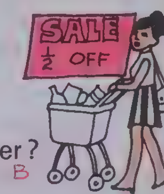
Preparation

For those children who experienced difficulty with inequalities in the last lesson, you might review basic fractional-number concepts, such as how to build sets of equivalent fractions, how fractions from one set of equivalent fractions name the same fractional number, and how fractional numbers may be compared as presented in the previous lesson.

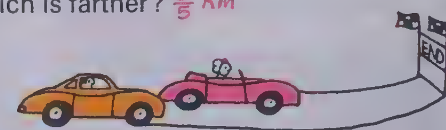
Fractional-Number Short Stories

1 Jim ate $\frac{1}{4}$ of the pie.
Joe ate $\frac{1}{5}$ of the pie.
Who ate more pie? Jim

2 Sale.
Store A: $\frac{1}{3}$ off.
Store B: $\frac{1}{2}$ off.
Which sale is better? B



3 $\frac{3}{5}$ kilometre. $\frac{4}{10}$ kilometre.
Which is farther? $\frac{3}{5}$ km

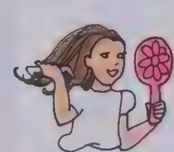


4 Halfway around the track.
Two thirds of the way around.
Which is less? Halfway around

5 Car A: finished $\frac{3}{4}$ of the race.
Car B: finished $\frac{5}{8}$ of the race.
Which went farther? Car A

6 Recipe 1: $\frac{7}{10}$ litre of milk.
Recipe 2: $\frac{3}{4}$ litre of milk.
Which recipe calls for more milk? 2

8 One board: 1 m 60 cm.
Other board: $\frac{15}{10}$ m.
Which board is longer?
1m 60cm



7 Jane took $\frac{1}{6}$ of an hour
to comb her hair.
Sue took 11 minutes.
Who took longer? Sue

10 Orchard:
$\frac{1}{5}$ apple trees, $\frac{6}{20}$ peach trees,
$\frac{4}{10}$ pear trees, $\frac{2}{20}$ apricot trees.
A Which kind of tree does the orchard have most of? Pear
B Which kind of tree does the orchard have fewest of? Apricot

9 T-bone steak: 480 grams.
Rib steak: $\frac{4}{10}$ kilogram.
Which weighs more?
T-bone

11 Number line.
Point A: $\frac{16}{40}$. Point B: $\frac{36}{80}$.
Which point is to the left of the other? A is left of B.

12 336-page book.
Pat read 150 pages.
Joe read $\frac{1}{2}$ of the book.
Who read more? Joe

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Discussion

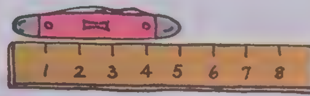
It is intended that children approach these problems in an informal, intuitive manner. It is not expected that any particular rules be formulated or applied. You might have children work in small groups to try to solve only four or five of the problems on each page. They might enjoy drawing pictures to represent the situation described in the problem. They might want to use a number line to associate each fractional number given in a problem with a point on the line and then compare the positions of the points. A few children may use the arithmetic skill of changing the given fractions to fractions with common denominators and then compare the numerators. Encourage the class to think through each problem in terms of the distinct, imaginable situation that it describes.

When the children have finished, ask them to explain to each other how they thought about each problem they worked on. Accept a variety of approaches to any given problem, so long as correct concepts are maintained. For example, in exercise 2, on page 217, the children have a choice of changing $\frac{20}{60}$ to $\frac{1}{3}$ or of noticing that $\frac{1}{4}$ hour is 15 minutes or $\frac{15}{60}$, so they can compare $\frac{1}{4}$ and $\frac{1}{3}$, or $\frac{20}{60}$ and $\frac{15}{60}$.

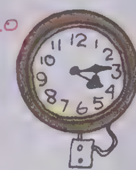
Comparing Fractional Numbers

For each exercise, give the missing number and answer the questions.

1. The length of the knife is $\frac{5}{9}$ of the ruler. The length of a second knife is $\frac{3}{5}$ of the ruler. Which knife is longer? Second Knife

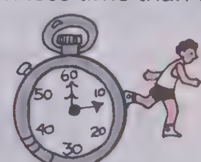


2. One trip took 20 minutes, or $\frac{\square}{60}$ 20 hour. A second trip took $\frac{1}{4}$ hour. Which trip took more time? 20-minute trip



3. The sack of candy on the scale weighs $\frac{5}{10}$ kg. Is this 5 more or less than $\frac{1}{4}$ kg? More

4. Joe ran 50 metres in $\frac{1}{6}$ of a minute. Tom ran 50 metres in 12 seconds, or 12 $\frac{\square}{60}$ of a minute. Which boy ran 50 metres in less time than the other? Joe



5. This jug is $\frac{\square}{4}$ 3 full. Would you have to add or pour out liquid to make the jug $\frac{5}{8}$ full? Pour out

6. The length of the shoe is $\square$ 13 cm. Is the length of the shoe more or less than $\frac{1}{10}$ metre? More

centimetres

7. A camel weighs $\frac{1}{2}$ tonne. A 400 moose weighs 400 kg, or $\frac{\square}{1000}$ tonne. Which animal weighs more? Camel

More practice, page A-21, Set 39

Although some of these problems may appear to be multiplication problems, they should be treated intuitively; any reference to multiplication of fractional numbers would be inappropriate at this time.

Follow-up

Problems such as those on these two pages should help children develop a creative approach to problem-solving situations. You might want to try other methods of developing such creativity. If so, present the class with a variety of questions which could stimulate a class project; for example, you might ask: "If we use as our unit the number of years between the time of the Founding of Quebec and now, what fraction of time passed before the British North America Act was signed? before the Riel Rebellion? before there were 10 provinces?" Such questions might stimulate children to make a time line, chart dates and events on it, and then show appropriate partitions and match dates with fractional numbers.

Workbook, page 72

Assignments (page 216)

Minimum: Odd-numbered problems.

Average: 1–10. Maximum: 1–12.

Assignments (page 217)

Minimum: 1–4. Average: 1–6.

Maximum: 1–7.

PAGES 218–219

Objective

Given a pair of numbers, the child will be able to write a ratio to compare one of the numbers to the other.

Preparation

Materials

colored strips

To prepare for this lesson, ask children to give the lengths of the red, the light green, and the purple strips when the white strip is the unit. These lengths, of course, will be the familiar whole-number lengths of these strips: red—$\frac{2}{1}$ or 2; light green—$\frac{3}{1}$ or 3; purple—$\frac{4}{1}$ or 4.

Investigation

Although children are working with comparisons much like those previously studied, the purpose of this lesson is to introduce them to the language of ratio.

Read through the investigation with the children. Since the word *ratio* will be new to most children you will need to emphasize that ratio is a special term used in mathematics to compare two numbers.

Allow sufficient time for the children to find different pairs of strips that suggest a ratio of 1 to 2. You may wish to list the pairs of strips they find on the chalkboard for reference. There are five different pairs of strips: white to red, red to purple, light green to dark green, purple to brown, and yellow to orange.

What is the meaning of ratio?

Investigating the Ideas

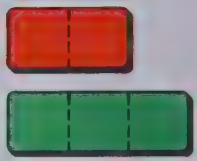
Since the purple strip is $\frac{1}{2}$ as long as the brown strip, we say the **ratio** of the length of the purple strip to the length of the brown strip is **1 to 2.**

Since the purple strip is 4 units and the brown strip is 8 units, we also say the ratio is **4 to 8.**

? How many other pairs of strips that have a ratio of 1 to 2 can you find? See Investigation.	Record your findings by giving the ratio suggested by the unit marks on the strips.

Discussing the Ideas

1. The ratio **4 to 8** is written as 4:8.
   A How can you think about the strips to show a ratio of 8 to 4? Compare brown strip to purple.
   B How would you write the ratio **8 to 4**? 8:4
2. The red strip is $\frac{2}{3}$ as long as the light green strip. What ratio does this fact suggest for comparing the red strip to the light green strip? 2:3



3. A What is the ratio of the number of boys to the number of girls in your room? Answers will vary from class to class.
   B What is the ratio of **girls to boys** in your room? Answers will vary.

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Discussion

The first discussion exercise introduces two important ideas concerning ratio. The first is that a colon is used between pairs of numbers to denote a ratio. Thus, the ratio of 4 *to* 8 is written as 4:8. The second idea to be stressed is that the order of the numbers in a ratio is important. A ratio of 4 to 8 is not the same as a ratio of 8 to 4.

Exercise 2 helps to show how closely the ideas of ratio are related to the ideas of fractional numbers. The fact that the red strip is $\frac{2}{3}$ as long as the light green strip means that the ratio of their lengths is 2 to 3.

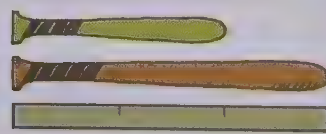
Exercise 3 gives the students an opportunity to apply ratio ideas to a familiar setting. (Of course, the ratios will vary with the composition of the children in the room.)

Using the Ideas

1. Copy the part in red and give the missing number.

A The ratio of the length of the toy bat to that of the big bat is 2 to ▒. 3

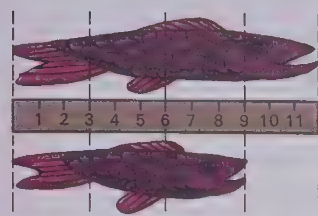
B The ratio of the length of the big bat to that of the toy bat is ▒ to ▒. 3, 2



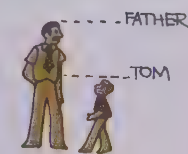
C The ratio of the length of the big bat to that of the toy bat is 36 to ▒. 24

D The ratio of the length of the large fish to the length of the small fish is 4 to ▒. 3

E The ratio of the length of the small fish to the length of the large fish is ▒ to ▒. 3, 4

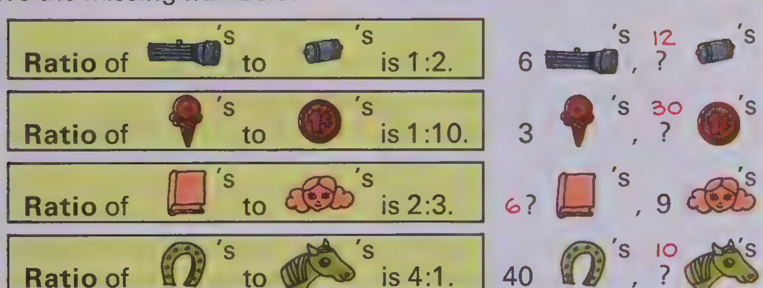


2. The ratio of Father's height to Tom's is 2 to 1. Father is 180 cm tall. How tall is Tom? 90 cm



3. We can use ratio to compare two sets. There is 1 tent for every 3 boys. The **ratio** of the tents to boys is **1 : 3**. If there are 4 tents, how many boys are there? 12

4. Give the missing numbers:



More practice, page A-21, Set 40

Resources for Active Learning

Franklin Series: *Probability*, "Sample Spaces and Events," pp. 18–36, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Freedom to Learn, "Ratio and Proportion," p. 147, Addison-Wesley.

SMSG: *Probability for Intermediate Grades*, "Ghosts, Goblins . . ." Lesson 12, Stanford University.

Workbook, page 73

Using the Exercises

Have children work through the exercises on page 219 independently or with two or three classmates. Point out that most of the exercises are related to the illustrations on the page. Give guidance to children who need it.

Remind the children that ratio can be expressed in any one of many ways. You might suggest to some more capable children that they write three expressions for each exercise.

Encourage children who have difficulty with exercise 2, 3, or 4 to draw pictures describing the ratio given for the troublesome problem.

When the children have finished, allow ample time for discussion and for questions.

Assignments (page 219)

Minimum: 1–3, oral.
Average: 1, oral; 2–4.
Maximum: 1–4.

PAGES 220–221

Objective

Given a map with a fractional scale the child will be able to find distances between specified points on the map.

Preparation

Materials

Centimetre ruler (1 per child); miscellaneous maps

To prepare for this lesson, simply show a variety of maps and point out, for example, how a map showing elevation differs from a map showing population density, or how a map showing mineral or agricultural resources differs from a road map. As you display road maps, discuss how colors and thicknesses of lines show different types of freeways, highways, county roads, and so on. Then explain to the children that in this lesson they will study a road map and the distances shown on it in relation to actual distances.

Investigation

Encourage the children to begin investigating the meaning of the map scale as soon as possible. Although most road maps have distances between points marked on the map, these distances have been omitted from the map shown here, so that the children will have to use the given scale to find distances.

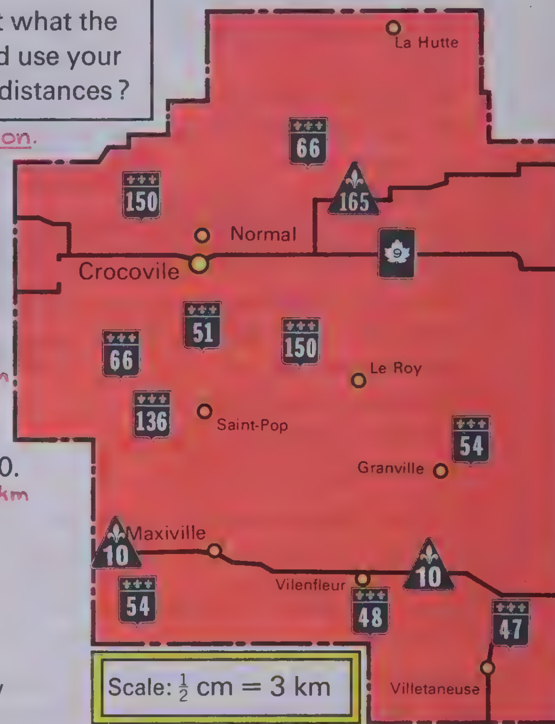
If you have directed the children to work in groups of two or three, suggest that each child make his own measurements and then compare and discuss his findings with others in his group. Also, encourage them to discuss the meaning of the phrase "as the crow flies." If this phrase is unfamiliar to them, explain that it is an idiomatic saying which refers to a path along a straight line between two places.

Let's explore map scales using fractional numbers.

Investigating the Ideas

? Can you figure out what the "scale" means and use your ruler to find these distances?

See Investigation.



A From Maxiville to Crocovile along route 51. 30 km

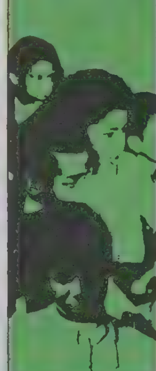
B From Normal to Vilenfleur, along routes 51 and 10. 48 km

C From Crocovile to LeRoy, along route 150. 18 km

D From Crocovile to Vilenfleur, "as the crow flies." 36 km

E From Villetaneuse to La Hutte "as the crow flies." 63 km

Discussing the Ideas



1. To show that $\frac{1}{2}$ centimetre on the map represents 3 kilometres on the earth, we wrote: $\frac{1}{2}$ cm = 3 km
   Complete the following to show other ways to indicate the scale for the map.
   A 1 cm = ▒ km 6 **B** ▒ cm = 12 km 2

2. What scale would you choose if you wanted to draw a map of your community on the chalkboard? on your paper?
   Answers may vary. See Discussion.

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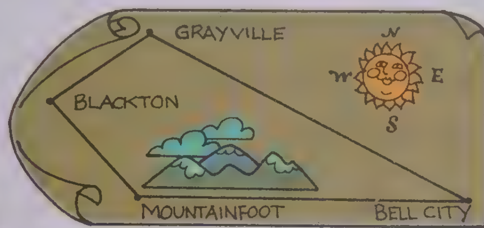
Discussion

Have children explain how they found the distances for each part of the investigation. Stress the meaning and use of the scale: every 2 centimetres represent 3 kilometres, so 1 centimetre represents 3 ÷ 2, or $1^1/_2$ centimetres, and so on.

To answer discussion exercise 2, children must decide what boundaries they would use for a map of their community. If you live in a large city, the map may show only the city; if you live in a small town the map may show neighboring towns or cities as well. This exercise would serve as an excellent group or class project. One group may use one set of boundaries and scale, and another group may use a different set of boundaries and a different scale. However, before children begin, discuss the convenience of some scales over others, for example, if 2 centimetres = 3 kilometres when they measure to the nearest 1 centimetre they will have to refer to $1^1/_2$ kilometres, but if 2 centimetres = 4 kilometres, then each centimetre will equal 2 kilometres, which is more convenient for adding.

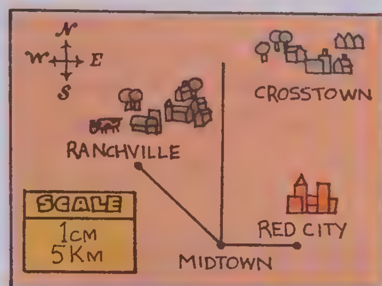
Using the Ideas

1. Since each segment on the map represents a certain distance on land, this map is drawn to scale. Use your centimetre ruler and give the missing numbers.
   - A If the distance from Blackton to Grayville is 2 kilometres, then the distance from Grayville to Bell City is ▒ kilometres. 6

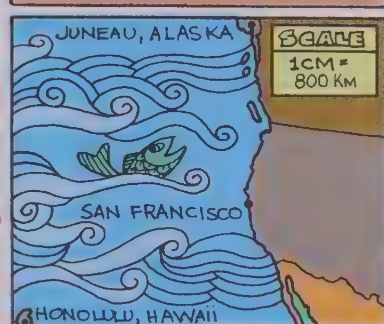


   - B According to exercise A, one centimetre on the map means ▒ kilometres on land. 1
   - C If the map scale is $\frac{1}{2}$ cm = 3 kilometres, what is the distance from Blackton through each city, and back to Blackton? 93 km

2. A How far is it from Red City to Midtown? 6 km
   - B How far is it from Midtown to Ranchville? 10 km
   - C It is 20 kilometres due north from Midtown to Crosstown. How far above the dot for Midtown should the dot for Crosstown be placed? 4 cm



3. Use the scale shown on the map to answer these questions.
   - A About how far is it from San Francisco to Juneau? 2400 km
   - B About how far is it from Juneau to Honolulu? About 4800 km
   - C About how far is it from Honolulu to San Francisco? About 3200 km



221

Follow-up

Children will benefit from work with actual maps. Official road maps of your province can be obtained from your provincial highway department, tourist information centres, and gasoline stations. Try to assemble enough maps from several different sources so that there is at least one map per pair of children. Direct the class to find and use the scale for their particular map. They should also learn to use the grid system to locate a given city or point of interest, learn to "tell directions" on the map, and learn to use the map key to decipher the symbols used on the map.

You might also suggest that they themselves choose a scale and draw a map showing their home, school, and neighborhood community. Note that the scale used will depend upon the distance between their home and the school.

Resources for Active Learning

Applied Mathematics Cards, "Maps and Scales," Group 2/25, Schofield and Sims. (Available from Mafex Associates, Willowdale)

Developmental Math Cards, I-14, J-8, Addison-Wesley.

Math Activity Cards, "Maps," C11, D10, Macmillan. [Using map scales]

Duplicator Masters, page 47

Using the Exercises

Have the children work on the exercises on page 221 on their own or in small groups. Stress the importance of measuring carefully and of approximating when necessary (as in exercise 3). As with previous problem sets using fractions, children should use an intuitive approach. For example, one approach to exercise 1C gives these results: measurement shows the round trip to consist of $15^1/_2$ cm. For $^1/_2$ cm, we think 3 kilometres. In 15 cm, there are 30 half centimetres, which represent 3 × 30, or 90 kilometres. The remaining $^1/_2$ cm gives us 3 more kilometres or 93 kilometres in all.

Assignments (page 221)

Minimum: 1–2. Average: 1–3. Maximum: 1–3.

PAGES 222–223

Objective

The child will demonstrate his ability to work with the concepts presented in this chapter.

Preparation

Review with the children any of the topics in the chapter with which they have had particular difficulty. One of the chief purposes of this chapter is to get children to think about the concept of fractional numbers; therefore, as part of your review, have the children build several sets of equivalent fractions and then associate these sets with one fractional number by identifying it on the number line for this set of equivalent fractions. If necessary, you might follow this activity by having the children answer true or false for various statements of equality or inequality concerning fractional numbers.

1. Give the correct point for the fractional number indicated by each set of equivalent fractions.

A 0 A B C 1

$\{\frac{5}{8}, \frac{10}{16}, \frac{15}{24}, \frac{20}{32}, \ldots\}$ B

B 0 A B C 1

$\{\frac{9}{10}, \frac{18}{20}, \frac{27}{30}, \frac{36}{40}, \ldots\}$ C

C 0 A B C 1

$\{\frac{4}{7}, \frac{8}{14}, \frac{12}{21}, \frac{16}{28}, \ldots\}$ A

D 0 A B C 1

$\{\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \ldots\}$ C

E 0 A B C 1

$\{\frac{1}{8}, \frac{2}{16}, \frac{3}{24}, \frac{4}{32}, \ldots\}$ A

F 0 A B C 1

$\{\frac{5}{9}, \frac{10}{18}, \frac{15}{27}, \frac{20}{36}, \ldots\}$ B

2. Use the lowest-terms fraction to name the fractional number for the point over the red arrow.

A 0 ↑ $\frac{1}{2}$ 1

B 0 ↑ $\frac{3}{4}$ 1

C 0 ↑ $\frac{2}{7}$ 1

D 0 ↑ $\frac{1}{10}$ 1

E 0 ↑ $\frac{5}{6}$ 1

F 0 ↑ $\frac{1}{3}$ 1

G 0 1 ↑ $\frac{7}{4}$ 2

H 0 1 2 ↑ $\frac{13}{6}$ 3

I 0 1 2 3 ↑ $\frac{7}{2}$ 4

J 0 1 2 3 4 ↑ $\frac{13}{3}$ 5

3. Give two other names for each fractional number. Answers may vary.

A $\frac{1}{2}$ $\frac{2}{4}, \frac{3}{6}$ B $\frac{1}{3}$ $\frac{2}{6}, \frac{3}{9}$ C $\frac{1}{4}$ $\frac{2}{8}, \frac{3}{12}$ D $\frac{2}{3}$ $\frac{4}{6}, \frac{6}{9}$ E $\frac{3}{4}$ $\frac{6}{8}, \frac{9}{12}$ F $\frac{5}{7}$ $\frac{10}{14}, \frac{15}{21}$ G $\frac{3}{10}$ $\frac{6}{20}, \frac{9}{30}$ H $\frac{4}{5}$ $\frac{8}{10}, \frac{12}{15}$

222

Discussion

Whether you use these pages as a class review or as an evaluation instrument, allow ample time for discussion of troublesome exercises. Encourage children to use number lines or pictures showing appropriate fractional parts for those exercises which they find difficult. Some children will be unable to complete all the exercises, so if you ask them to do a representative sampling of problems from each exercise, you will get a better picture of their understanding of the ideas presented in this chapter.

4. Give the correct sign (=, ≠) for each ▒.

A $\frac{3}{4}$ ▒ $\frac{75}{100}$ = C $\frac{4}{6}$ ▒ $\frac{12}{15}$ ≠ E $\frac{5}{7}$ ▒ $\frac{70}{80}$ ≠ G $\frac{30}{5}$ ▒ $\frac{6}{1}$ =

B $\frac{15}{20}$ ▒ $\frac{3}{4}$ = D $\frac{11}{15}$ ▒ $\frac{20}{30}$ ≠ F $\frac{56}{80}$ ▒ $\frac{60}{90}$ ≠ H $\frac{0}{7}$ ▒ $\frac{0}{2}$ =

5. Give the missing numerator or denominator.

A $\frac{4}{10} = \frac{▒}{20}$ 8 C $\frac{3}{4} = \frac{▒}{8}$ 6 E $\frac{5}{2} = \frac{▒}{8}$ 20 G $\frac{60}{60} = \frac{5}{▒}$ 5

B $\frac{15}{6} = \frac{5}{▒}$ 2 D $\frac{4}{6} = \frac{▒}{18}$ 12 F $\frac{7}{10} = \frac{▒}{100}$ 70 H $\frac{0}{8} = \frac{▒}{7}$ 0

6. Give the correct sign (< or >) for each ▒.

A $\frac{1}{2}$ ▒ $\frac{2}{3}$ < C $\frac{1}{8}$ ▒ $\frac{1}{4}$ < E $\frac{3}{5}$ ▒ $\frac{2}{5}$ > G $\frac{4}{5}$ ▒ $\frac{7}{10}$ >

B $\frac{1}{2}$ ▒ $\frac{1}{4}$ > D $\frac{6}{10}$ ▒ $\frac{11}{20}$ > F $\frac{5}{8}$ ▒ $\frac{5}{6}$ < H $\frac{4}{6}$ ▒ $\frac{3}{4}$ <

7. In each exercise, list the numbers in order from smallest to largest.

A $\frac{7}{2}, \frac{4}{2}, \frac{3}{2}, \frac{5}{2}, \frac{8}{2}, \frac{12}{2}, \frac{9}{2}, \frac{1}{2}$ $\frac{1}{2}, \frac{3}{2}, \frac{4}{2}, \frac{5}{2}, \frac{7}{2}, \frac{8}{2}, \frac{9}{2}, \frac{12}{2}$

C $\frac{8}{8}, \frac{1}{2}, \frac{3}{8}, \frac{3}{4}, \frac{5}{8}, \frac{1}{8}, \frac{1}{4}, \frac{0}{8}$ $\frac{0}{8}, \frac{1}{8}, \frac{1}{4}, \frac{3}{8}, \frac{1}{2}, \frac{5}{8}, \frac{3}{4}, \frac{8}{8}$

B $\frac{5}{6}, \frac{1}{3}, \frac{7}{6}, \frac{2}{3}, \frac{5}{3}, \frac{4}{3}, \frac{0}{3}$ $\frac{0}{3}, \frac{1}{3}, \frac{2}{3}, \frac{5}{6}, \frac{7}{6}, \frac{4}{3}, \frac{5}{3}$

D $\frac{2}{3}, \frac{1}{2}, \frac{5}{6}, \frac{1}{3}, \frac{1}{6}, \frac{1}{4}, \frac{3}{4}$ $\frac{1}{6}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}$

8. Each whole number is also a fractional number. Some fractional numbers are whole numbers. Give the whole number for each of the following fractional numbers.

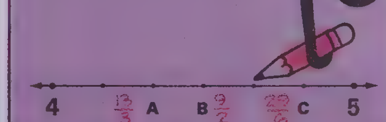
A $\frac{12}{3}$ 4 D $\frac{28}{4}$ 7 G $\frac{56}{8}$ 7

B $\frac{25}{5}$ 5 E $\frac{14}{7}$ 2 H $\frac{80}{10}$ 8

C $\frac{16}{2}$ 8 F $\frac{36}{4}$ 9 I $\frac{40}{5}$ 8

think

Give the lowest-terms improper fractions for points A, B, and C.



9. Write a fraction for each of the following whole numbers.

Answers may vary.

A 2 $\frac{2}{1}$ C 3 $\frac{3}{1}$ E 4 $\frac{4}{1}$ G 5 $\frac{5}{1}$

B 1 $\frac{1}{1}$ D 0 $\frac{0}{1}$ F 6 $\frac{6}{1}$ H 7 $\frac{7}{1}$

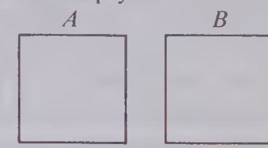
223

Follow-up

To help children with concepts they found difficult, prepare a worksheet similar to the one below.

For each pair of fractions, name the fraction which is greater. Use the directions to help you.

$\frac{1}{3}, \frac{1}{2}$

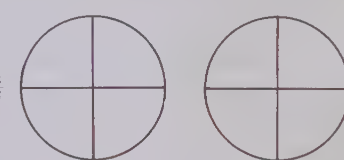


Color $\frac{1}{2}$ of square *A*. Color $\frac{1}{3}$ of square *B*. Which region is larger, $\frac{1}{2}$ or $\frac{1}{3}$?

$\frac{5}{6}, \frac{4}{5}$

Draw a number line so that the distance from 0 to 1 is 5 cm. Count each cm as $\frac{1}{5}$. Divide this same number line as best you can into sixths. Remember that $\frac{3}{6}$ is halfway between 0 and 1. Use it to figure which fraction is larger, $\frac{5}{6}$ or $\frac{4}{5}$.

$\frac{7}{4}, \frac{3}{2}$



Color $\frac{3}{2}$ blue. Have you colored $\frac{7}{4}$ also? Which is larger, $\frac{7}{4}$ or $\frac{3}{2}$?

Workbook, page 74

Objective

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

Review with the children any topics from the text with which they may have had particular difficulty in the past. Since in this lesson only one exercise is devoted to using the addition, subtraction, multiplication, and division algorithms, you may find it helpful in this review period to provide additional practice by having the children do several exercises of this type on the chalkboard and then explain the various steps to the class.

Keeping in Touch with Computing, Place value, Measurement, Number theory, Geometry

1. A List the multiples of 4 (less than 50). {0,4,8,12,16,20,24,28,32,36,40,44,48}
   B List the multiples of 6 (less than 50). {0,6,12,18,24,30,36,42,48}
   C List the common multiples of 6 and 4. 0,12,24,36,48
   D What is the least common multiple of 6 and 4? 12

2. Find the least common multiple of the two numbers given.
   A 3, 4 12   B 6, 9 18   C 5, 3 15   D 3, 8 24   E 12, 20 60

3. Solve the equations.
   34 A $42 = n + 8$ E $352 = n + 50 + 2$ 300   I $8 \div 8 = n$ 1
   51 B $100 - 49 = n$ F $486 = 400 + n + 6$ 80   J $93 \times 0 = n$ 0
   8 C $720 = n \times 90$ G $974 = (n \times 100) + (7 \times 10) + 4$ 9   K $17 \times 1 = n$ 17
   90 D $270 \div 3 = n$ H $6037 = (6 \times n) + 30 + 7$ 1000   L $0 \div 9 = n$ 0

4. Find the sum, difference, product, and quotient.
   A $9659 + 7838 = $ 17497   B $683 \times 246 = $ 168018   C $8002 - 4526 = $ 3476   D $63\overline{)3030}$ 48 R6

5. Use your protractor to find the degree measure of each angle.
   A 30°   B 135°   C 90°

6. A What is the area of rectangle *ABCD*? 24 sq units
   B What fraction of the rectangular region is shaded pink? $\frac{15}{24}$
   C What is the area of the pink region? 15 sq units

You are invited to explore

ACTIVITY CARD 10 Page 338

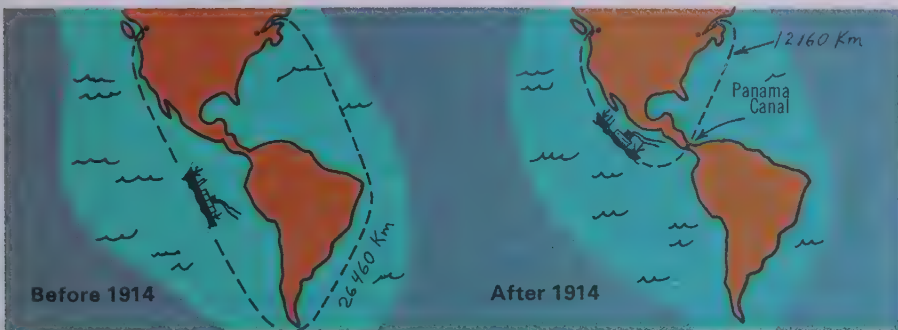
224

Discussion

Page 224 is intended for independent work, unless you work through the exercises as a class review. When the children finish, discuss any difficult exercises. It may also be helpful to particularly emphasize exercises 1 and 2 in preparation for the material in the next chapter, which utilizes the concept of least common multiple. Have exercises 1 and 2 presented on the chalkboard and ask the children to explain how they arrived at their lists of multiples and their list of common multiples. Once the common multiples are listed, it is easy to give the least common multiple.

THE PANAMA CANAL

The Panama Canal is a waterway that crosses Central America to connect the Atlantic and Pacific oceans. Before it was completed in 1914, a ship sailing from Montreal to Vancouver had to travel about 24 640 kilometres. After 1914, the distance was cut to 12 160 kilometres.



A The Panama Canal is about 80 kilometres long.
B Average time for passing through is about 7 hours.
C 1962—10 866 commercial vessels passed through.
1969—14 606 commercial vessels passed through.

1. How many kilometres are saved by passing through the canal between Montreal and Vancouver ? 12 480 km
2. If a ship makes 6 round trips from Montreal to Vancouver, how many kilometres does the canal save the ship ? 149 760 km
3. How many more ships went through the canal in 1969 than in 1962 ? 3740
4. If the average amount of cargo carried by each ship in 1969 was 7000 tonnes, what was the total tonnage carried by the ships ? 102 242 000 tonnes
★ 5. About what is the average speed (kilometres per hour) of a ship moving through the canal ? About 11 km/h

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Using the Exercises
Page 225 provides ample material for an introductory discussion. Keep in mind that one of the intended outcomes for problem sets of this nature is to provide an interesting application of arithmetic for the children. Therefore, it is very helpful to allow considerable discussion of the factual material presented in these problem sets. Following a discussion, have the children do the exercises. When they have finished, allow time for checking papers and further discussion of these ideas.

Follow-up
Other statistics pertinent to commerce, transportation, imports and exports, and the like can provide data for word problems. You may wish to give the children a list of information such as that in the following tables, and ask them to make up related problems, including several which require more than one operation to solve.

Ocean and River Ports	Total Tonnage (to nearest 1000)
Vancouver, B.C.	26 518 000
Sept-Iles, P.Q.	24 241 000
Montreal, P.Q.	22 376 000
Thunder Bay, Ont.	20 754 000
Port Cartier, P.Q.	16 017 000
Hamilton, Ont.	12 881 000
Halifax, N.S.	11 072 000

Immigration to Canada	Total from each country (nearest 100)
United Kingdom & Ireland	27 600
France	4 400
Germany	4 200
Italy	8 500
Asia	21 500
Australasia	4 400
United States	24 400
West Indies	12 500

Pages 226–251

General Objectives

To provide additional experience in working with fractional numbers

To introduce addition of fractional numbers

To introduce subtraction of fractional numbers

To provide skills in finding sums of fractional numbers

To provide skills in finding differences of fractional numbers

To introduce mixed-numeral notation

To provide word-problem experiences in work with addition and subtraction of fractional numbers

The first lesson of this chapter utilizes the colored strips, number lines, and regions to provide a concrete and semi-concrete introduction to addition and subtraction of fractional numbers. In the next two lessons, the approach is more abstract: first the children list sets of equivalent fractions for each of the fractions which represent the addends; then from these lists, they choose fractions with a common denominator so that they can find the sum or difference. Subsequent lessons provide experiences in working with the familiar least-common-denominator method.

Each whole number is identified as a fractional number; then mixed numerals are introduced as a convenient notation for fractional numbers, and the skill of changing improper fractions to mixed numerals, and vice versa, is studied. The remaining lessons of the chapter present material which emphasizes and utilizes the fact that the basic principles which hold for the whole numbers also hold for the fractional numbers. Using these basic principles, subsequent examples develop an understanding of how we find the sum of two fractional numbers when mixed-numeral notation is used for the numbers. The same principles provide the basis for the regrouping used in subtraction that involves similar mixed-numeral notation.

Mathematics

One of the simplest and most straightforward definitions for the sum of any two fractional numbers is:

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two fractional numbers, then

(A) $\frac{a}{b}+\frac{c}{d}=\frac{(a\times d)+(b\times c)}{b\times d}$.

With this definition and a few basic concepts of fractional numbers, we can deduce this familiar rule:

(B) $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$ in this manner:

$$\frac{a}{c}+\frac{b}{c}=\frac{(a\times c)+(b\times c)}{c\times c}$$
$$=\frac{c\times(a+b)}{c\times c}$$
$$=\frac{a+b}{c}$$

Using definition (B), we could have deduced (A) as follows.

$$\frac{a}{b}+\frac{c}{d}=\frac{a\times d}{b\times d}+\frac{b\times c}{b\times d}$$
$$=\frac{(a\times d)+(b\times c)}{b\times d}$$

Note that in both instances we use the fact that for any fractional number $\frac{a}{b}$, if $k\neq 0$, then $\frac{a}{b}=\frac{a\times k}{b\times k}$.

These ideas are presented to children informally. For example, it is a simple matter to explain a sum such as $\frac{3}{8}+\frac{2}{8}=\frac{5}{8}$, using the number line or regions. Once children understand this idea, they can then be led to find the sum of any two fractional numbers regardless of the fractions used to denote them. That is, when the fractions have different denominators, the task of finding the sum is reduced to that of finding equivalent fractions for the two numbers that *do* have the same denominator.

$$\frac{3}{8}+\frac{1}{6}$$
$$\frac{9}{24}+\frac{4}{24}=\frac{13}{24}$$

Once addition concepts and skills are established, it is easy to move to subtraction by using the already familiar relationship between addition and subtraction of whole numbers. We point out that the same relationship exists between addition and subtraction of fractional numbers. If we accept this relationship, we can justify the following:

$$\frac{a}{b}-\frac{c}{b}=\frac{a-c}{b};$$
$$\frac{a}{b}-\frac{c}{d}=\frac{(a\times d)-(b\times c)}{b\times d}$$

Teaching the Chapter

Materials

Colored strips
Demonstration number line
Centimetre rulers of clear plastic for use on an overhead projector
Regions cut into fractional parts
Centimetre rulers (one for each child)
Slips of paper approximately 10 by 15 centimetres, or paper to be cut into slips
Toothpicks or matches

Vocabulary

least common denominator
line graph
mixed numeral
pictograph

The colored strips are the most essential materials used by the children in this chapter. You will prob-

ably find a demonstration number line and cutouts of regions helpful in some discussions. The lesson on fractional parts of units of measure will be greatly facilitated by demonstrations with clear plastic rulers which show the measuring marks quite clearly on an overhead projector. You should encourage children to use any materials which help them gain a clearer understanding of the concepts or skills studied, but it is hoped that most will be able to work with minimal use of such materials in this chapter.

Notice the term *mixed numeral* in the vocabulary list. Since it is a symbol for a whole number and a fraction, it is preferable to refer to this as a mixed *numeral* rather than as a mixed number.

Lesson Schedule

Plan to spend three to three-and-one-half weeks on this chapter, but do not hesitate to adjust the schedule to the abilities and needs of your children. One of the principal objectives of the chapter is to help children understand addition and subtraction of rational numbers, but it is also important to lead them toward a moderate degree of skill in computing with some simple fractional numbers.

Evaluation

Children's achievement in this chapter should be evaluated in terms of both skills and understanding. It is easy to give a routine test to find out whether or not the children can add and subtract fractional numbers, but it is difficult to evaluate whether or not the children understand the concepts involved in the development of the various techniques for adding and subtracting fractional numbers. Therefore, you should include as part of your evaluation a day-to-day observation of the children's participation in class discussions of the various ideas presented in the chapter.

Pages 248 and 249 offer a chapter review which you may use either for evaluation or for review, as you prefer. Pages 250 and 251 should help you evaluate the children's retention of skills and concepts previously taught.

Resources for Active Learning

General References

A Cloudburst, Vol. 2, Nos. 3213, 4334, Midwest Publications

Franklin Series: *Making and Using Graphs and Nomographs,* "Nomograph graphs . . . ," pp. 35-36, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Mathex: Numeration No. 7, "Fraction Dominoes," p. 46 (pupil pages 42–46), Encyclopaedia Britannica Publications Ltd. [An addition game]

Manipulative Devices

Cuisenaire Rods (Cuisenaire Co.)

Commercial Games

Action Fraction Games (CCM School Materials; Lakeshore; Math Media)

Come Out Even (Holt, Rinehart and Winston)

Competitive Fractions (Selective Educational Equipment)

Imout—fractions (Imout)

Spinner Number Games—fractions (Heath)

PAGES 226–227

Objective

Given fractional numbers expressed with like denominators, the child will be able to add and subtract them.

Preparation

Materials

colored strips

To prepare for this lesson, review lengths of the colored strips when the brown strip is the unit. For example, ask the children to give the length of the red strip if the brown strip is the unit ($\frac{2}{8}$). Work through the lengths of all the strips, particularly including the light green and purple strips ($\frac{3}{8}$ and $\frac{4}{8}$, respectively). Familiarity with these lengths will facilitate children's use of them in the investigation.

Investigation

Encourage the children to work independently on this investigation. Point out that the equations they write should all relate to the brown strip as the unit. After they have completed this activity, if you would like to extend the investigation, direct children to use other strips as the unit and show other fractions with the strips. For example, ask them to show with strips and to solve equations such as $\frac{3}{10} + \frac{5}{10} = n$, $\frac{2}{7} + \frac{1}{7} = n$, $\frac{1}{6} + \frac{3}{6} = n$, and so on. Although these examples have a sum that is less than one, equations in which the answer is an improper fraction are acceptable. However, children would have to use a combination of strips to show their answer if the numerator is greater than 10, as in $\frac{5}{8} + \frac{7}{8} = \frac{12}{8}$.

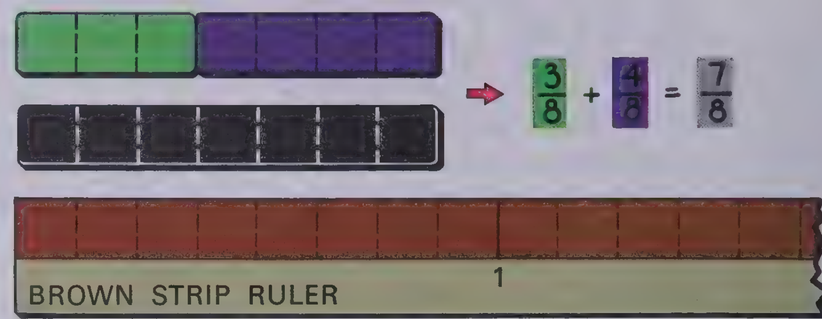
Although the intent of the investigation is to have the children write addition equations, subtraction equations are also acceptable, but they are treated more specifically in the discussion section.

11 Addition and Subtraction of Fractional Numbers

Can fractional numbers be added and subtracted?

Investigating the Ideas

Think of a ruler with the brown strip as the unit.



$\frac{3}{8} + \frac{4}{8} = \frac{7}{8}$

? Can you use your strips to help you write some other equations about eighths?

$\frac{1}{8}+\frac{2}{8}=\frac{3}{8}$; $\frac{2}{8}+\frac{3}{8}=\frac{5}{8}$; $\frac{3}{8}+\frac{6}{8}=\frac{9}{8}$; $\frac{3}{8}+\frac{1}{8}=\frac{4}{8}$; etc.
See Investigation.

Discussing the Ideas

1. Here is one subtraction equation related to the addition equation above.
   $\frac{7}{8} - \frac{4}{8} = \frac{3}{8}$ Can you find another one? $\frac{7}{8}-\frac{3}{8}=\frac{4}{8}$

2. Give some subtraction equations related to some of the addition equations you found with your strips.
   $\frac{3}{8}-\frac{1}{8}=\frac{2}{8}$; $\frac{5}{8}-\frac{2}{8}=\frac{3}{8}$; $\frac{4}{8}-\frac{2}{8}=\frac{2}{8}$; etc.

3. Do you know a rule for adding and subtracting when the fractions have the same denominator?
   Sample answer: Add or subtract the numerators.
   See Discussion.

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Discussion

The addition and subtraction of fractional numbers expressed with like denominators is not a difficult process for children to grasp. Use some of their equations from the investigation to point out that all the fractions in each equation have the same denominator. Have volunteers give related subtraction equations which consist of fractions with the same denominator.

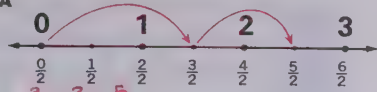
As you discuss exercise 3, help children verbalize the rule for adding or subtracting, making sure they realize that they may add or subtract numerators only when the denominators are alike.

Since the chief concern of this lesson is to introduce the adding and subtracting of fractional numbers using fractions that have the same denominator, do not ask children to change the answers to lowest terms. All the numbers involved should be represented by fractions having the same denominator.

Using the Ideas

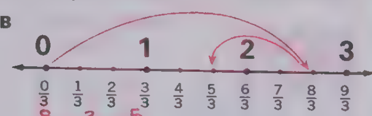
1. Write an equation for each number-line picture.

A



$\frac{3}{2} + \frac{2}{2} = \frac{5}{2}$

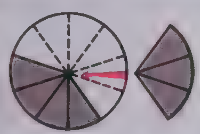
B



$\frac{8}{3} - \frac{3}{3} = \frac{5}{3}$

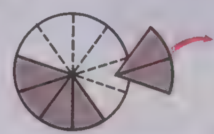
2. Find the sum and difference. The red arrows help you think about putting things together or taking them away.

A



$\frac{4}{10} + \frac{3}{10} = n$ $\frac{7}{10}$

B



$\frac{6}{10} - \frac{2}{10} = a$ $\frac{4}{10}$

3. Find the sums and differences.

A $\frac{3}{8} + \frac{1}{8}$ $\frac{4}{8}$	**D** $\frac{8}{3} + \frac{9}{3}$ $\frac{17}{3}$	**G** $\frac{2}{5} + \frac{2}{5}$ $\frac{4}{5}$	**J** $\frac{3}{7} - \frac{1}{7}$ $\frac{2}{7}$
B $\frac{6}{7} + \frac{5}{7}$ $\frac{11}{7}$	**E** $\frac{10}{20} - \frac{1}{20}$ $\frac{9}{20}$	**H** $\frac{7}{3} - \frac{7}{3}$ $\frac{0}{3}$	**K** $\frac{7}{50} - \frac{6}{50}$ $\frac{1}{50}$
C $\frac{15}{100} - \frac{9}{100}$ $\frac{6}{100}$	**F** $\frac{7}{10} - \frac{2}{10}$ $\frac{5}{10}$	**I** $\frac{3}{4} + \frac{0}{4}$ $\frac{3}{4}$	**L** $\frac{3}{10} + \frac{7}{10}$ $\frac{10}{10}$

Short Stories

1. Diane ate $\frac{3}{12}$ of the cookies. Paula ate $\frac{4}{12}$ of them. What fraction of the cookies were eaten? $\frac{7}{12}$

2. Mother had $\frac{1}{6}$ of a pie left after we ate $\frac{4}{6}$ of the pie. She started with what part of a pie? $\frac{5}{6}$

3. Jack grew $\frac{7}{10}$ cm while Bob grew $\frac{3}{10}$ cm. How much more did Jack grow? $\frac{4}{10}$ cm

4. Dennis painted $\frac{4}{10}$ of the fence, while Craig painted $\frac{3}{10}$ of it. What part of the fence did the boys paint? $\frac{7}{10}$

5. Kurt mowed $\frac{3}{8}$ of the yard in the morning and $\frac{5}{8}$ in the afternoon. Did he mow the whole yard? Yes

6. Susan lives $\frac{9}{10}$ km from school. Carol lives only $\frac{6}{10}$ km away. How much farther does Susan live? $\frac{3}{10}$ km

More practice, page A-22, Set 41

Using the Exercises
Assign the exercises on page 227 as individual or small-group work. Have number lines and cutouts available for those who wish to use them. When they have finished, allow time for discussion and checking papers. If time permits, you may find it helpful to demonstrate several parts of exercise 3 and the short stories, using either number lines or regions as models.

Assignments (page 227)
Minimum: 1–3.
Average: 1–3, Short Stories 1–3.
Maximum: 1–2, oral; 3; Short Stories 1–6.

Workbook, page 75

PAGES 228–229

Objective

Given two fractional numbers expressed with unlike denominators, and related sets of equivalent fractions, the child will be able to add or subtract them after first expressing them with the same denominator.

Preparation

Materials

colored strips

To prepare for this lesson, you might write some addition and subtraction equations with fractions which have like denominators and review the process of adding numerators when the denominators are the same. As always, such a review should be brief, but it is important for children to know how to add and subtract fractions with a common denominator before they attempt these operations with fractions which have unlike denominators.

Investigation

Before children begin, point out the partitions dividing the brown strip into eighths. The goal of this investigation is to have the children discover that $\frac{1}{2}$ and $\frac{1}{4}$ may be thought of in terms of eighths when necessary for addition and subtraction.

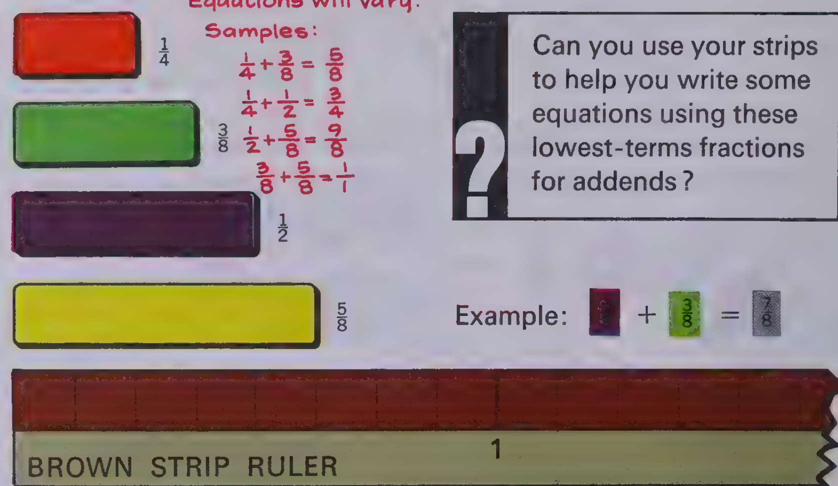
Encourage children to work independently on the investigation. Remind them to consider each strip in relation to the brown strip as the unit, as illustrated in the text. However, note that the children are instructed to use lowest-terms fractions in the addends of the equations they write. To challenge them, write this equation on the chalkboard:

$$\frac{1}{2}+\frac{1}{4}=\frac{?}{8}$$

They will have to think of $\frac{1}{2}$ and $\frac{1}{4}$ in terms of eighths in order to find the correct numerator.

● *Let's explore sums and differences.*

Investigating the Ideas



Equations will vary. Samples:

$\frac{1}{4}+\frac{3}{8}=\frac{5}{8}$

$\frac{1}{4}+\frac{1}{2}=\frac{3}{4}$

$\frac{1}{2}+\frac{5}{8}=\frac{9}{8}$

$\frac{3}{8}+\frac{5}{8}=\frac{1}{1}$

? Can you use your strips to help you write some equations using these lowest-terms fractions for addends?

Example: $\frac{1}{2} + \frac{3}{8} = \frac{7}{8}$

Discussing the Ideas

1. Why do you think these two sums are the same? ➡ $\frac{4}{8}+\frac{3}{8}$ | $\frac{1}{2}+\frac{3}{8}$

   $\frac{4}{8}=\frac{1}{2}$ and $\frac{3}{8}=\frac{3}{8}$

2. Can you find another pair of fractions that have the same sum as $\frac{1}{2}+\frac{1}{4}$? $\frac{2}{4}+\frac{2}{8}$; $\frac{3}{6}+\frac{2}{8}$; $\frac{4}{8}+\frac{3}{12}$; etc.

3. Explain each example below and give the sum or difference.

   $\boxed{A}$ To find $\frac{1}{4}+\frac{1}{3}$, we think $\frac{3}{12}+\frac{4}{12}=\square$. $\frac{7}{12}$

   $\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \ldots\}$ $\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots\}$

   $\boxed{B}$ To find $\frac{1}{2}-\frac{1}{5}$, we think $\frac{5}{10}-\frac{2}{10}=\square$. $\frac{3}{10}$

   $\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \ldots\}$ $\{\frac{1}{5}, \frac{2}{10}, \frac{3}{15}, \frac{4}{20}, \frac{5}{25}, \ldots\}$

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Discussion

During your discussion of exercise 1, stress the role of sets of equivalent fractions. Have children list the first several members of the set of fractions for $\frac{1}{2}$: $\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \ldots\}$. Point out that any fraction in this set may be used to represent $\frac{1}{2}$. Since the denominators must be the same for addition of fractional numbers, $\frac{4}{8}$ is an appropriate choice to replace $\frac{1}{2}$. Similarly, in exercise 2, $\frac{4}{8}+\frac{2}{8}$ may replace $\frac{1}{2}+\frac{1}{4}$. Some children may also realize that $\frac{1}{2}+\frac{1}{4}$ may be replaced with $\frac{2}{4}+\frac{1}{4}$.

As you work through exercise 3, continue to stress two important points: we can add or subtract fractional numbers only when the fractions used have like denominators, and if two fractions are in the same set of equivalent fractions, one can replace the other.

Use other examples, directing children to build sets of equivalent fractions for the fractions used in the problem. For instance, write $\frac{1}{2}+\frac{2}{3}$ on the chalkboard, and have children build the two sets of equivalents: $\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \ldots\}$ and $\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \ldots\}$. Direct them to find fractions with the least common denominators. When they are satisfied that $\frac{1}{2}=\frac{3}{6}$ and $\frac{2}{3}=\frac{4}{6}$, they should realize that $\frac{3}{6}+\frac{4}{6}=\frac{7}{6}$ is a solution to the equation $\frac{1}{2}+\frac{2}{3}=?$

Using the Ideas

1. For each exercise, write an equation using fractions that have the same denominator.

A To find $\frac{1}{2} + \frac{1}{3}$, $\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots\}$

we think ▮▮▮▮. $\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots\}$

$\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$

B To find $\frac{3}{4} - \frac{1}{6}$, $\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \ldots\}$

we think ▮▮▮▮. $\{\frac{1}{6}, \frac{2}{12}, \frac{3}{18}, \frac{4}{24}, \ldots\}$

$\frac{9}{12}-\frac{2}{12}=\frac{7}{12}$

C To find $\frac{2}{3} + \frac{1}{4}$, $\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \ldots\}$

we think ▮▮▮▮. $\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \ldots\}$

$\frac{8}{12}+\frac{3}{12}=\frac{11}{12}$

D To find $\frac{5}{6} - \frac{1}{4}$, $\{\frac{5}{6}, \frac{10}{12}, \frac{15}{18}, \frac{20}{24}, \ldots\}$

we think ▮▮▮▮. $\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \ldots\}$

$\frac{10}{12}-\frac{3}{12}=\frac{7}{12}$

2. Find the sums and differences.

A Since $\frac{1}{2} = \frac{3}{6}$ and $\frac{1}{3} = \frac{2}{6}$, we know that $\frac{1}{2} + \frac{1}{3} = \boldsymbol{a}$. $\frac{5}{6}$

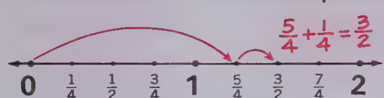
B Since $\frac{3}{4} = \frac{9}{12}$ and $\frac{1}{6} = \frac{2}{12}$, we know that $\frac{3}{4} - \frac{1}{6} = \boldsymbol{c}$. $\frac{7}{12}$

C Since $\frac{2}{3} = \frac{8}{12}$ and $\frac{1}{4} = \frac{3}{12}$, we know that $\frac{2}{3} + \frac{1}{4} = \boldsymbol{x}$. $\frac{11}{12}$

D Since $\frac{5}{6} = \frac{10}{12}$ and $\frac{1}{4} = \frac{3}{12}$, we know that $\frac{5}{6} - \frac{1}{4} = \boldsymbol{b}$. $\frac{7}{12}$

3. For each number-line picture, write an addition or subtraction equation.

A



B

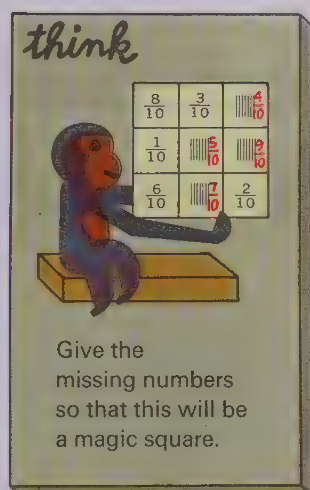
★ **4.** Give the numbers for ***a***, ***b***, and ***c***.

A To find $\frac{1}{2} - \frac{1}{8}$, we think $\frac{a}{8} - \frac{b}{8} = \frac{c}{8}$. 4 1 3

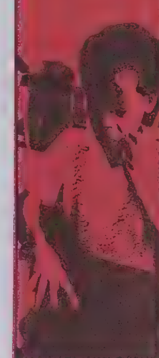
B To find $\frac{3}{8} - \frac{1}{4}$, we think $\frac{a}{8} - \frac{b}{8} = \frac{c}{8}$. 3 2 1

C To find $\frac{1}{3} - \frac{1}{6}$, we think $\frac{a}{6} - \frac{b}{6} = \frac{c}{6}$. 2 1 1

D To find $\frac{3}{4} - \frac{2}{3}$, we think $\frac{a}{12} - \frac{b}{12} = \frac{c}{12}$. 9 8 1



229



Mathematics

Throughout this program, the relationship between addition and subtraction of whole numbers has been made clear and emphasized for the children. By now, there should be little question about what is meant by finding a difference by thinking about a missing addend.

Subtraction of fractional numbers is defined in almost the same words as was subtraction of whole numbers. The only difference is that we refer to fractional numbers rather than *whole* numbers.

For fractional numbers a, b, and c, if $a + b = c$, then $a = c - b$, and $b = c - a$.

Resources for Active Learning

Discovery, Section II, Unit 21/6, Encyclopaedia Britannica Educational Corp.

Workbook, page 76

Using the Exercises

Direct the children to try the exercises on page 229 on their own.

Do not insist that the children write their answers in lowest terms. The goal for this lesson is to have the children begin to understand the idea of finding the sum or difference of two rational numbers even when the fractions given do not have the same denominator. Sufficient experience in writing sums in lowest terms will be provided later.

Assignments (page 229)

Minimum: 1–2. Average: 1–3. Maximum: 1–4.

PAGES 230–231

Objective

Given two fractional numbers expressed with unlike denominators, the child will be able to build related sets of equivalent fractions and add or subtract the numbers using fractions with a common denominator.

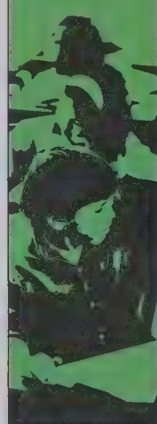
Preparation

Unless the children are adept at building sets of equivalent fractions, spend a few minutes reviewing this process. For example, ask children to list the first five members of the set whose lowest-terms fraction is $\frac{5}{6}$; or $\frac{8}{9}$; or $\frac{5}{7}$; or $\frac{2}{5}$. Then have them list their answers to correspond with their explanation of how they found the equivalent fractions for each, as follows:

$\frac{5\times1}{6\times1}, \frac{5\times2}{6\times2}, \frac{5\times3}{6\times3}, \frac{5\times4}{6\times4}, \frac{5\times5}{6\times5}, \ldots$

$\frac{5}{6}, \frac{10}{12}, \frac{15}{18}, \frac{20}{24}, \frac{25}{30}, \ldots$

If children understand this process well, the development in the present lesson will be less difficult.



Let's find sums and differences.

Discussing the Ideas

1. For each example below, give the missing number for the ▯. Then solve for *n*. Explain the steps used in finding the number for *n*.

A
$\frac{1}{2}$ $\{\frac{1}{2}, \frac{2}{4}, \;, \frac{4}{8}, \ldots\}$ $\frac{3}{6}$
$+\frac{1}{3}$ $\{\frac{1}{3}, \;, \frac{3}{9}, \frac{4}{12}, \ldots\}$ $+\frac{2}{6}$
$\frac{5}{6}$ n ← ▯ $\frac{5}{6}$

B
$\frac{1}{8}$ $\{\;, \frac{2}{16}, \frac{3}{24}, \frac{4}{32}, \ldots\}$ $\frac{1}{8}$
$+\frac{1}{2}$ $\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \;, \ldots\}$ $+\frac{4}{8}$
$\frac{5}{8}$ n ← ▯ $\frac{5}{8}$

C
$\frac{1}{4} + \frac{3}{10} = n$ $\frac{11}{20}$
$\frac{2}{8}$
$\frac{3}{12}$ $\frac{9}{30}$
$\frac{4}{16}$ $\frac{12}{40}$
$\frac{15}{50}$
⋮ ⋮
$\frac{5}{20} + \frac{6}{20} =$ ▯ $\frac{11}{20}$

2. Study the following examples and give the missing numbers. Explain how the process of finding a difference is much the same as finding a sum.

A
$\frac{1}{2}$ $\{\frac{1}{2}, \frac{2}{4}, \;, \frac{4}{8}, \ldots\}$ $\frac{3}{6}$
$-\frac{1}{3}$ $\{\frac{1}{3}, \;, \frac{3}{9}, \frac{4}{12}, \ldots\}$ $-\frac{2}{6}$
$\frac{1}{6}$ n ← ▯ $\frac{1}{6}$

B
$\frac{5}{6}$ $\{\frac{5}{6}, \;, \frac{15}{18}, \frac{20}{24}, \ldots\}$ $\frac{10}{12}$
$-\frac{1}{4}$ $\{\frac{1}{4}, \frac{2}{8}, \;, \frac{4}{16}, \ldots\}$ $-\frac{3}{12}$
$\frac{7}{12}$ n ← ▯ $\frac{7}{12}$

C
$\frac{3}{10} - \frac{1}{4} = n$ $\frac{1}{20}$
$\frac{2}{8}$
$\frac{9}{30}$ $\frac{3}{12}$
$\frac{12}{40}$ $\frac{4}{16}$
$\frac{15}{50}$
⋮ ⋮
$\frac{6}{20} - \frac{5}{20} =$ ▯ $\frac{1}{20}$

3. For each part, make your own lists of equivalent fractions until you find two fractions with the same denominator, one for each fraction given. Then give the sum or difference.

A $\frac{1}{2} + \frac{1}{8}$ $\frac{5}{8}$ B $\frac{1}{4} - \frac{1}{10}$ $\frac{3}{20}$ C $\frac{2}{5} + \frac{1}{4}$ $\frac{13}{20}$ D $\frac{1}{3} + \frac{3}{4}$ $\frac{13}{12}$ E $\frac{3}{4} - \frac{1}{10}$ $\frac{13}{20}$

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Discussion

The goal of this lesson is for children to develop the power skill of choosing from sets of equivalent fractions appropriate common-denominator fractions in order to add or subtract. Exercises 1 and 2 show two ways of listing the equivalent fractions. Note that the lists of equivalents in exercise 1A are extended farther than necessary, so that children can get a clearer idea of what is being done.

In exercise 1B, point out that it is not really necessary to list the fractions equivalent to $\frac{1}{8}$, since we can arrive at the appropriate fraction, $\frac{4}{8}$, simply by listing some of the fractions which are equivalent to $\frac{1}{2}$.

Work through parts of exercise 3 together, helping children set up their lists in a convenient form, as in exercises 1 and 2. For these exercises, you should not require that the children change the sums or differences to lowest terms or change improper-fraction sums or differences to mixed numerals.

Using the Ideas

1. Copy each exercise (without the arrows) on your paper and give the fraction, in the order shown, for ***a***, ***b***, and ***c***.

A $\frac{3}{4} + \frac{2}{3} = (c)$ $\frac{17}{12}$

$\frac{6}{8}$	$\frac{4}{6}$
$\frac{9}{12}$	$\frac{8}{12}$
⋮	⋮

$(a) + \frac{8}{12} = (b)$

$\frac{9}{12}$ $\frac{17}{12}$

B $\frac{3}{5} \rightarrow \{\frac{3}{5}, \frac{6}{10}, \frac{9}{15}, \ldots\} \rightarrow (a)$ $\frac{6}{10}$

$-\frac{1}{10} \rightarrow \{\frac{1}{10}, \frac{2}{20}, \frac{3}{30}, \ldots\} \rightarrow (b)$ $\frac{1}{10}$

$\frac{5}{10}$ $(d) \leftarrow (c)$ $\frac{5}{10}$

$\frac{2}{16}$ $\frac{3}{24}$

C $\frac{1}{2} \rightarrow \{\frac{1}{2}, (a), (b), \ldots\} \rightarrow (c)$ $\frac{3}{6}$

$+\frac{2}{3} \rightarrow \{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \ldots\} \rightarrow (d)$ $\frac{4}{6}$

$\frac{7}{6}$ $(f) \leftarrow (e)$ $\frac{7}{6}$

2. Make your own lists of equivalent fractions until you find two with the same denominator. Then find the sum or difference.

A $\frac{1}{4} + \frac{1}{10}$ $\frac{7}{20}$ **B** $\frac{3}{4} - \frac{1}{2}$ $\frac{1}{4}$ **C** $\frac{5}{6} - \frac{3}{8}$ $\frac{11}{24}$ **D** $\frac{1}{2} + \frac{3}{10}$ $\frac{8}{10}$

E $\frac{1}{4} - \frac{1}{10}$ $\frac{3}{20}$ **F** $\frac{1}{4} + \frac{1}{5}$ $\frac{9}{20}$ **G** $\frac{3}{4} + \frac{2}{5}$ $\frac{23}{20}$ **H** $\frac{3}{4} - \frac{1}{10}$ $\frac{13}{20}$

I $\frac{3}{4} - \frac{1}{3}$ $\frac{5}{12}$ **J** $\frac{3}{5} + \frac{1}{10}$ $\frac{7}{10}$ **K** $\frac{2}{5} + \frac{1}{3}$ $\frac{11}{15}$ **L** $\frac{1}{3} + \frac{1}{4}$ $\frac{7}{12}$ **M** $\frac{2}{3} - \frac{1}{4}$ $\frac{5}{12}$ **N** $\frac{1}{4} + \frac{1}{5}$ $\frac{9}{20}$

O $\frac{63}{100} + \frac{1}{5}$ $\frac{83}{100}$ **P** $\frac{11}{50} - \frac{1}{10}$ $\frac{1}{50}$ **Q** $\frac{3}{2} + \frac{1}{4}$ $\frac{7}{4}$ **R** $\frac{6}{5} - \frac{3}{10}$ $\frac{9}{10}$ **S** $\frac{1}{3} + \frac{7}{10}$ $\frac{31}{30}$ **T** $\frac{83}{100} + \frac{1}{2}$ $\frac{133}{100}$

3. Copy each exercise on your paper and give the numbers for ***a*** and ***b***. Then find the sum or difference of the two numbers.

A $\frac{1}{2} + \frac{1}{3} \rightarrow$ 3 $\frac{a}{6} + \frac{b}{6}$ 2 $\quad \frac{5}{6}$

B $\frac{2}{3} + \frac{1}{6} \rightarrow$ 4 $\frac{a}{6} + \frac{b}{6}$ 1 $\quad \frac{5}{6}$

C $\frac{1}{8} + \frac{1}{4} \rightarrow$ 1 $\frac{a}{8} + \frac{b}{8}$ 2 $\quad \frac{3}{8}$

D $\frac{1}{4} - \frac{1}{6} \rightarrow$ 3 $\frac{a}{12} - \frac{b}{12}$ 2 $\quad \frac{1}{12}$

E $\frac{5}{8} - \frac{1}{6} \rightarrow$ 15 $\frac{a}{24} - \frac{b}{24}$ 4 $\quad \frac{11}{24}$

F $\frac{8}{15} - \frac{1}{5} \rightarrow$ 8 $\frac{a}{15} - \frac{b}{15}$ 3 $\quad \frac{5}{15}$

think

Find fractional numbers for ***a*** and ***b*** so that

$a + b = 1$

and

$a - b = \frac{1}{2}$.

$a = \frac{3}{4}$, $b = \frac{1}{4}$

More practice, page A-22, Set 42

Using the Exercises

For exercise 1 on page 231, note with the children that it is important that they give the missing fractions in the order indicated by the shaded letters. For example, in exercise 1B, when subtracting $\frac{1}{10}$ from $\frac{3}{5}$, they should first give the fraction $\frac{6}{10}$ for a, then the fraction $\frac{1}{10}$ for b, then the difference $\frac{5}{10}$ for c, and, finally, the difference $\frac{5}{10}$ for d. Encourage the children to write their work for exercises 2 and 3 in the forms illustrated in exercise 1.

Assignments (page 231)
Minimum: 1–2N. Average: 1–3.
Maximum: 1–3.

Duplicator Masters, page 48
Workbook, page 77
Skill Masters, page 48

Objective

Given two fractional numbers expressed with unlike denominators, the child will be able to find the sums or differences after choosing the least common multiple to use as the least common denominator.

Preparation

To prepare for this lesson, review building sets of equivalent fractions and finding the least common denominators, as studied in the previous lesson. For example, ask the children to list the sets of equivalent fractions for $\frac{2}{5}$ and $\frac{1}{3}$ and to choose the fractions which have the same denominator.

● *What is the least common denominator for two fractions?*

Discussing the Ideas

1. Find each sum, A, B, and C.

$\frac{3}{4} \rightarrow \{\frac{3}{4}, \frac{6}{8}, \boxed{\frac{9}{12}}, \frac{12}{16}, \frac{15}{20}, \boxed{\frac{18}{24}}, \frac{21}{28}, \frac{24}{32}, \boxed{\frac{27}{36}}, \ldots\}$

$+\frac{1}{6} \rightarrow \{\frac{1}{6}, \boxed{\frac{2}{12}}, \frac{3}{18}, \boxed{\frac{4}{24}}, \frac{5}{30}, \boxed{\frac{6}{36}}, \frac{7}{42}, \frac{8}{48}, \frac{9}{54}, \ldots\}$

A $\frac{9}{12} + \frac{2}{12} = ▥$ $\frac{11}{12}$

B $\frac{18}{24} + \frac{4}{24} = ▥$ $\frac{22}{24}$

C $\frac{27}{36} + \frac{6}{36} = ▥$ $\frac{33}{36}$

2. Do you think each sum you found is the sum of $\frac{3}{4}$ and $\frac{1}{6}$? Yes

3. Which of the additions would you be most likely to use to find $\frac{3}{4} + \frac{1}{6}$? $\frac{9}{12} + \frac{2}{12}$

4. The denominator 12 is the least common denominator for $\frac{3}{4}$ and $\frac{1}{6}$. You can find the least common denominator for $\frac{3}{4}$ and $\frac{1}{6}$ without writing out the sets of equivalent fractions.

> When you find the **least common multiple** of **4** and **6**, you have found the **least common denominator** for $\frac{3}{4}$ and $\frac{1}{6}$.

Why would the least common denominator for $\frac{1}{4}$ and $\frac{5}{6}$ also be 12?
12 is the least common multiple of the denominators 4 and 6.

5. A What is the least common multiple of 5 and 3? 15

B What is the least common denominator for $\frac{1}{5}$ and $\frac{1}{3}$? 15

6. Give two fractions that have the denominator you found in exercise 5B and could be used to find the sum $\frac{1}{5} + \frac{1}{3}$. $\frac{3}{15}$, $\frac{5}{15}$

7. Find the sum. $\frac{1}{5} + \frac{1}{3} = n$ $\frac{8}{15}$

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Discussion

After children have found the sums in exercise 1, and as you discuss exercises 2 and 3, stress that, although $\frac{11}{12}$, $\frac{22}{24}$, and $\frac{33}{36}$ are all correct representations of the same fractional number, the equation $\frac{9}{12} + \frac{2}{12} = \frac{11}{12}$ is the most likely to be used, because 12 is the *least* common denominator of $\frac{3}{4}$ and $\frac{1}{6}$. Emphasize with the children that the least common denominator for two fractions is the least common multiple of the two denominators. The children should see this easily if you list the multiples of 4 and 6 (4, 8, 12, 16, . . . and 6, 12, 18, 24, . . .) and observe that 12 is the least number in both lists, so 12 is the least common multiple. Carefully work through exercises 5 through 7, stressing the idea of least common denominator and its connection to least common multiple. Use other examples similarly to strengthen these ideas.

Using the Ideas

1. A What is the least common multiple of 2 and 5? 10
   B What is the least common denominator for $\frac{2}{5}$ and $\frac{1}{2}$? 10
   C Give the numerators. $\frac{2}{5} = \frac{\square}{10}$ 4 $\frac{1}{2} = \frac{\square}{10}$ 5
   D Find the sum. $\frac{4}{10} + \frac{5}{10} = a$ $\frac{9}{10}$ E Find the sum. $\frac{2}{5} + \frac{1}{2} = b$ $\frac{9}{10}$

2. A What is the least common multiple of 2 and 3? 6
   B What is the least common denominator for $\frac{1}{2}$ and $\frac{1}{3}$? 6
   C Give the numerators. $\frac{1}{2} = \frac{\square}{6}$ 3 $\frac{1}{3} = \frac{\square}{6}$ 2
   D Find the sum. $\frac{3}{6} + \frac{2}{6} = r$ $\frac{5}{6}$ E Find the sum. $\frac{1}{2} + \frac{1}{3} = s$ $\frac{5}{6}$

3. A What is the least common multiple of 8 and 4? 8
   B What is the least common denominator for $\frac{3}{8}$ and $\frac{1}{4}$? 8
   C Give the numerators. $\frac{3}{8} = \frac{\square}{8}$ 3 $\frac{1}{4} = \frac{\square}{8}$ 2
   D Find the sum. $\frac{3}{8} + \frac{2}{8} = c$ $\frac{5}{8}$ E Find the sum. $\frac{3}{8} + \frac{1}{4} = d$ $\frac{5}{8}$

4. A What is the least common multiple of 9 and 6? 18
   B What is the least common denominator for $\frac{4}{9}$ and $\frac{1}{6}$? 18
   C Give the numerators. $\frac{4}{9} = \frac{\square}{18}$ 8 $\frac{1}{6} = \frac{\square}{18}$ 3
   D Find the difference. $\frac{8}{18} - \frac{3}{18} = m$ $\frac{5}{18}$
   E Find the difference. $\frac{4}{9} - \frac{1}{6} = n$ $\frac{5}{18}$

★ 5. Find the sums and differences.

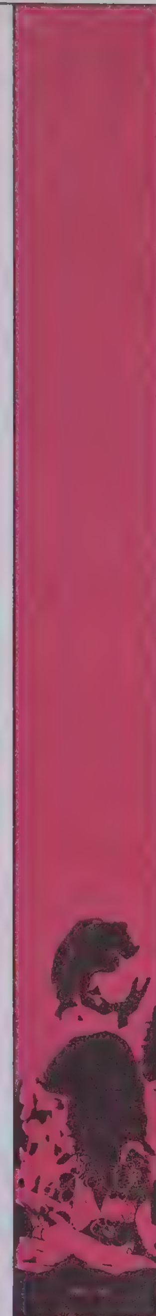
A $\frac{1}{3} + \frac{1}{4}$ $\frac{7}{12}$	G $\frac{5}{8} + \frac{1}{4}$ $\frac{7}{8}$
B $\frac{1}{3} - \frac{1}{4}$ $\frac{1}{12}$	H $\frac{5}{8} - \frac{1}{4}$ $\frac{3}{8}$
C $\frac{2}{5} - \frac{1}{4}$ $\frac{3}{20}$	I $\frac{7}{8} - \frac{1}{4}$ $\frac{5}{8}$
D $\frac{1}{2} + \frac{2}{3}$ $\frac{7}{6}$	J $\frac{2}{3} + \frac{1}{6}$ $\frac{5}{6}$
E $\frac{2}{3} - \frac{1}{2}$ $\frac{1}{6}$	K $\frac{2}{3} - \frac{1}{6}$ $\frac{3}{6}$
F $\frac{7}{6} - \frac{1}{2}$ $\frac{4}{6}$	L $\frac{7}{10} - \frac{1}{6}$ $\frac{16}{30}$

think

Arrange 24 matches or sticks into 9 small squares as in the figure.

1. Remove 8 matches so you have 5 small squares left.
2. Try forming 5 small squares by removing only 4 matches.

See Sample Solutions, T.E. page 233.



Sample solutions, Think, page 233
Remove the sticks indicated by the dashed lines

1. 

2. 

Workbook, page 78

Using the Exercises
Have the children do the exercises on page 233. When they have finished, allow time for discussion and checking papers. It might be helpful to discuss portions of the five exercises with some groups. You might have the children read the problems one at a time and provide time for any questions which the other children may have after the correct answer is given.

Allow the children considerable freedom in the methods they use for completing exercise 5. You might encourage the faster children to solve these problems by finding the least common denominator and then finding the two fractions for the two numbers that have this denominator.

Supply sticks or toothpicks so that children can make a model of the pattern shown in the *Think* problem. Give them plenty of time to manipulate the toothpicks before any solutions are given.

Assignments (page 233)
Minimum: 1–3. Average: 1–5F. Maximum: 1–5.

PAGES 234–235

Objective

Given two fractional numbers, the child will be able to find the sum or difference by using the least common denominator.

Preparation

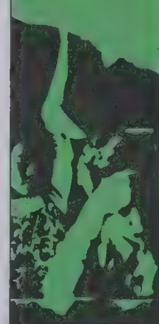
To prepare for this lesson, you might ask the children whether any of them have seen a computer. Encourage them to share whatever they may know about computers. Then explain that computers can be programmed with instructions to perform various operations and that the persons who write such instructions often use *flow charts* to put the instructions in order. In the present lesson, the children will follow a set of instructions or a flow chart for adding fractions.

Investigation

It would be appropriate for children to work together on this investigation. They may use any of a variety of methods for finding the least common denominator. Encourage them to discuss which method is best, for example, building sets of equivalent fractions or listing common multiples. Also, encourage them to make up a rule by which they can find the numerator of a fraction, given the common denominator. During the discussion section, you will probably want to suggest the technique of thinking of missing factors, such as, since $4 \times 3 = 12$, 3×3 will give the new numerator, 9.

Stimulate discussion in a group by asking key questions like these: "What number times the 'old' denominator will give the 'new' denominator?" "If you've multiplied the old denominator by 2, what must you multiply the old numerator by?" Challenge the children with other sums also:

$\frac{3}{5} + \frac{1}{10}$; $\frac{2}{3} + \frac{4}{5}$; $\frac{1}{2} + \frac{5}{7}$.



Is there a shortcut for adding and subtracting?

Investigating the Ideas

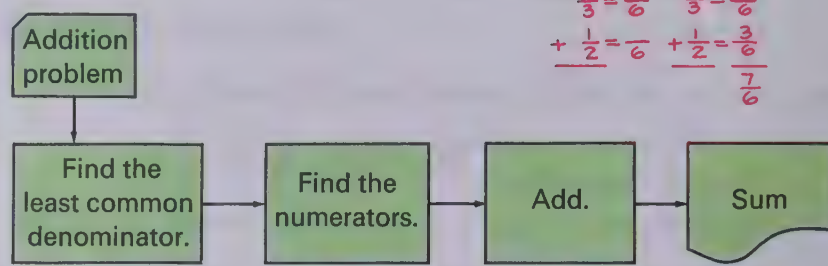
Study the flow chart for finding $\frac{3}{4} + \frac{1}{6}$.

To find $\frac{2}{3} + \frac{1}{2}$:

$\frac{2}{3} = \frac{}{6}$ $\quad$ $\frac{2}{3} = \frac{4}{6}$

$+\frac{1}{2} = \frac{}{6}$ $\quad$ $+\frac{1}{2} = \frac{3}{6}$

$\frac{7}{6}$



$\frac{3}{4} = \frac{}{12}$, $+\frac{1}{6} = \frac{}{12}$ $\quad$ $\frac{3}{4} = \frac{9}{12}$, $+\frac{1}{6} = \frac{2}{12}$ $\quad$ $\frac{3}{4} = \frac{9}{12}$, $+\frac{1}{6} = \frac{2}{12}$ $\quad$ $\frac{3}{4} = \frac{9}{12}$, $+\frac{1}{6} = \frac{2}{12}$, $\frac{11}{12}$

? Can you follow the flow chart to find the sum of $\frac{2}{3}$ and $\frac{1}{2}$? See above and Investigation.

Discussing the Ideas

Explain each step in the examples below. See Discussion.
Then find the sum or difference.

A

$\frac{5}{8}$ $+\frac{1}{6}$

Think: The least common denominator is 24.

Write: $\frac{5}{8} = \frac{}{24}$ $+\frac{1}{6} = \frac{}{24}$

Think: $\frac{5}{8} = \frac{15}{24}$ $\quad \frac{1}{6} = \frac{4}{24}$

Write: $\frac{5}{8} = \frac{15}{24}$ $+\frac{1}{6} = \frac{4}{24}$ $\quad \frac{19}{24}$

B

$\frac{5}{4} - \frac{1}{6}$

Think: The least common denominator is 12.

Write: $\frac{5}{4} - \frac{1}{6}$ $\quad \frac{}{12} - \frac{}{12}$

Think: $\frac{5}{4} = \frac{15}{12}$ $\quad \frac{1}{6} = \frac{2}{12}$

Write: $\frac{5}{4} - \frac{1}{6}$ $\quad \frac{15}{12} - \frac{2}{12} = $ $\frac{13}{12}$

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Discussion

Have children explain the steps they used to find the sum $\frac{2}{3} + \frac{1}{2}$. Then discuss the best procedures as you work through charts A and B. Note that A and B differ in written form. It is not essential that you teach both methods, but each may be convenient in a given situation, depending on the form in which the problem is presented. The main idea to stress is the use of the least common multiple as the least common denominator. However, make sure children develop a technique for finding the numerator for the fraction of a least common denominator. For example, with

$$\frac{5}{8} = \frac{?}{24},$$

they may think: "What number times 8 will give 24? Since $3 \times 8 = 24$, we must multiply 3×5 to get the new numerator, 15." Similarly, with

$$\frac{1}{6} = \frac{?}{12},$$

they may think: "What number times 6 will give 12? Since $2 \times 6 = 12$, then 2×1, or 2, will be the new numerator." Remind the children that they can always use the cross-product check for equivalent fractions.

Resources for Active Learning

Discovery, Section II, Unit 5/6, 7, Encyclopaedia Britannica Educational Corp.

Duplicator Masters, page 49
Workbook, page 79
Skill Masters, page 49

Using the Ideas

1. Find the least common multiple of the numbers shown in red. This is the least common denominator of the two fractions.

A $\frac{1}{2}$ $\frac{1}{3}$ 6 C $\frac{1}{8}$ $\frac{1}{4}$ 8 E $\frac{3}{8}$ $\frac{1}{2}$ 8 G $\frac{5}{8}$ $\frac{7}{12}$ 20 I $\frac{2}{3}$ $\frac{3}{7}$ 21
B $\frac{1}{6}$ $\frac{5}{9}$ 18 D $\frac{3}{4}$ $\frac{5}{6}$ 12 F $\frac{1}{12}$ $\frac{1}{4}$ 20 H $\frac{11}{12}$ $\frac{1}{6}$ 20 J $\frac{1}{5}$ $\frac{7}{10}$ 10

2. Give the missing numerators.

A $\frac{1}{3} = \frac{\square}{6}$ 2 C $\frac{5}{10} = \frac{\square}{30}$ 15 E $\frac{1}{4} = \frac{\square}{100}$ 25 G $\frac{1}{2} = \frac{\square}{8}$ 4 I $\frac{1}{3} = \frac{\square}{21}$ 7 K $\frac{7}{10} = \frac{\square}{100}$ 70
B $\frac{1}{2} = \frac{\square}{6}$ 3 D $\frac{5}{8} = \frac{\square}{8}$ 5 F $\frac{7}{8} = \frac{\square}{8}$ 7 H $\frac{3}{5} = \frac{\square}{20}$ 12 J $\frac{1}{7} = \frac{\square}{21}$ 3 L $\frac{21}{50} = \frac{\square}{100}$ 42

3. Find the sums and differences.

A $\frac{1}{3} + \frac{1}{2}$ $\frac{5}{6}$ B $\frac{1}{2} + \frac{1}{4}$ $\frac{3}{4}$ C $\frac{1}{2} - \frac{1}{3}$ $\frac{1}{6}$ D $\frac{1}{2} - \frac{1}{4}$ $\frac{1}{4}$ E $\frac{2}{5} - \frac{1}{10}$ $\frac{3}{10}$ F $\frac{2}{3} - \frac{1}{4}$ $\frac{5}{12}$ G $\frac{5}{7} + \frac{1}{4}$ $\frac{27}{28}$

H $\frac{3}{2} + \frac{1}{3}$ $\frac{11}{6}$ I $\frac{3}{2} - \frac{1}{3}$ $\frac{7}{6}$ J $\frac{7}{10} - \frac{1}{2}$ $\frac{2}{10}$ K $\frac{7}{10} + \frac{1}{2}$ $\frac{12}{10}$ L $\frac{3}{2} - \frac{1}{4}$ $\frac{5}{4}$ M $\frac{3}{4} + \frac{2}{5}$ $\frac{23}{20}$ N $\frac{3}{4} - \frac{2}{5}$ $\frac{7}{20}$

4. Find the sums and differences.

A $\frac{1}{3} + \frac{1}{6}$ $\frac{3}{6}$ D $\frac{1}{3} - \frac{1}{6}$ $\frac{1}{6}$ G $\frac{1}{3} + \frac{1}{7}$ $\frac{10}{21}$ J $\frac{1}{3} - \frac{1}{7}$ $\frac{4}{21}$ M $\frac{7}{10} - \frac{3}{5}$ $\frac{1}{10}$
B $\frac{1}{8} + \frac{1}{4}$ $\frac{3}{8}$ E $\frac{1}{4} - \frac{1}{8}$ $\frac{1}{8}$ H $\frac{1}{5} + \frac{7}{10}$ $\frac{9}{10}$ K $\frac{1}{2} - \frac{1}{10}$ $\frac{4}{10}$ N $\frac{9}{10} - \frac{25}{100}$ $\frac{65}{100}$
C $\frac{2}{3} + \frac{1}{6}$ $\frac{5}{6}$ F $\frac{2}{3} - \frac{1}{6}$ $\frac{3}{6}$ I $\frac{7}{10} - \frac{5}{100}$ $\frac{65}{100}$ L $\frac{5}{10} + \frac{1}{5}$ $\frac{7}{10}$ O $\frac{3}{10} + \frac{18}{100}$ $\frac{48}{100}$

Short Stories

1 Walked $\frac{7}{10}$ km. Ran $\frac{1}{2}$ km
How far in all? $\frac{12}{10}$ km

2 $\frac{3}{10}$ centimetre of rain.
Another $\frac{1}{4}$ centimetre of rain. How much rain? $\frac{11}{20}$ cm

3 First box: $\frac{2}{3}$ full.
Second box: $\frac{3}{4}$ full.
How much more is in the second box? $\frac{1}{12}$

4 Joe ate $\frac{1}{2}$ of the candy. Jim ate $\frac{1}{3}$ of it. How much was left for Tom? $\frac{1}{6}$

5 Jack's step: $\frac{7}{10}$ metre. Sue's step: $\frac{3}{5}$ metre.
A How much longer is Jack's step? $\frac{1}{10}$ m
B How long are two of Jack's steps? $\frac{14}{10}$ m

More practice, page A-23, Set 43

Using the Exercises
Use some parts of the exercises on page 235 for further discussion, or assign them as independent work. When the children have finished, allow time for discussion and checking papers.

Assignments (page 235)
Minimum: 1–2, oral; 3.
Average: 1–2, oral; 3–4.
Maximum: 1–2, oral; 3–4; Short Stories 1–5.

PAGES 236–237

Objectives

Given a fractional representation of a whole number, the child will be able to name the whole number.

Given a whole number and a chosen denominator, the child will be able to express the whole number as a fraction with that denominator.

Preparation

Materials

slips of paper approximately 10 by 15 centimetres, or paper from which children can cut such slips

You may wish to review a few addition problems, or changing fractions to equivalent fractions in higher terms, before beginning the investigation.

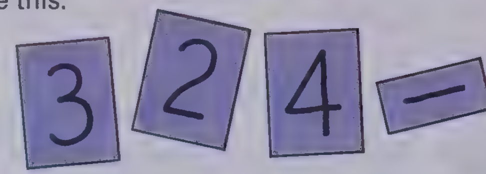
Investigation

Direct the children to work independently on this investigation. Note that the bar line may be thought of either as a symbol for subtraction or as a bar line in a fraction. Also, point out that the slip of paper for the bar line is of different shape and size than for the digits. Remind the children that they are to record each numeral or number expression which they find. You might point out that they may make expressions using either two or three of the digit symbols. Thus, $4 - 3$, $2\frac{3}{4}$, etc., are all acceptable.

What are some other names for fractional numbers?

Investigating the Ideas

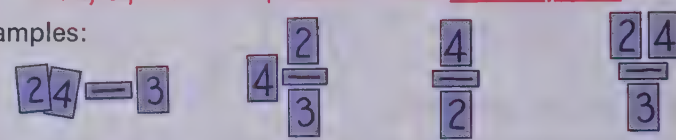
Cut out 4 slips of paper and label them like this.



? How many different number symbols can you make with these slips?	Record your findings.

Many symbols are possible. See Investigation.

Examples:



Discussing the Ideas

1. Numerals such as $4\frac{2}{3}$ and $5\frac{3}{8}$ are called mixed numerals
   $4\frac{2}{3}$ means $4 + \frac{2}{3}$. Which of your numerals in the Investigation were mixed numerals? Answers will vary. See Discussion.

2. Fractions such as $\frac{4}{2}$ and $\frac{5}{1}$ are names for whole numbers. What whole numbers do these numerals name? 2, 5

3. Were any of your fraction symbols in the Investigation names for whole numbers? Answers will vary.

4. Give the whole number for each set of equivalent fractions.

   A $\{\frac{3}{1}, \frac{6}{2}, \frac{9}{3}, \ldots\}$ 3　C $\{\frac{10}{1}, \frac{20}{2}, \frac{30}{3}, \ldots\}$ 10　E $\{\frac{9}{1}, \frac{18}{2}, \frac{27}{3}, \ldots\}$ 9

   B $\{\frac{6}{1}, \frac{12}{2}, \frac{18}{3}, \ldots\}$ 6　D $\{\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \ldots\}$ 1　F $\{\frac{0}{1}, \frac{0}{2}, \frac{0}{3}, \ldots\}$ 0

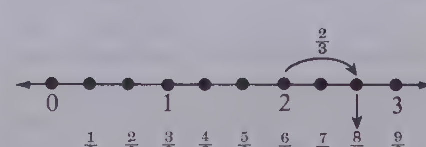
236

Discussion

Use several examples which children found in the investigation to teach them how to read mixed numerals. The mixed numeral $5\frac{3}{8}$ means $5 + \frac{3}{8}$; we say, "five and three eighths."

For discussion exercises 2 and 3, give children an opportunity to determine which whole numbers the fractions represent, before giving any particular rule for determining this whole number. At this stage, do not point out the fact that they can find the whole number by dividing numerator by denominator. Give them a chance to discover this from the other exercises.

One of the most important points to stress throughout your entire discussion is that a mixed numeral is a symbol for a fractional number. Although the symbol, a whole number combined with a fraction, may be new, the numbers are the same; the children are still working with fractional numbers. It would be helpful to use a number line to show that numerals such as $\frac{8}{3}$ and $2\frac{2}{3}$ represent the same fractional number:



Using the Ideas

1. Each of these fractions names a fractional number that you can think of as a whole number. Give the whole number for each fraction.

A $\frac{2}{1}$ 2 B $\frac{8}{2}$ 4 C $\frac{3}{1}$ 3 D $\frac{14}{2}$ 7 E $\frac{20}{2}$ 10 F $\frac{15}{3}$ 5 G $\frac{12}{3}$ 4 H $\frac{6}{3}$ 2

I $\frac{6}{1}$ 6 J $\frac{24}{2}$ 12 K $\frac{24}{3}$ 8 L $\frac{18}{3}$ 6 M $\frac{0}{7}$ 0 N $\frac{0}{1}$ 0 O $\frac{1}{1}$ 1 P $\frac{7}{7}$ 1

2. Each of these sums is a whole number. Give the whole number for each sum.

A $\frac{2}{3}+\frac{4}{3}$ 2 C $\frac{2}{3}+\frac{1}{3}$ 1 E $\frac{7}{5}+\frac{8}{5}$ 3 G $\frac{10}{4}+\frac{2}{4}$ 3 H $\frac{7}{6}+\frac{11}{6}$ 3 I $\frac{10}{8}+\frac{6}{8}$ 2

B $\frac{5}{2}+\frac{3}{2}$ 4 D $\frac{1}{2}+\frac{1}{2}$ 1 F $\frac{7}{2}+\frac{9}{2}$ 8

3. Give the correct mixed numeral for each sum.

A $1+\frac{1}{3}$ $1\frac{1}{3}$ C $1+\frac{1}{5}$ $1\frac{1}{5}$ E $4+\frac{2}{3}$ $4\frac{2}{3}$ G $5+\frac{1}{4}$ $5\frac{1}{4}$ I $3+\frac{1}{10}$ $3\frac{1}{10}$ K $1+\frac{1}{2}$ $1\frac{1}{2}$

B $\frac{3}{3}+\frac{1}{3}$ $1\frac{1}{3}$ D $\frac{5}{5}+\frac{1}{5}$ $1\frac{1}{5}$ F $\frac{12}{3}+\frac{2}{3}$ $4\frac{2}{3}$ H $\frac{20}{4}+\frac{1}{4}$ $5\frac{1}{4}$ J $\frac{30}{10}+\frac{1}{10}$ $3\frac{1}{10}$ L $\frac{100}{100}+\frac{3}{100}$ $1\frac{3}{100}$

4. Give the correct whole number for n.

A $1=\frac{n}{2}$ 2 C $2=\frac{n}{2}$ 4 E $3=\frac{n}{1}$ 3 G $3=\frac{n}{7}$ 21 I $6=\frac{n}{2}$ 12

B $1=\frac{n}{5}$ 5 D $2=\frac{n}{3}$ 6 F $3=\frac{n}{2}$ 6 H $4=\frac{n}{3}$ 12 J $7=\frac{n}{5}$ 35

5. In each exercise, give the correct mixed numeral for n.

$2\frac{1}{4}$ A Since $\frac{8}{4}=2$, we know that $\frac{9}{4}=n$.
$3\frac{1}{2}$ B Since $\frac{6}{2}=3$, we know that $\frac{7}{2}=n$.
$1\frac{2}{5}$ C Since $\frac{5}{5}=1$, we know that $\frac{7}{5}=n$.
$5\frac{1}{2}$ D Since $\frac{10}{2}=5$, we know that $\frac{11}{2}=n$.
$4\frac{2}{3}$ E Since $\frac{12}{3}=4$, we know that $\frac{14}{3}=n$.
$3\frac{3}{4}$ F Since $\frac{12}{4}=3$, we know that $\frac{15}{4}=n$.
$6\frac{1}{3}$ G Since $\frac{18}{3}=6$, we know that $\frac{19}{3}=n$.
$2\frac{9}{10}$ H Since $\frac{20}{10}=2$, we know that $\frac{29}{10}=n$.
$4\frac{3}{4}$ I Since $\frac{16}{4}=4$, we know that $\frac{19}{4}=n$.

think

Find the whole numbers for a and b that make the sentences true.

a=4 b=8

$\frac{a}{8}=\frac{2}{a}$ $\frac{b}{16}=\frac{4}{b}$

237

Mathematics

In the set of fractional numbers, there is a subset of numbers that can be represented by fractions which have a numerator that is a multiple of the denominator (e.g., the fractions $\frac{0}{2}$, $\frac{9}{3}$, $\frac{12}{3}$, $\frac{5}{1}$, $\frac{24}{3}$, $\frac{16}{2}$, $\frac{2}{1}$, and so on). We have introduced addition, and will in a later chapter introduce multiplication, in such a way that we can show that this set of numbers is arithmetically identical to the set of whole numbers. Because of this, we take the point of view that each whole number is also a fractional number—that the whole numbers constitute a subset of the fractional numbers. This, of course, is brought out when we make such statements as $2=\frac{6}{3}$, $4=\frac{12}{3}$, $5=\frac{10}{2}$, $7=\frac{7}{1}$, and so on.

Note that $\frac{8}{3}$, which is called an improper fraction, represents a particular fractional number. Note also that $2\frac{2}{3}$, which is called a mixed numeral, represents the same fractional number. We bring out this point simply to observe that both $\frac{8}{3}$ and $2\frac{2}{3}$ represent the same fractional number; only the notation, or symbols, are different. It should be noted, therefore, that the terms *improper fraction* and *mixed numeral* refer to the symbols we write for the number and not to the number itself. Although, in traditional arithmetic, the term *mixed number* was used to refer to the symbol that represents the fractional number, we choose in this program to speak of *mixed numerals* rather than of mixed numbers. The number is the same whether an improper fraction or a mixed numeral is used to name it.

Workbook, page 80

Using the Exercises

Have the children do the exercises on page 237 on their own. Then use the exercises as a basis for discussion. Have children share the rules they developed for changing a fractional representation of a whole number to the whole-number symbol. The process of changing a mixed numeral to an improper fraction will be treated in the next lesson.

By using the cross-product check, the children should see that a doubles combination with product 16 will complete the first part of the *Think* problem. A doubles with product 64 completes the second.

Assignments (page 237)

Minimum: 1, oral; 2–3.
Average: 1–4. Maximum: 1–5.

PAGES 238–239

Objectives

Given a mixed numeral, the child will be able to write an improper fraction for it.

Given an improper fraction, the child will be able to write a mixed numeral for it.

Preparation

To prepare for this lesson, review fractional representation of whole numbers. For example, write $\frac{10}{2}$, $\frac{12}{3}$, $\frac{9}{3}$, $\frac{15}{5}$, $\frac{14}{7}$, $\frac{24}{8}$ on the chalkboard, and ask children to give the whole number each fraction represents. If time permits, you might also review mixed numerals as sums. For example, write $1 + \frac{1}{3}$, $2 + \frac{3}{4}$, $1 + \frac{4}{5}$, $4 + \frac{1}{2}$ on the chalkboard and ask children to name and write the mixed numeral for each sum.

● *Let's change fractional number names.*

Discussing the Ideas

1. Give the whole number for a. Then give the whole number for b.

A $1\frac{1}{2} = \frac{a}{2} + \frac{1}{2} \rightarrow 1\frac{1}{2} = \frac{b}{2}$ 2,3   C $5\frac{1}{4} = \frac{a}{4} + \frac{1}{4} \rightarrow 5\frac{1}{4} = \frac{b}{4}$ 20,21

B $2\frac{1}{3} = \frac{a}{3} + \frac{1}{3} \rightarrow 2\frac{1}{3} = \frac{b}{3}$ 6,7   D $6\frac{1}{2} = \frac{a}{2} + \frac{1}{2} \rightarrow 6\frac{1}{2} = \frac{b}{2}$ 12,13

2. Study this flow chart: **Mixed Numerals to Improper Fractions**

Any mixed numeral	→	Multiply whole number by the denominator.	→	Add the numerator.	→	Write the sum over denominator.
↓		↓		↓		↓
$2\frac{3}{4}$		$2 \times 4 = 8$		$8 + 3 = 11$		$\frac{11}{4}$

Now find improper fractions for the following mixed numerals.

A $3\frac{1}{5}$ $\frac{16}{5}$ B $7\frac{1}{2}$ $\frac{15}{2}$ C $3\frac{2}{3}$ $\frac{11}{3}$ D $4\frac{1}{4}$ $\frac{17}{4}$ E $8\frac{7}{10}$ $\frac{87}{10}$ F $9\frac{3}{4}$ $\frac{39}{4}$

3. Give the whole number for a and the whole number for b. Then write a mixed numeral for each improper fraction.

A $\frac{5}{2} = \frac{a}{2} + \frac{1}{2} \rightarrow \frac{5}{2} = b + \frac{1}{2} \rightarrow \frac{5}{2} = \square$

Answer: $a = 4, b = 2, \frac{5}{2} = 2\frac{1}{2}$

B $\frac{10}{3} = \frac{a}{3} + \frac{1}{3} \rightarrow \frac{10}{3} = b + \frac{1}{3} \rightarrow \frac{10}{3} = \square$ 9 3 $3\frac{1}{3}$

C $\frac{15}{4} = \frac{a}{4} + \frac{3}{4} \rightarrow \frac{15}{4} = b + \frac{3}{4} \rightarrow \frac{15}{4} = \square$ 12 3 $3\frac{3}{4}$

4. Study this flow chart: **Improper Fractions to Mixed Numerals**

Any improper fraction	→	Divide numerator by denominator.	→	Use quotient as whole number part.	→	Write remainder over denominator.
↓		↓		↓		↓
$\frac{17}{5}$		$5\overline{)17}$ = 3 R2		$\frac{17}{5} = 3\frac{\square}{\square}$		$\frac{17}{5} = 3\frac{2}{5}$

Now find mixed numerals for the following improper fractions.

A $\frac{9}{2}$ $4\frac{1}{2}$ B $\frac{13}{4}$ $3\frac{1}{4}$ C $\frac{22}{7}$ $3\frac{1}{7}$ D $\frac{33}{10}$ $3\frac{3}{10}$ E $\frac{250}{100}$ $2\frac{50}{100}$ or $2\frac{1}{2}$ F $\frac{43}{5}$ $8\frac{3}{5}$ G $\frac{29}{3}$ $9\frac{2}{3}$

238

Discussion

This lesson extends development of the power skills treated in the previous lesson and introduces the algorithm for the speed-skill of writing improper fractions for mixed numerals, and vice versa. Discussion exercises 1 and 3 concentrate on building understanding. Each whole number is seen as a certain number of the fractional parts named by the denominator. Stress for the children that in exercise 1A, for example, they are writing the number 1 as $\frac{2}{2}$ and then observing that $1\frac{1}{2} = \frac{2}{2} + \frac{1}{2}$, or $\frac{3}{2}$; in part B, they are writing the number 2 as $\frac{6}{3}$ and, finally, observing that $2\frac{1}{3} = \frac{7}{3}$. Then, work through several examples using the speed-skill algorithm illustrated by the flow chart in exercise 2.

In exercise 3, point out that the children must first find out how many whole numbers or units are in the fractional parts and then express the remaining fractional parts as a fraction. Thus, in 3B, $\frac{10}{3}$ contains $\frac{9}{3}$ or 3 whole numbers, plus $\frac{1}{3}$, so $\frac{10}{3} = 3\frac{1}{3}$.

The flow chart in exercise 4 shows the steps of the algorithm for finding mixed numerals for improper fractions. Give children an opportunity to work through the examples and discuss them.

Using the Ideas

1. Write an improper fraction for each mixed numeral.
   A $1\frac{2}{3}$ $\frac{5}{3}$ B $3\frac{4}{5}$ $\frac{19}{5}$ C $2\frac{1}{2}$ $\frac{5}{2}$ D $1\frac{3}{10}$ $\frac{13}{10}$ E $3\frac{4}{5}$ $\frac{19}{5}$ F $4\frac{3}{4}$ $\frac{19}{4}$ G $5\frac{1}{8}$ $\frac{41}{8}$
   H $2\frac{9}{10}$ $\frac{29}{10}$ I $6\frac{7}{10}$ $\frac{67}{10}$ J $1\frac{75}{100}$ $\frac{175}{100}$ K $4\frac{1}{10}$ $\frac{41}{10}$ L $7\frac{2}{5}$ $\frac{37}{5}$ M $6\frac{50}{100}$ $\frac{650}{100}$ N $9\frac{7}{10}$ $\frac{97}{10}$

2. Write a mixed numeral for each improper fraction.
   A $\frac{10}{3}$ $3\frac{1}{3}$ B $\frac{26}{3}$ $8\frac{2}{3}$ C $\frac{34}{10}$ $3\frac{4}{10}$ D $\frac{12}{7}$ $1\frac{5}{7}$ E $\frac{15}{4}$ $3\frac{3}{4}$ F $\frac{13}{4}$ $3\frac{1}{4}$ G $\frac{8}{3}$ $2\frac{2}{3}$
   H $\frac{5}{2}$ $2\frac{1}{2}$ I $\frac{17}{4}$ $4\frac{1}{4}$ J $\frac{27}{5}$ $5\frac{2}{5}$ K $\frac{16}{10}$ $1\frac{6}{10}$ L $\frac{11}{10}$ $1\frac{1}{10}$ M $\frac{115}{100}$ $1\frac{15}{100}$ N $\frac{264}{100}$ $2\frac{64}{100}$

Give your answer to each exercise as a mixed numeral.

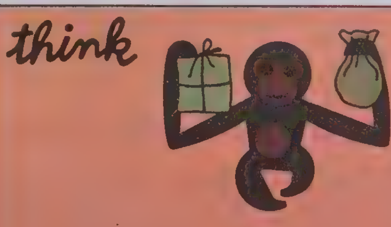
3. Janet and Rita walked to their grandfather's house. It took them $\frac{3}{4}$ of an hour to get there and $\frac{1}{2}$ hour to get back home.
   A How much time did they spend going both ways? $1\frac{1}{4}$ h
   B If they spent $\frac{1}{3}$ of an hour visiting, how long were they gone? $1\frac{7}{12}$ h

4. If butter and margarine come in $\frac{1}{4}$-kilogram sticks
   A How much would 10 sticks of butter weigh? $2\frac{1}{2}$ kg
   B How much would 17 sticks of butter weigh? $4\frac{1}{4}$ kg

5. A If a car race is 14 laps around a $\frac{1}{4}$-km track, how many km long is the race? $3\frac{1}{2}$ km
   B How many km long is a race that is 5 laps around a $\frac{1}{2}$-km track? $2\frac{1}{2}$ km

6. One bunch of grapes weighs $\frac{3}{10}$ of a kilogram. Add another, and the scale shows $1\frac{3}{4}$ kg. How much does the second bunch weigh? $1\frac{9}{20}$ kg

think



A boy spends $\frac{1}{2}$ of his money in a store. Then he goes to another store and spends $\frac{1}{2}$ of what is left. After that he has 24 cents. With how much did he start?

More practice, page A-24, Set 44

239

Using the Exercises

You might have the children work on the exercises on page 239 independently or allow them to work with one another in small groups. In particular, children might benefit from sharing ideas about the word problems. In checking papers with the children, do not insist that their answers be expressed in one specific form. For example, if an answer to a problem is $\frac{8}{3}$, the children might well write it as $2\frac{2}{3}$ or as $\frac{8}{3}$. Even an answer such as $\frac{16}{6}$ in place of $\frac{8}{3}$ would be satisfactory; however, children should realize why the form $2\frac{2}{3}$ is preferred.

Assignments (page 239)
Minimum: 1–3. Average: 1–5.
Maximum: 1–6.

Follow-up

Combo, a game similar to Bingo, will help the children review fractions and mixed numerals. The object of the game is to cover five answers in any row, column, or diagonal as the caller shows addition or subtraction problems on flash cards and calls them out. Winners call "Combo" and verify the answers from the caller's master sheet.

To make the game, cut 10-by-12-cm cards and rule 30 two-centimetre boxes on them. Fill in "Combo" and "Free" in the proper squares, and then fractions and mixed numbers at random (see illustration). However, make sure the answers correspond to the problems on the flash cards. The players should have cork or paper discs to cover answers.

C	O	M	B	O
$4\frac{3}{4}$	$1\frac{1}{2}$	$\frac{3}{5}$	$\frac{11}{16}$	$\frac{4}{3}$
$\frac{9}{2}$	$\frac{2}{2}$	$2\frac{4}{5}$	$4\frac{1}{4}$	$\frac{1}{8}$
$\frac{1}{3}$	$\frac{12}{6}$	FREE	$3\frac{5}{8}$	$1\frac{2}{5}$
$1\frac{5}{6}$	$\frac{4}{5}$	$3\frac{1}{2}$	$\frac{8}{5}$	$\frac{3}{4}$
$\frac{1}{2}$	$2\frac{1}{4}$	$\frac{4}{5}$	$\frac{9}{3}$	$5\frac{1}{8}$

Duplicator Masters, page 50
Workbook, page 81
Skill Masters, page 50

PAGES 240–241

Objective

Given an addition problem involving mixed numerals, the child will find the sum by applying the basic principles for fractional numbers.

Preparation

Materials

colored strips

To prepare for this lesson, review the basic principles for addition of whole numbers. For example, write sample equations on the chalkboard:

$23 + 15 = 15 + 23$ *(commutative)*
$42 + 0 = 42$ *(zero principle)*
$31 + (42 + 27) = (31 + 42) + 27$ *(associative)*

Then ask children if they remember the principle each illustrates.

You might also want to review lengths of the strips when the purple strip is the unit, particularly for the white, the red, and the brown strips.

Investigation

Although children might not be aware of this fact until you bring it out in the discussion, they are investigating the addition, not only of four addends, but also of two mixed numerals, $1\frac{1}{4} + 2\frac{1}{2}$. Manipulation with the strips should help them see that, just as with whole numbers, rearranging the addends does not change the sum. There are 24 possible arrangements of the four addends taken separately. However, some children may combine some of these to get other equations. For example, $1 + \frac{1}{4} + 2 + \frac{1}{2}$ might be thought of as $(1 + \frac{1}{4}) + (2 + \frac{1}{2})$ or as $1\frac{1}{4} + 2\frac{1}{2}$. And $1 + 2 + \frac{1}{4} + \frac{1}{2}$ might be thought of as $(1 + 2) + (\frac{1}{4} + \frac{1}{2})$ or as $3 + \frac{3}{4}$.

● *Can the basic principles be used for fractional numbers?*

Investigating the Ideas

How long (in purple units) will the "train" be if you place these strips end-to-end?

? How many addition equations can you write by using all the strips above?

Sample answers:
$1\frac{1}{4} + 2\frac{1}{2} = 3\frac{3}{4}$
$\frac{1}{2} + \frac{1}{4} + 3 = 3\frac{3}{4}$
See Investigation.

Discussing the Ideas

1. Does the length of your train depend upon the order in which you place your strips? No

2. What basic principle does this equation show? Commutative +

$$\tfrac{1}{5} + \tfrac{3}{5} = \tfrac{3}{5} + \tfrac{1}{5}$$

3. What principle does this equation show? Associative +

$$(\tfrac{1}{10} + \tfrac{3}{10}) + \tfrac{5}{10} = \tfrac{1}{10} + (\tfrac{3}{10} + \tfrac{5}{10})$$

4. Can you state a principle illustrated by this equation?

$$\tfrac{2}{3} + 0 = \tfrac{2}{3}$$

The sum of any number and zero is the number itself.

5. Explain the steps in each example. Give the sum.

A $2\frac{1}{4} + 1\frac{1}{2} = (2 + 1) + (\frac{1}{4} + \frac{1}{2}) = 3 + \frac{3}{4} =$ ▥ $3\frac{3}{4}$

B $4\frac{1}{4} + 1\frac{1}{3} = (4 + 1) + (\frac{3}{12} + \frac{4}{12}) = 5 + \frac{7}{12} =$ ▥ $5\frac{7}{12}$

C
$$\begin{array}{rcl} 4\frac{1}{2} & = & 4\frac{3}{6} \\ +7\frac{1}{3} & = & 7\frac{2}{6} \\ \hline & & ▥\ 11\frac{5}{6} \end{array}$$

See Discussion.

Discussion

One of the main points of this lesson is that the order and the grouping of addends may be rearranged in any manner without changing the sum. Use a discussion of the investigation to extend this idea. Help children see that different trains of the strips give different equations but that these equations have the same sum. Then stress how the commutative and associative principles, treated in exercises 2 and 3, allow addends to be rearranged without changing the sum. Exercise 4 illustrates that the zero principle applies to fractional numbers as well as whole numbers.

Develop exercise 5 carefully; it is directly related to the investigation. You might explain to the children how we separate the fraction and the whole number and rearrange them:

$$2\tfrac{1}{4} + 1\tfrac{1}{2} = 2 + (\tfrac{1}{4} + 1) + \tfrac{1}{2} = (2 + 1) + (\tfrac{1}{4} + \tfrac{1}{2})$$

Use exercise 5C to illustrate how this arrangement principle allows the convenient vertical notation: the fractions are added first, then the whole numbers.

1. Copy each exercise and give the numbers for ***a***, ***b***, and ***c***.

A $1\frac{1}{7} + 5\frac{3}{7} = (\boldsymbol{a} + \boldsymbol{b}) + (\frac{1}{7} + \frac{3}{7}) = 6 + \frac{4}{7} = \boldsymbol{c}$ $a=1, b=5, c=6\frac{4}{7}$

B $5\frac{1}{2} + 3\frac{1}{4} = (5 + \boldsymbol{a}) + (\frac{2}{4} + \boldsymbol{b}) = 8 + \boldsymbol{c} = 8\frac{3}{4}$ $a=3, b=\frac{1}{4}, c=\frac{3}{4}$

C $6\frac{1}{5} + 1\frac{7}{10} = (6 + 1) + (\boldsymbol{a} + \frac{7}{10}) = \boldsymbol{b} + \frac{9}{10} = \boldsymbol{c}$ $a=\frac{2}{10}, b=7, c=7\frac{9}{10}$

2. Copy and complete each exercise.

A $\begin{array}{r} 2\frac{1}{3} = 2\frac{\square}{6} \;2 \\ +5\frac{1}{6} = 5\frac{\square}{6} \;1 \\ \hline 7\frac{\square}{6} \;3 \end{array}$	B $\begin{array}{r} 8\frac{1}{3} = 8\frac{\square}{12} \;4 \\ +7\frac{1}{4} = 7\frac{\square}{12} \;3 \\ \hline 15\frac{\square}{12} \;7 \end{array}$	C $\begin{array}{r} 2\frac{1}{3} = 2\frac{\square}{6} \;2 \\ +5\frac{1}{2} = 5\frac{\square}{6} \;3 \\ \hline 7\frac{\square}{6} \;5 \end{array}$
D $\begin{array}{r} 3\frac{1}{4} = 3\frac{\square}{4} \;1 \\ +9\frac{1}{2} = 9\frac{\square}{4} \;2 \\ \hline 12\frac{\square}{4} \;3 \end{array}$	E $\begin{array}{r} 5\frac{1}{8} = 5\frac{\square}{24} \;3 \\ +7\frac{1}{6} = 7\frac{\square}{24} \;4 \\ \hline 12\frac{\square}{24} \;7 \end{array}$	F $\begin{array}{r} 3\frac{2}{3} = 3\frac{\square}{12} \;8 \\ +1\frac{1}{4} = 1\frac{\square}{12} \;3 \\ \hline 4\frac{\square}{12} \;11 \end{array}$

3. Find the sums.

A $\begin{array}{r} 6\frac{1}{5} \\ +1\frac{7}{10} \\ \hline 7\frac{9}{10} \end{array}$	B $\begin{array}{r} 5\frac{1}{2} \\ +3\frac{3}{10} \\ \hline 8\frac{4}{5} \end{array}$	C $\begin{array}{r} 1\frac{1}{7} \\ +5\frac{3}{7} \\ \hline 6\frac{4}{7} \end{array}$	D $\begin{array}{r} 5\frac{1}{2} \\ +2\frac{1}{3} \\ \hline 7\frac{5}{6} \end{array}$	E $\begin{array}{r} 8\frac{1}{3} \\ +7\frac{1}{4} \\ \hline 15\frac{7}{12} \end{array}$	F $\begin{array}{r} 3\frac{1}{2} \\ +4\frac{1}{4} \\ \hline 7\frac{3}{4} \end{array}$	G $\begin{array}{r} 6\frac{3}{4} \\ +2\frac{1}{8} \\ \hline 8\frac{7}{8} \end{array}$
H $\begin{array}{r} 5\frac{3}{10} \\ +3\frac{23}{50} \\ \hline 8\frac{19}{25} \end{array}$	I $\begin{array}{r} 8\frac{7}{10} \\ +1\frac{1}{5} \\ \hline 9\frac{9}{10} \end{array}$	J $\begin{array}{r} 7\frac{1}{5} \\ +8\frac{1}{4} \\ \hline 15\frac{9}{20} \end{array}$	K $\begin{array}{r} 9\frac{1}{2} \\ +\frac{1}{5} \\ \hline 9\frac{7}{10} \end{array}$	L $\begin{array}{r} \frac{1}{10} \\ +6\frac{1}{5} \\ \hline 6\frac{3}{10} \end{array}$	M $\begin{array}{r} 9\frac{7}{10} \\ +8\frac{2}{30} \\ \hline 17\frac{23}{30} \end{array}$	N $\begin{array}{r} 6\frac{3}{5} \\ +7\frac{1}{10} \\ \hline 13\frac{7}{10} \end{array}$

★ **4.** Solve the equations.

A $3\frac{1}{2} + 5\frac{3}{4} = 9 + n$ $\frac{1}{4}$

B $6\frac{5}{8} + 7\frac{3}{4} = 14 + n$ $\frac{3}{8}$

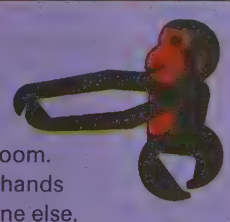
C $9\frac{2}{3} + 7\frac{1}{2} = n + \frac{1}{6}$ 17

D $3\frac{7}{8} + 1\frac{3}{4} = 5 + n$ $\frac{5}{8}$

E $9\frac{5}{6} + 7\frac{1}{4} = n + \frac{1}{12}$ 17

F $15\frac{7}{10} + 29\frac{2}{3} = 45 + n$ $\frac{11}{30}$

think



Five people in a room.
Everyone shakes hands
once with everyone else.
How many handshakes? 10
See Using the Exercises.

More practice, page A-24, Set 45

Using the Exercises
You might want to work through parts of the exercises on page 241 together with some groups of children. For example, have volunteers explain how they would complete exercise 1. In exercise 2, the least common denominator is provided so that children may concentrate their attention on the addition process with vertical notation. But in the remaining exercises they must work the problem completely. Children need not express their answers as lowest-terms fractions. Concentrate on the task of finding sums using mixed numerals, not on the form in which the sum is written.

Children might enjoy working out the *Think* problem by forming groups of five and recording the number of handshakes. As long as we consider a handshake between person A and person B the same as between B and A, the following explanation holds.

A shakes hands with B, C, D, E.
B shakes hands with C, D, E. (He has already shaken hands with A.)
C shakes hands with D, E.
D shakes hands with E.
Total handshakes: 10.

Assignments (page 241)
Minimum: 1–2. Average: 1–3.
Maximum: 1–4.

Workbook, page 82

PAGES 242–243

Objective

Given a mixed numeral, such as $4\frac{9}{6}$, the child will be able to rename the numeral as $5\frac{1}{2}$.

Preparation

To prepare for this lesson, have the children practice writing mixed numerals for improper fractions and improper fractions for mixed numerals. You might also review addition of mixed numerals in which the fractional part of the answer represents a fractional number less than one.

Changing Numerals

1. Give the missing numerators.

A $\frac{5}{3} = \frac{3}{3} + \frac{▯}{3}$ 2
E $\frac{9}{7} = \frac{7}{7} + \frac{▯}{7}$ 2
I $\frac{3}{2} = \frac{2}{2} + \frac{▯}{2}$ 1
M $\frac{11}{10} = \frac{10}{10} + \frac{▯}{10}$ 1

B $\frac{6}{5} = \frac{5}{5} + \frac{▯}{5}$ 1
F $\frac{12}{7} = \frac{7}{7} + \frac{▯}{7}$ 5
J $\frac{10}{8} = \frac{8}{8} + \frac{▯}{8}$ 2
N $\frac{15}{10} = \frac{10}{10} + \frac{▯}{10}$ 5

C $\frac{7}{4} = \frac{4}{4} + \frac{▯}{4}$ 3
G $\frac{17}{10} = \frac{10}{10} + \frac{▯}{10}$ 7
K $\frac{6}{4} = \frac{4}{4} + \frac{▯}{4}$ 2
O $\frac{16}{10} = \frac{10}{10} + \frac{▯}{10}$ 6

D $\frac{8}{5} = \frac{5}{5} + \frac{▯}{5}$ 3
H $\frac{12}{10} = \frac{10}{10} + \frac{▯}{10}$ 2
L $\frac{10}{6} = \frac{6}{6} + \frac{▯}{6}$ 4
P $\frac{120}{100} = \frac{100}{100} + \frac{▯}{100}$ 20

2. Give the missing numerators.

A $\frac{7}{4} = 1 + \frac{▯}{4}$ 3
E $\frac{14}{10} = 1 + \frac{▯}{5}$ 2
I $\frac{13}{10} = 1 + \frac{▯}{10}$ 3
M $\frac{9}{7} = 1 + \frac{▯}{7}$ 2

B $\frac{17}{10} = 1 + \frac{▯}{10}$ 7
F $\frac{8}{6} = 1 + \frac{▯}{3}$ 1
J $\frac{6}{4} = 1 + \frac{▯}{2}$ 1
N $\frac{5}{3} = 1 + \frac{▯}{3}$ 2

C $\frac{9}{6} = 1 + \frac{▯}{2}$ 1
G $\frac{8}{5} = 1 + \frac{▯}{5}$ 3
K $\frac{15}{10} = 1 + \frac{▯}{2}$ 1
O $\frac{12}{7} = 1 + \frac{▯}{7}$ 5

D $\frac{16}{10} = 1 + \frac{▯}{5}$ 3
H $\frac{6}{5} = 1 + \frac{▯}{5}$ 1
L $\frac{9}{8} = 1 + \frac{▯}{8}$ 1
P $\frac{10}{8} = 1 + \frac{▯}{4}$ 1

3. Solve the equations. (All fractions should be in lowest terms.)

A $3\frac{9}{6} = 4 + n$ $\frac{1}{2}$
F $4\frac{15}{10} = 5 + n$ $\frac{1}{2}$
K $2\frac{10}{6} = n + \frac{2}{3}$ 3
P $33\frac{7}{4} = n + \frac{3}{4}$ 34

B $5\frac{8}{5} = n + \frac{3}{5}$ 6
G $16\frac{9}{5} = n + \frac{4}{5}$ 17
L $1\frac{10}{8} = 2 + n$ $\frac{1}{4}$
Q $41\frac{14}{10} = 42 + n$ $\frac{2}{5}$

C $7\frac{15}{10} = 8 + n$ $\frac{1}{2}$
H $7\frac{17}{10} = 8 + n$ $\frac{7}{10}$
M $4\frac{9}{7} = 5 + n$ $\frac{2}{7}$
R $27\frac{13}{10} = 28 + n$ $\frac{3}{10}$

D $9\frac{12}{7} = n + \frac{5}{7}$ 10
I $8\frac{16}{10} = 9 + n$ $\frac{3}{5}$
N $6\frac{6}{4} = n + \frac{1}{2}$ 7
S $84\frac{9}{8} = n + \frac{1}{8}$ 85

E $15\frac{5}{3} = 16 + n$ $\frac{2}{3}$
J $3\frac{6}{5} = 4 + n$ $\frac{1}{5}$
O $19\frac{8}{6} = n + \frac{1}{3}$ 20
T $65\frac{12}{8} = 66 + n$ $\frac{1}{2}$

4. Solve the equations. (All fractions should be in lowest terms.)

A $\frac{9}{5} = 1 + a$ $\frac{4}{5}$
F $75\frac{6}{5} = 76 + a$ $\frac{1}{5}$

B $2\frac{7}{6} = a + \frac{1}{6}$ 3
G $56\frac{3}{2} = 57 + a$ $\frac{1}{2}$

C $4\frac{6}{4} = 5 + a$ $\frac{1}{2}$
H $83\frac{11}{10} = 84 + a$ $\frac{1}{10}$

D $2\frac{10}{3} = a + \frac{1}{3}$ 5
I $96\frac{12}{10} = 97 + a$ $\frac{1}{5}$

E $5\frac{13}{10} = 6 + a$ $\frac{3}{10}$
J $38\frac{7}{4} = 39 + a$ $\frac{3}{4}$

think

a=? b=?

Find the whole numbers for *a* and *b* that make the sentences true.

$\frac{a}{3} = \frac{4}{a+4}$ $a = 2$

$\frac{b}{4} = \frac{9}{b+9}$ $b = 3$

242

Discussion

Although the exercises on page 242 are straightforward and children might work on them independently, they follow a progressive development, which should be pointed out to the children. You might work through a few examples from each exercise. Be sure to help the children see that they are learning to simplify certain types of mixed numerals – those whose fractional part represents a fractional number greater than one. Also, they are strengthening their skill in writing improper fractions as mixed numerals.

Thus, this page is actually a readiness page for regrouping in addition using mixed numerals. It could easily be used as a whole day's lesson.

Assignments (page 242)

Minimum: 1A–H, 2A–H, 3A–J.
Average: 1, oral; 2A–L; 3A–O.
Maximum: 1–2, oral; 3–4, alternate parts.

Adding and Renaming

1. Give each sum in simplest form.

A $\frac{1}{2} = \frac{2}{4}$	**B** $7\frac{5}{8} = 7\frac{15}{24}$	**C** $4\frac{1}{2} = 4\frac{3}{6}$
$+\frac{3}{4} = \frac{3}{4}$	$+9\frac{2}{3} = 9\frac{16}{24}$	$+2\frac{5}{6} = 2\frac{5}{6}$
$\frac{5}{4} = ▮$ $1\frac{1}{4}$	$16\frac{31}{24} = ▮$ $17\frac{7}{24}$	$6\frac{8}{6} = 7\frac{2}{6} = ▮$ $7\frac{1}{3}$

2. Find the sums. Use mixed numerals for your answers.

A	**B**	**C**	**D**	**E**	**F**	**G**
$\frac{2}{3}$	$\frac{5}{6}$	$\frac{7}{10}$	$\frac{7}{10}$	$\frac{7}{10}$	$\frac{7}{8}$	$\frac{3}{10}$
$+\frac{2}{3}$	$+\frac{2}{3}$	$+\frac{3}{5}$	$+\frac{2}{3}$	$+\frac{5}{4}$	$+\frac{1}{4}$	$+\frac{4}{5}$
$1\frac{1}{3}$	$1\frac{1}{2}$	$1\frac{3}{10}$	$1\frac{11}{30}$	$1\frac{19}{20}$	$1\frac{1}{8}$	$1\frac{1}{10}$

3. Find the sums. Use mixed numerals for your answers.

A	**B**	**C**	**D**	**E**	**F**
$7\frac{2}{3}$	$6\frac{5}{6}$	$4\frac{7}{10}$	$6\frac{7}{10}$	$19\frac{7}{10}$	$27\frac{7}{10}$
$+8\frac{2}{3}$	$+9\frac{2}{3}$	$+8\frac{4}{5}$	$+7\frac{2}{3}$	$+2\frac{3}{4}$	$+8\frac{3}{5}$
$16\frac{1}{3}$	$16\frac{1}{2}$	$13\frac{1}{2}$	$14\frac{11}{30}$	$22\frac{9}{20}$	$36\frac{3}{10}$
G	**H**	**I**	**J**	**K**	**L**
$17\frac{7}{8}$	$54\frac{3}{10}$	$85\frac{1}{2}$	$37\frac{3}{5}$	$86\frac{7}{10}$	$92\frac{3}{5}$
$+26\frac{1}{4}$	$+37\frac{4}{5}$	$+61\frac{5}{6}$	$+84\frac{3}{4}$	$+25\frac{11}{20}$	$+88\frac{5}{7}$
$44\frac{1}{8}$	$92\frac{1}{10}$	$147\frac{1}{3}$	$122\frac{7}{20}$	$112\frac{1}{4}$	$181\frac{11}{35}$

4. Find the sums. Use mixed numerals for your answers.

A	**B**	**C**	**D**	**E**	**F**
$5\frac{5}{8}$	$8\frac{1}{8}$	$9\frac{4}{5}$	$13\frac{9}{10}$	$27\frac{3}{4}$	$36\frac{95}{100}$
$+6\frac{3}{4}$	$+7\frac{9}{10}$	$+2\frac{13}{15}$	$+6\frac{1}{2}$	$+8\frac{5}{7}$	$+28\frac{1}{5}$
$12\frac{3}{8}$	$16\frac{1}{40}$	$12\frac{2}{3}$	$20\frac{2}{5}$	$36\frac{13}{28}$	$65\frac{3}{20}$

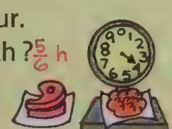
Short Stories

1 Live $1\frac{9}{10}$ km from school.
Walked both ways.
Walked how far? $3\frac{4}{5}$ km

2 Normal body temperature: 37 °C.
Caught the flu: up $1\frac{1}{2}$ C°.
What was the temperature? $38\frac{1}{2}$ °C

3 Height in June: $124\frac{1}{2}$ cm.
Grew $3\frac{2}{10}$ cm during summer.
How tall in September? $127\frac{7}{10}$ cm

4 Music lesson: $\frac{1}{3}$ hour.
Homework: $\frac{1}{2}$ hour.
How long for both? $\frac{5}{6}$ h



5 Pork chops: $2\frac{3}{4}$ kilograms.
Hamburger: $3\frac{1}{2}$ kilograms.
How much is total weight? $6\frac{1}{4}$ kg

6 Recipe: Need $1\frac{7}{10}$ litres milk. Double batch. How much milk for both? $3\frac{2}{5}$ ℓ

More practice, page A-25, Set 46

243

Using the Exercises

Exercise 1 on page 243 provides examples for demonstration and discussion.

Be sure the children see the relationship between these problems and the ones they worked on page 242. Certainly, they should understand the simplifications involved here if they were able to work the exercises on page 242.

When the children have finished the exercises, allow time for discussion, particularly of their solutions to the short stories.

Page 243 might well constitute an entire day's lesson. However, the only difference between these problems and those in previous lessons is in renaming the sums.

Assignments (page 243)

Minimum: 1–2, Short Stories 1–3.
Average: 1–3, Short Stories 1–4.
Maximum: 1–4, Short Stories 1–6.

Duplicator Masters, page 51
Workbook, page 83

PAGES 244–245

Objective

Given subtraction problems with mixed numerals, in which renaming is necessary, the child will be able to find the differences.

Preparation

To prepare for this lesson, give children practice with renaming mixed numerals similar to those in exercise 1. For example, ask them to find the new numerators for $7\frac{1}{2} = 7\frac{2}{4} = 6\frac{6}{4}$; $6\frac{1}{6} = 6\frac{2}{12} = 5\frac{14}{12}$; $8\frac{1}{3} = 8\frac{3}{9} = 7\frac{12}{9}$. For some groups you may find it helpful to show the following extra steps:

$$7\tfrac{1}{2} = 7\tfrac{2}{4} = 6 + 1 + \tfrac{2}{4} = 6 + \tfrac{4}{4} + \tfrac{2}{4} = 6\tfrac{6}{4}$$

$$6\tfrac{1}{6} = 6\tfrac{2}{12} = 5 + 1 + \tfrac{2}{12} = 5 + \tfrac{12}{12} + \tfrac{2}{12} = 5\tfrac{14}{12}$$

$$8\tfrac{1}{3} = 8\tfrac{3}{9} = 7 + 1 + \tfrac{3}{9} = 7 + \tfrac{9}{9} + \tfrac{3}{9} = 7\tfrac{12}{9}$$

However, if children understood the process on page 242, they should not have much difficulty with the concepts presented in this lesson.

Subtracting with Renaming

1. Some of the numerators are covered by screens.
Copy each exercise on your paper and give the missing numerators.

A $6\frac{1}{2} = 6\frac{\square}{4} = 5\frac{6}{4}$ 2	E $9\frac{1}{2} = 9\frac{5}{10} = 8\frac{\square}{10}$ 15	I $4\frac{3}{10} = 4\frac{\square}{100} = 3\frac{\square}{100}$ 30,130
B $7\frac{1}{4} = 7\frac{5}{20} = 6\frac{\square}{20}$ 25	F $7\frac{1}{4} = 7\frac{\square}{8} = 6\frac{\square}{8}$ 2,10	J $2\frac{1}{2} = 2\frac{\square}{4} = 1\frac{\square}{4}$ 2,6
C $3\frac{1}{3} = 3\frac{3}{9} = 2\frac{\square}{9}$ 12	G $8\frac{3}{5} = 8\frac{\square}{10} = 7\frac{\square}{10}$ 6,16	K $5\frac{1}{2} = 5\frac{\square}{6} = 4\frac{\square}{6}$ 3,9
D $5 = 5\frac{0}{8} = 4\frac{\square}{8}$ 8	H $1 = 1\frac{\square}{10} = \frac{\square}{10}$ 0,10	L $3\frac{3}{5} = 3\frac{\square}{15} = 2\frac{\square}{15}$ 9,24

Study these examples.

$\boxed{A}$
$$\begin{array}{r} 7\frac{1}{8} = 7\frac{1}{8} = 6\frac{9}{8} \\ \underline{-2\frac{1}{4}} = \underline{2\frac{2}{8}} = \underline{2\frac{2}{8}} \\ 4\frac{7}{8} \end{array}$$

$\boxed{B}$
$$\begin{array}{r} 4\frac{1}{2} = 4\frac{3}{6} = 3\frac{9}{6} \\ \underline{-1\frac{5}{6}} = \underline{1\frac{5}{6}} = \underline{1\frac{5}{6}} \\ 2\frac{4}{6} = 2\frac{2}{3} \end{array}$$

2. Copy and complete each exercise.

A
$$\begin{array}{r} 6\frac{1}{2} = 6\frac{2}{4} = 5\frac{\square}{4} \quad 6 \\ \underline{-1\frac{3}{4}} = \underline{1\frac{3}{4}} = \underline{1\frac{\square}{4}} \quad 3 \\ 4\frac{\square}{4} \quad 3 \end{array}$$

B
$$\begin{array}{r} 7\frac{1}{4} = 7\frac{3}{12} = 6\frac{\square}{\square} \quad \frac{15}{12} \\ \underline{-1\frac{1}{3}} = \underline{1\frac{4}{12}} = \underline{1\frac{\square}{\square}} \quad \frac{4}{12} \\ 5\frac{\square}{\square} \quad \frac{11}{12} \end{array}$$

C
$$\begin{array}{r} 4\frac{1}{3} = 4\frac{3}{9} = \square\frac{\square}{9} \quad 3\frac{12}{9} \\ \underline{-2\frac{5}{9}} = \underline{2\frac{5}{9}} = \underline{\square\frac{\square}{9}} \quad 2\frac{5}{9} \\ \square\frac{\square}{9} \quad 1\frac{7}{9} \end{array}$$

3. Find the differences. Give the differences in lowest terms.

A $8\frac{1}{4} - 2\frac{1}{2}$ = $5\frac{3}{4}$	B $9\frac{2}{5} - 6\frac{7}{10}$ = $2\frac{7}{10}$	C $8\frac{1}{2} - 4\frac{3}{4}$ = $3\frac{3}{4}$	D $7\frac{3}{5} - 6\frac{2}{3}$ = $\frac{14}{15}$	E $19\frac{7}{10} - 2\frac{3}{4}$ = $16\frac{19}{20}$	F $28\frac{7}{10} - 7\frac{5}{8}$ = $21\frac{3}{40}$	G $26 - 17\frac{7}{8}$ = $8\frac{1}{8}$
H $54\frac{3}{10} - 37\frac{4}{5}$ = $16\frac{1}{2}$	I $6\frac{3}{10} - 5$ = $1\frac{3}{10}$	J $8\frac{3}{4} - 7\frac{4}{5}$ = $\frac{19}{20}$	K $9\frac{7}{10} - 2\frac{11}{15}$ = $6\frac{29}{30}$	L $13\frac{3}{5} - 6\frac{13}{15}$ = $6\frac{11}{15}$	M $85\frac{5}{8} - 61\frac{3}{4}$ = $23\frac{7}{8}$	N $84\frac{1}{8} - 37$ = $47\frac{1}{8}$
O $91\frac{1}{2} - 88\frac{3}{5}$ = $2\frac{9}{10}$	P $19\frac{2}{3} - 5\frac{9}{10}$ = $13\frac{23}{30}$	Q $36\frac{1}{3} - 29\frac{4}{5}$ = $6\frac{8}{15}$	R $83 - 42\frac{7}{8}$ = $40\frac{1}{8}$	S $61\frac{4}{5} - 39\frac{9}{10}$ = $21\frac{9}{10}$	T $86\frac{7}{15} - 59\frac{9}{10}$ = $26\frac{17}{30}$	U $93 - 39\frac{9}{10}$ = $53\frac{1}{10}$

You are invited to explore

ACTIVITY CARD 11
Page 338

244

Discussion

Parts A and B of exercise 1 should be used as a basis for discussion with most children. (You might let more capable children study these examples among themselves.) Explain that the first step is always to find a common denominator. Then the children must examine the fractions to see whether or not they need to regroup. Only then, if necessary, do they regroup the mixed numeral so that it will have enough fractional parts from which to subtract.

You will probably want to spend extra time working on both exercises 1 and 2 prior to having children complete exercise 3. Note how regrouping here is much like that done for whole numbers. Hence, there is really no new concept involved, just different notation.

Assignments (page 244)

Minimum: 1A–H, 2.

Average: 1–3G. Maximum: 1–3.

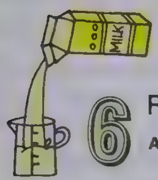
Short Stories

1 $6\frac{1}{2}$ km going. $9\frac{7}{10}$ km returning a different way. How far? $16\frac{1}{5}$ km

2 New candle: 20 cm. Burns $2\frac{1}{4}$ centimetres. How long now? $17\frac{3}{4}$ cm

3 Normal body temperature: 37 °C. Caught the flu: $39\frac{3}{4}$ °C. Temperature was how many degrees above normal? $2\frac{3}{4}$°

4 Average yearly rainfall: $35\frac{1}{2}$ cm. This year: $29\frac{3}{4}$ cm. How much is this below average? $5\frac{3}{4}$ cm



5 Jim: $34\frac{2}{10}$ kg. Joe: $36\frac{3}{4}$ kg. On the scales together. How many kg? $70\frac{19}{20}$ kg

6 Recipe A: $1\frac{1}{5}$ litres milk. Recipe B: $\frac{3}{4}$ litre milk.

A How much more milk for A than for B? $\frac{9}{20}$ ℓ

B How much milk is needed for both recipes? $1\frac{19}{20}$ ℓ

7 15 pies for the party. Eat $12\frac{5}{6}$ of the pies. How much pie is left? $2\frac{1}{6}$ pies

9 Triangular area: $17\frac{1}{2}$ units. Rectangular area: $15\frac{7}{8}$ units.

A How much smaller is the rectangle? $1\frac{5}{8}$ units

B What is the total area of the two figures together? $33\frac{3}{8}$ units

8 Number-line point A: $7\frac{5}{6}$. Point B is $3\frac{1}{3}$ units to the right of A. Where is B? $11\frac{1}{6}$

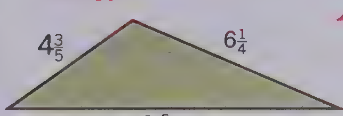
10 $6\frac{7}{12}$ dozen eggs. $5\frac{1}{2}$ dozen eggs. How many dozen eggs? $12\frac{1}{12}$ doz

11 Flight time: $5\frac{3}{4}$ hours. How long for two such trips? $11\frac{1}{2}$ h

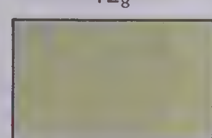
12 One box of Brand X: $376\frac{3}{4}$ grams. One box of Brand Y: $348\frac{1}{2}$ grams. How much more does a box of Brand X weigh than a box of Brand Y? $28\frac{1}{4}$ g

★ **13** Give the perimeter for each figure.

A $20\frac{7}{20}$



B $41\frac{1}{12}$



More practice, page A-25, Set 47

Using the Exercises

The problems on page 245 may be assigned as independent work or you may have children work together in groups to solve them. Once the situation in each problem is understood, it simply becomes an exercise in addition or subtraction with mixed numerals. Problem 7 may need special explanation:

$$\begin{array}{rcl} 15 & = & 14\frac{6}{6} \\ -\,12\frac{5}{6} & = & 12\frac{5}{6} \\ \hline & & 2\frac{1}{6} \end{array}$$

Assignments (page 245)

Minimum: 1–5.

Average: Even-numbered problems.

Maximum: 1–13.

Duplicator Masters, pages 52, 53

Workbook, pages 84, 85

Objective

The child will be able to use mixed numerals to measure lengths to the nearest fractional part of a centimetre and to add or subtract these measures.

Preparation

Materials

transparency of a clear plastic ruler suitable for use on an overhead projector

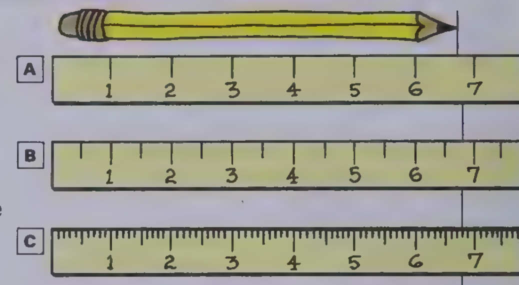
Unless you wish to review measurement as approximation or some other related topic with which the children have had difficulty, begin immediately with this investigation.

Investigation

The children have had many experiences in using centimetre rulers to determine the length of a given object, but, in this lesson, they are required to measure with a greater degree of precision. This investigation gives them an opportunity to study centimetre rulers with different scales and to compare measurements made to the nearest centimetre, 1/2 centimetre, and 1/10 centimetre. It would be appropriate for children to work in groups of two or three so that they can help each other make some measurements. Have available several objects which children can measure. They, of course, might measure books, parts of their desks or chairs, shelves, etc., but other objects such as dominoes, dice, egg cartons, a checkerboard, a playing card, aquarium, bulletin board, pictures, and so on might stimulate more interest. Direct the children to carefully name each object and record its length to the three specified degrees of precision. They might find it helpful to make a table on which they can record this information.

How can you use fractional numbers in measurement?

Investigating the Ideas

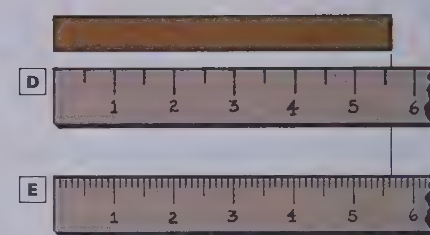


Using ruler A, the length of the pencil is closer to 7 centimetres than to 6 centimetres. The length of the pencil to the nearest centimetre is 7 centimetres. Using ruler B, the length of the pencil to the nearest half centimetre is still 7 centimetres. What does ruler C show? The length to the nearest tenth centimetre is $6\frac{8}{10}$ cm.

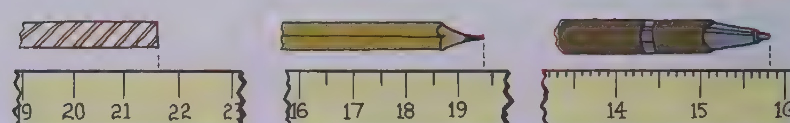
See Investigation.

? Can you measure some objects and record their lengths to the nearest cm, nearest $\frac{1}{2}$ cm, and nearest $\frac{1}{10}$ cm?

Discussing the Ideas



1. Use ruler D to explain how to find the length of the bar to the nearest half centimetre.
The $\frac{1}{2}$ cm marks show it is closer to $5\frac{1}{2}$ cm than to 6 cm.
2. Use ruler E to find the length of the bar to the nearest tenth of a centimetre. $5\frac{6}{10}$ cm
3. Give the length of each object, according to the directions.
A nearest centimetre 22 cm B nearest $\frac{1}{2}$ centimetre $19\frac{1}{2}$ cm C nearest $\frac{1}{10}$ centimetre $15\frac{8}{10}$ cm



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Discussion

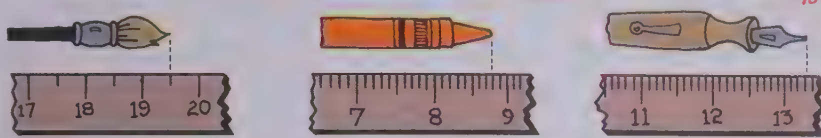
As you discuss the measurements children made in the investigation, point out that in each case the three measurements differ in degree of precision. We get closer to the actual length of an object when we measure it to the nearest tenth of a centimetre than when we measure it to the nearest centimetre. Continue this development in your discussion of exercises 1 and 2.

Explain that each type of measurement is useful for certain purposes. However, there are times when we want a more precise measurement. You might also mention to the children that for some measurements we use hundredths, thousandths, and even millionths of a centimetre in order to measure an object with great precision.

Use transparencies or clear plastic rulers on the overhead projector during your discussion, with examples to remind the children how the 1/2-cm mark on a ruler helps us to measure to the nearest centimetre, and so on. Continue to stress that the smaller the unit in which a measurement is given, the closer that measurement is to the actual length of an object.

Using the Ideas

1. Give the length of each object according to the directions.
   A nearest $\frac{1}{2}$ cm $19\frac{1}{2}$ cm B nearest $\frac{1}{10}$ cm $8\frac{8}{10}$ cm C nearest $\frac{1}{10}$ cm $13\frac{3}{10}$ cm



2. Find the length of the segment to the nearest $\frac{1}{2}$ centimetre.

   A ______________ $4\frac{1}{2}$ cm
   B ______________ 7 cm ($6\frac{1}{2}$ cm is also acceptable),

3. A Draw a segment that is as long as 2A and 2B together. (About 11.2 cm)
   B What is the sum of their lengths to the nearest $\frac{1}{10}$ centimetre? $11\frac{1}{2}$ cm
   (11 cm is also acceptable.)

4. Draw segments of the following lengths.

A $2\frac{1}{2}$ cm	C $3\frac{8}{10}$ cm	E $\frac{15}{10}$ cm	G $9\frac{1}{2}$ cm
B $7\frac{1}{2}$ cm	D 6 cm	F $4\frac{4}{10}$ cm	H $1\frac{5}{10}$ cm

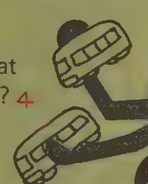
5. Use the segments in exercise 4. Find the sums and differences of each pair of lengths.
   A A and C $6\frac{3}{10}$, $1\frac{3}{10}$
   B B and G 17, 2
   C C and F $8\frac{2}{10}$, $\frac{6}{10}$
   D F and H $5\frac{9}{10}$, $2\frac{9}{10}$

6. Tell which is longer.
   A $\frac{3}{2}$ cm, $\frac{5}{10}$ cm
   B $\frac{7}{10}$ cm, $\frac{1}{2}$ cm
   C $1\frac{19}{10}$ cm, $1\frac{1}{2}$ cm
   D $\frac{21}{10}$ cm, 2 cm

think

There are 16 girls and 14 boys in Miss Gray's class. 20 of the children ride the bus.

1. Is it possible that all the girls ride the bus? Yes
2. What is the least number of girls that might ride the bus? 6
3. What is the least number of boys that might ride the bus? 4



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measurement is to the actual length of an object.

You might also point out how fractions in measurements need not be expressed in lowest terms if they indicate what the unit of measure is. For example, if a measure is reported as $2^5/_{10}$ cm, this may help to indicate that the measurement was made to the nearest $^1/_{10}$ centimetre.

Using the Exercises

As children work on the exercises on page 247, be prepared to help any who have difficulty using the smaller fractional units on the ruler. Remind them that the fractional part of the unit used should be stated ("... to the nearest ____") when they report the measure of an object. Stress that all measurement is approximate and that the fractional part of a unit is used to give measurements with greater precision.

Assignments (page 247)

Minimum: 1–2, 4. Average: 1–5. Maximum: 1–6.

Mathematics

The ideas of this lesson lay the groundwork for children to understand the ideas of precision and accuracy of measurement in their study of measurement in junior or senior high school.

The *precision* of a measurement depends upon the unit selected; of any two measurements of an object, the one which uses the smaller unit is the more precise. Precision is usually described in terms of the *greatest possible error* of a measure, which is defined as one half of the smallest unit used in the measurement process. Thus, if we measure an object to the nearest centimetre, the greatest possible error is $^1/_2$ centimetre; when we measure to the nearest $^1/_{10}$ centimetre the greatest possible error is $^1/_{20}$ centimetre, and so on.

Accuracy in measurement is a slightly more abstract concept and depends upon the *relative error* of the measurement. Relative error is defined to be the ratio of the greatest possible error to the measure of the object. Thus, the relative error made in measuring an object that is 5 centimetres long to the nearest centimetre is $^1/_2$ to 5, or $^1/_{10}$. If a 4-metre rug were measured to the nearest centimetre, the relative error would be small: $^1/_2$ to 400, or $^1/_{800}$. This small relative error denotes great *accuracy* even though the *precision* of the measurement is the same as that in the measurement of the 5-cm object.

Resources for Active Learning

Applied Mathematics Cards, Group 2/29, Schofield and Sims. (Available from Mafex Associates, Willowdale)

Objective

The child will demonstrate his ability to work with the concepts presented in this chapter.

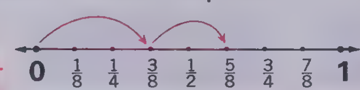
Preparation

Review any topics in this chapter with which the children have had special difficulty. In particular, review problems of the type found in exercise 5 on page 248, for these are perhaps the most difficult problems, with respect to the mechanics of operations, in the chapter. Give the children problems to work at the chalkboard, and have them explain the steps to each other.

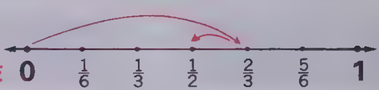
Reviewing the Ideas

1. Write the addition or subtraction equation suggested by each number-line picture.

A $\frac{3}{8}+\frac{2}{8}=\frac{5}{8}$ or $\frac{3}{8}+\frac{1}{4}=\frac{5}{8}$



B $\frac{2}{3}-\frac{1}{6}=\frac{1}{2}$



C $2\frac{1}{2}-\frac{3}{4}=1\frac{3}{4}$

2. Give the correct whole number for each exercise.

A $\frac{9}{3}$ 3 B $\frac{12}{3}$ 4 C $\frac{16}{2}$ 8 D $\frac{10}{6}+\frac{8}{6}$ 3 E $\frac{12}{5}+\frac{8}{5}$ 4

3. Solve the equations.

A $2+\frac{1}{3}=n$ $2\frac{1}{3}$ B $3+n=3\frac{3}{4}$ $\frac{3}{4}$ C $n+\frac{6}{7}=4\frac{6}{7}$ 4 D $5\frac{7}{8}=5+n$ $\frac{7}{8}$

4. Find the sums and differences. Give your answers in lowest terms.

A $\frac{2}{5}+\frac{1}{5}$ $\frac{3}{5}$ B $\frac{3}{7}-\frac{2}{7}$ $\frac{1}{7}$ C $\frac{3}{10}+\frac{7}{10}$ $\frac{1}{1}$ or 1 D $\frac{9}{10}-\frac{3}{10}$ $\frac{3}{5}$ E $\frac{5}{8}+\frac{1}{4}$ $\frac{7}{8}$ F $\frac{3}{4}-\frac{1}{2}$ $\frac{1}{4}$ G $\frac{5}{6}-\frac{1}{4}$ $\frac{7}{12}$

H $\frac{3}{10}+\frac{1}{10}$ I $\frac{8}{15}-\frac{2}{15}$ $\frac{2}{5}$ J $\frac{7}{20}+\frac{3}{20}$ $\frac{1}{2}$ K $\frac{7}{20}-\frac{3}{20}$ $\frac{1}{5}$

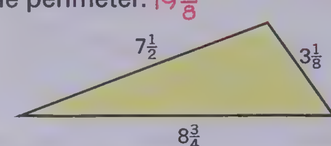
L $\frac{3}{8}+\frac{1}{3}$ $\frac{17}{24}$ M $\frac{3}{8}-\frac{1}{3}$ $\frac{1}{24}$ N $\frac{4}{5}-\frac{1}{3}$ $\frac{7}{15}$ O $\frac{9}{10}+\frac{1}{4}$ $\frac{23}{20}$ or $1\frac{3}{20}$

5. Find the sums and differences. Give your answers in lowest terms.

A $7\frac{1}{2}+3\frac{1}{4}$ $10\frac{3}{4}$ B $8\frac{1}{3}-4\frac{1}{6}$ $4\frac{1}{6}$ C $3\frac{7}{10}+9\frac{4}{5}$ $13\frac{1}{2}$

D $15\frac{4}{5}-8\frac{7}{10}$ $7\frac{1}{10}$ E $13\frac{3}{5}+9\frac{9}{10}$ $23\frac{1}{2}$ F $58\frac{1}{4}-12\frac{5}{6}$ $45\frac{5}{12}$

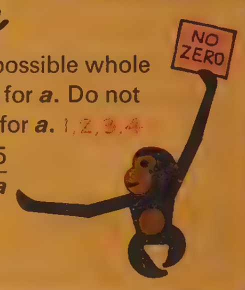
★ **6.** Give the perimeter. $19\frac{3}{8}$



think

Give all possible whole numbers for *a*. Do not use zero for *a*. 1, 2, 3, 4

$\frac{a}{5}<\frac{5}{a}$



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Discussion

If you choose to use page 248 as an evaluation instrument, have the children do the exercises, and then discuss the page with them after you have checked their work. If you prefer to use this page strictly as review, work through several problems together and stress concepts which seemed particularly difficult during study of the chapter. Remind children that although it is preferred that they use fractions with lowest terms in their answers, improper fractions (which are correct) are acceptable.

Orbital Speeds of the Planets

The table shows the symbols for the planets and the speed at which some of the planets go around the sun. The picture above shows the orbital paths of some of the planets around the sun. The sun is much larger than any of the planets. If you think of Jupiter, the largest planet, as being the size of a pea, then the sun would be about the size of a softball and Earth would be only a tiny speck.

Planet	Symbol	Approximate orbital speed (Km per second)
Mercury	☿	
Venus	♀	$34\frac{4}{5}$
Earth	⊕	$29\frac{3}{5}$
Mars	♂	24
Jupiter	♃	$12\frac{9}{10}$
Saturn	♄	$9\frac{3}{5}$
Uranus	⛢	$6\frac{3}{4}$
Neptune	♆	$5\frac{2}{5}$
Pluto	♇	

1. About how many kilometres per second faster is:
   A Mars than Saturn? C Mars than Jupiter? E Uranus than Neptune?
   B Venus than Earth? D Saturn than Uranus? F Jupiter than Neptune?
   A $14\frac{2}{5}$ B $5\frac{1}{5}$ C $11\frac{1}{10}$ D $2\frac{17}{20}$ E $1\frac{7}{20}$ F $7\frac{1}{2}$
2. Mercury is about $40\frac{4}{5}$ kilometres per second faster than Uranus. About what is the orbital speed of Mercury? $47\frac{11}{20}$ Km/s
3. The moon orbits Earth at about $1\frac{1}{5}$ kilometres per second. The speed of Pluto is about $3\frac{3}{5}$ kilometres per second more than that. About what is the orbital speed of Pluto? $4\frac{4}{5}$ Km/s
4. Manned satellites orbit Earth at about 27 200 kilometres per hour, or $2\frac{3}{20}$ kilometres per second faster than Neptune orbits the sun. About what is the speed of these satellites in kilometres per second? $7\frac{11}{20}$ Km/s

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Follow-up
If the children are interested, provide more space data, such as the following, and ask them to create word problems which involve whole-number operations and addition and subtraction of fractional numbers.

Celestial Bodies	Fractional Part of Earth's Weight
Moon	$\frac{1}{6}$
Venus	$\frac{5}{6}$
Mars	$\frac{2}{5}$
Jupiter	$2\frac{2}{3}$
Sun	28

Workbook, page 86

Using the Exercises
Before you assign page 249, read and study the introductory information with the children. You might point out that they have learned how widely the planets vary in size; emphasize that the sun is so much larger than any of the planets that if we attempted to show the planets in the proper scale on this drawing, they would be nothing more than tiny specks. Give the children considerable latitude in discussing the facts presented in the chart, and then have them do the exercises. When they have finished, allow time for discussion and for checking papers.

PAGES 250–251

Objective

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

Review with the children any topics with which they have had difficulty. Since a considerable amount of this "Keeping in Touch" lesson is devoted to place value, as preparation for the introduction of decimals in the next chapter, you will perhaps want to include in your review a considerable amount of work on place-value names. Note that exercise 6 is also, in a sense, preparation for decimals. You may choose, during your preparation period or as a follow-up to this lesson, to provide additional exercises like exercise 6.

Keeping in Touch with

Computing
Measurement
Geometry
Place value
Functions
Fractional numbers

1. For the numeral 96 321 587, tell which digit is in each of these places.

A thousands' 1
B ten millions' 9
C hundred thousands' 3
D ten thousands' 2
E millions' 6
F hundreds' 5

2. Copy each sentence. Give the missing words and numbers.

6 709 483 875

A The 3 in the _?_ place means 3 × ▥. thousands', 1000
B The 9 in the _?_ place means 9 × ▥. millions', 1 000 000
C The 6 in the _?_ place means 6 × ▥. billions', 1 000 000 000

3. Give the correct sign (<, =, >) for each ▥.

A 53 680 ▥ 50 000 + 3000 + 500 + 80 >
B 657 009 ▥ 600 000 + 50 000 + 7000 + 90 <

4. Each book costs \$2.49. Bought 6 books. Paid how much for books? \$14.94

5. Think about the function machine and complete these tables.

Function Rule $(2 \times n) + 8$

n	Output
4	A 16
24	B 56
359	C 726
0	D 8
5 E	18

Function Rule $(n \times n) - n$

n	Output
0	F 0
1	G 0
10	H 90
99	I 9702
325	J 105 300

6. Solve each equation.

A $\frac{1}{10} = \frac{n}{100}$ 10
B $\frac{3}{10} = \frac{n}{100}$ 30
C $\frac{6}{10} = \frac{n}{100}$ 60
D $\frac{1}{10} = \frac{n}{1000}$ 100
E $\frac{1}{100} = \frac{n}{1000}$ 10
F $\frac{5}{100} = \frac{n}{1000}$ 50

7. Find the length (to the nearest centimetre) of each segment.

A ———————— *B* 10 cm

C ———————— *D* 7 cm

E ———————— *F* 9 cm

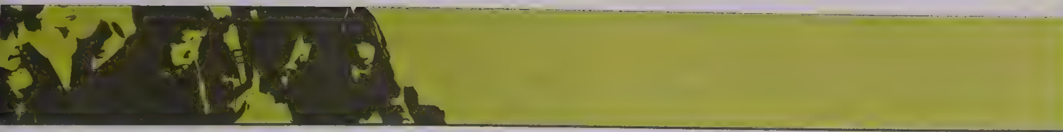
You are invited to explore

ACTIVITY CARD 12
Page 339

Discussion

Have the children do the exercises on page 250 independently. When they have finished, allow time for checking papers and discussion of the exercises. Be sure, during the discussion, to emphasize place value and the names of the various places. Have several children exhibit and explain to the class their work for exercise 6.

You may find it helpful, since the next chapter is on decimals, to make up several additional problems of this type and have them presented to the class.



THE WORLD

Africa ♀♀♀♀ About 350 million (375 also acceptable)

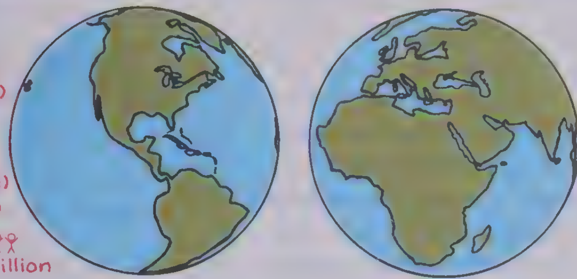
North America ♀♀♀ About 300 million

South America ♀♀ About 175 million (150 also acceptable)

Europe ♀♀♀♀♀♀ About 625 million

Asia ♀♀♀♀♀♀♀♀♀♀♀♀♀♀♀♀♀♀♀♀ About 2 billion, or 2000 million

Each ♀ represents 100 million people.



This **pictograph** shows the population of five of the seven large blocks of land (continents) on the earth's surface. Antarctica is the only continent that is not populated. The population of Australia and the islands of the Pacific Ocean (Oceania) is about nineteen million and is too small to show on the pictograph.

1. List the continents given in the pictograph. Beside each continent give the approximate population. See above.
2. Use the pictograph to tell how many times as many people live in Asia as in North America. Almost 7 times as many
3. Use the pictograph to tell how many more people live in Asia than on the other four continents combined. 500-600 million more
4. The total population of the world was about three billion six hundred thirty-two million people in 1970. Write the numeral for this population. 3 632 000 000
5. ★ The approximate area (in square kilometres) of each continent is given in the table. Let the symbol ■ represent 1 million square kilometres and make a pictograph that shows the sizes of the continents. See Answers, T.E. page 251.

Continent	Area
N. America	24 500 000
S. America	17 800 000
Europe	10 500 000
Asia	44 600 000
Africa	30 300 000
Australia	7 700 000
Antarctica	13 300 000

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Using the Exercises

Before assigning page 251, read and study the material at the top of the page with the children. Be sure to give the children adequate opportunity to discuss the pictograph and the meaning of the portions of symbols. Specifically, the children should recognize that, since each symbol represents 100 000 000 people, fractional parts of these symbols would represent fractional parts of 100 000 000 people. In a lesson like this, much value is derived from stimulating the children's interest concerning the topic being presented as well as from just working on the arithmetic.

Following this discussion, have the children do the exercises. When they have finished, allow time for more discussion and for checking papers.

Follow-up

You may choose to extend exercise 5, page 251, into a class activity, having children prepare charts for display. You might suggest that they add to such a display any pictographs which they can find in magazines or newspapers. Also, if you can find statistics suitable for a pictograph, you might have the children use these facts in constructing their own pictograph as part of the activity.

Answers, exercise 5, page 251

The children's pictographs should show the following number of symbols:

N. America—$24\frac{1}{2}$
S. America—$17\frac{4}{5}$
Europe—$10\frac{1}{2}$
Asia—$44\frac{3}{5}$
Africa—$30\frac{1}{3}$
Australia—$7\frac{3}{4}$
Antarctica—$13\frac{1}{3}$

CHAPTER 12 | Decimals

Pages 252–267

General Objectives

To introduce decimal notation
To provide additional experience in working with fractional numbers
To extend concepts of place value to include tenths, hundredths, and thousandths
To introduce addition and subtraction of fractional numbers using decimal notation
To relate notation for money to decimal notation
To provide experience in using decimals in measurement

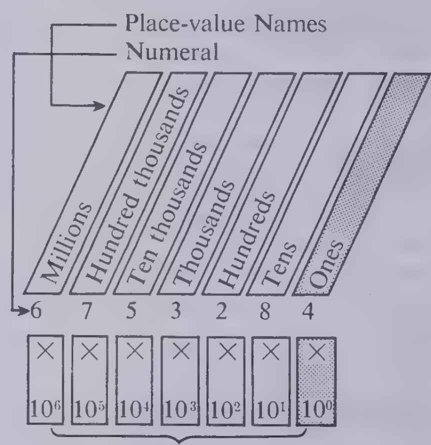
The beginning pages of this chapter define decimals and introduce the tenths', hundredths', and thousandths places. Subsequent material prepares the children for the necessary steps involved in adding and subtracting fractional numbers using decimal notation. The next pages of the chapter provide: (1) a lesson dealing with the relationship between work with fractional numbers using decimal notation and the familiar work with the usual notation for money and some word-problems; (2) a final lesson relating decimals to metric units of measurement; (3) cumulative and chapter reviews.

Mathematics

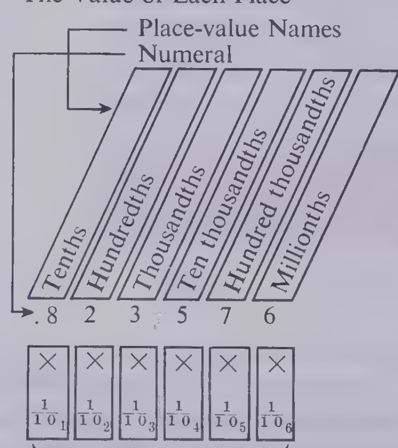
A discussion of decimals is meaningful only if we carefully distinguish between the two concepts involved:

1. The notation (the *decimal symbols* that we write).
2. The set of *numbers* represented by the decimal symbols.

When we use *decimal notation,* we are merely choosing an efficient way of writing symbols for *fractional numbers.* It may help you to think of decimal notation as a way of simplifying the use of fractions. Fractions whose denominators are any whole number except zero may be written in fractional notation, whereas only special fractions whose denominators are 1, 10, 100, 1000, . . . may be written in decimal notation. We can extend our place-value system in the following way to give meaning to decimal notation.



The Value of Each Place



The Value of Each Place

Once we understand decimal notation we can derive rules for computing with these new symbols *because we understand fractional numbers.* To develop skill with decimal notation it is necessary to have an

1. understanding of computation with fractional numbers, using fraction notation.
2. understanding of the relationship between decimal notation and fraction notation.

The following equation is a special case to illustrate the general definition of the use of decimal notation.

$$3.672 = 3 + \tfrac{6}{10} + \tfrac{7}{100} + \tfrac{2}{1000}$$

In order to read 3.672 as three and six hundred seventy-two thousandths, we must show that, given the basic definition of 3.672 above,

$$3 + \tfrac{6}{10} + \tfrac{7}{100} + \tfrac{2}{1000} = 3\tfrac{672}{1000}.$$

The proof is simple, but nevertheless, it is an important step and should not be omitted:

$$\begin{aligned} &3 + \tfrac{6}{10} + \tfrac{7}{100} + \tfrac{2}{1000} \\ &\qquad = 3 + \tfrac{600}{1000} + \tfrac{70}{1000} + \tfrac{2}{1000} \\ &\qquad = 3\tfrac{672}{1000} \end{aligned}$$

Teaching the Chapter

Materials

Centimetre ruler, clear plastic, for use on overhead projector
Centimetre rulers (1 per child)
Metre stick
Overhead projector (if available) and transparencies
Slips of paper, 7 by 12 centimetres

Vocabulary

centimetre	metre
decimal	millimetre
decimetre	tenth
hundredth	thousandth

Although children may have been introduced to the terms *tenths, hundredths,* and *thousandths* prior to this chapter, we include these words in the new vocabulary because they have not been presented in the context of decimals. Point out for the children that the difference between tenths written as a fraction and tenths written as a decimal is simply a matter of notation; do not present tenths as something really new in this chapter. The only thing that is new is

the way we write tenths or hundredths or thousandths.

The children are not expected to memorize the words for metric units of measurement. However, if you point out to them that *deci-* means one tenth, *centi-* means one hundredth, and *milli-* means one thousandth, they will be more likely to remember these names. Explain to the children that, since *centi-* means one hundredth, a hundredth of a metre is called a centimetre; and since *deci-* means a tenth, a tenth of a metre, or ten times one centimetre is a decimetre. Similar statements apply to *millimetre.* Do not insist that children master these words. The important objective is to acquaint the children with the idea of work with decimals associated with measurement concepts.

Lesson Schedule

Plan to spend about a week and a half to two weeks on this chapter. Of course, you may wish to adjust your schedule to the special needs and abilities of your children and to the amount of time left in the school year. All the material covered in the next three chapters—decimals, multiplication and division of fractional numbers, and graphing—will be covered in detail in Book 6. Therefore, you may exercise some freedom in your selection of topics if the time left in the school year is limited.

Evaluation of Progress

Since one of the chief objectives of this chapter is to help children gain an understanding of work with decimal notation, you will want to focus much of your evaluation on the degree to which the children have attained this understanding. Children's abilities to add and subtract using decimal notation and to work with money problems and word problems in general are important skills developed in this chapter and should be a part of your evaluation. However, a much more vital area for evaluation is the children's understanding of the concepts with which they are working as they use this notation. Usually this type of learning is best evaluated on a day-to-day basis, by continual observation of the children's comments and reactions during discussion periods.

Your overriding concern should be to convince children that they are working with exactly the same fractional numbers that they have worked with previously and that the only difference is in the marks they write on their papers. Once you have convinced them of this, it is easy to point out to them the convenience of working with decimals. That is, when working with decimals, they can think of working with fractions that have common denominators (assuming that they line up the decimal points); hence, they can add and subtract with such notation rather easily.

The discussion section on page 258 emphasizes that the children may think about converting decimals to fractional notation to compute. Naturally, they will not always want to think this way, but it is important as a first step toward understanding the use of decimal notation in adding and subtracting rational numbers.

Pages 264 and 265 provide material to use either in reviewing the chapter or in evaluating progress, as you prefer. The cumulative review on pages 266 and 267 may help you check the children's retention of concepts previously taught.

Resources for Active Learning

General Activities

Discovery, Section II, Unit 14/5,6, Encyclopaedia Britannica Educational Corp.

Mathex: Numeration No. 7, "Decimals," pp. 47–48 (pupil pages 52–60), Encyclopaedia Britannica Publications Ltd.

Nuffield Project: *Computation and Structure* 4, "Extension of Place Value," pp. 11–12, Wiley.

Manipulative Devices

Abacus or abacus board (school supplier)

Centimetre Decimal Set (Math Media)

Metre Stick (Geyer Instructional Aids)

Objective

Given a fraction with a denominator of ten, the child will be able to write a decimal numeral for the fraction.

Preparation

To prepare for this lesson, you might briefly review how different fractions can be used to represent the same fractional number and how to write pairs of equivalent fractions. For example, write pairs of fractions such as the following and ask children to supply the missing number for n.

$$\frac{2}{5} = \frac{n}{15} \qquad \frac{1}{5} = \frac{n}{20}$$

$$\frac{7}{2} = 3\frac{n}{2} \qquad \frac{3}{8} = \frac{n}{24}$$

$$\frac{7}{5} = 1\frac{n}{5} \qquad \frac{1}{4} = \frac{n}{12}$$

Such a review, using familiar denominators, should help the children begin the investigation of this lesson with greater understanding.

Investigation

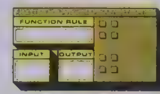
Direct the children to study the function machines carefully. They should understand the rule for function machine A before they proceed to study function machine B. Some will find it helpful to share with each other what they think the rule for each function machine is. Help any children who seem confused with function machine A, but, for those who understand A, give little guidance concerning function machine B. Encourage them to apply what they think is the rule; then tell them whether or not they are correct.

12 Decimals

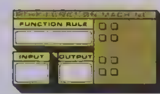
● *What is a decimal numeral?*

Investigating the Ideas

Each output from function machine A becomes an input for function machine B.

A 

n	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{7}{10}$	$\frac{3}{5}$	$\frac{2}{10}$	$2\frac{1}{2}$	$7\frac{1}{5}$	$35\frac{1}{10}$	$\frac{3}{2}$	$\frac{13}{5}$
	$\frac{5}{10}$	$\frac{2}{10}$	$\frac{7}{10}$	$\frac{6}{10}$	$\frac{2}{10}$	$2\frac{5}{10}$	$7\frac{2}{10}$	$35\frac{1}{10}$	$1\frac{5}{10}$	$2\frac{6}{10}$

B 

n	$\frac{5}{10}$	$\frac{2}{10}$	$\frac{7}{10}$	$\frac{6}{10}$	$\frac{2}{10}$	$2\frac{5}{10}$	$7\frac{2}{10}$	$35\frac{1}{10}$	$1\frac{5}{10}$	$2\frac{6}{10}$
	0.5	0.2	0.7	0.6	0.2	2.5	7.2	35.1	1.5	2.6

? Can you figure out how these two function machines work? **Show you know** by giving output B for each of these inputs for A. 0.4 $\frac{2}{5}$, 0.8 $\frac{4}{5}$, 3.5 $3\frac{1}{2}$, 4.2 $4\frac{1}{5}$, 0.8 $\frac{8}{10}$, 6.1 $6\frac{1}{10}$, 7.4 $7\frac{2}{5}$

Discussing the Ideas

1. Explain each function rule in the Investigation.
Rule A: Give each input number as a fraction or mixed numeral with a denominator of 10. Rule B: Express each input number as a decimal.
2. Study the table and give the missing **decimals**.

	We see	We think	We write		We say
			fractions	decimals	
A		2 tens, 3 ones, and 8 tenths	$23\frac{8}{10}$	23.8	twenty-three and eight tenths
B		1 ten, 8 ones, and 4 tenths	$18\frac{4}{10}$	18.4	eighteen and four tenths
C		1 and 2 tenths	$1\frac{2}{10}$	1.2	one and two tenths
D		7 tenths	$\frac{7}{10}$	0.7	seven tenths

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Discussion

The main point of this lesson is for children to write and read tenths in decimal notation and to understand that decimal notation is simply another way to represent fractional numbers. As you discuss the chart for exercise 2, point out to the children that in the first column they see the objects; in the second column they think about the number of tens, ones, and the number of tenths; and in the third column they see two different ways to write this number (with fractions and with decimals). Emphasize for the children that they are learning a new way to write a symbol for fractional numbers. For example, the old form was $23\frac{8}{10}$, and the new form is 23.8. Actually, we read both notations the same way: twenty-three and eight tenths.

Using the Ideas

1. For each exercise, give the correct mixed numeral and the correct decimal.
   - A For 8 tens, 3 ones, and 6 tenths, we write ▮ or ▮. Answer: $83\frac{6}{10}$, 83.6
   - B For 1 ten, 2 ones, and 5 tenths, we write ▮ or ▮. $12\frac{5}{10}$, 12.5
   - C For 3 tens, 0 ones, and 7 tenths, we write ▮ or ▮. $30\frac{7}{10}$, 30.7
   - D For 9 tens, 7 ones, and 1 tenth, we write ▮ or ▮. $97\frac{1}{10}$, 97.1
   - E For 8 tens, 3 ones, and 9 tenths, we write ▮ or ▮. $83\frac{9}{10}$, 83.9
   - F For 1 ten, 4 ones, and 3 tenths, we write ▮ or ▮. $14\frac{3}{10}$, 14.3
   - G For 5 tens, 1 one, and 8 tenths, we write ▮ or ▮. $51\frac{8}{10}$, 51.8
   - H For 6 tens, 2 ones, and 4 tenths, we write ▮ or ▮. $62\frac{4}{10}$, 62.4

2. Copy each exercise and give the missing numerator or denominator.

A $6.8 = 6 + \frac{▮}{10}$ 8	D $23.9 = 23 + \frac{▮}{10}$ 9	G $2.2 = 2 + \frac{▮}{10}$ 2
B $17.5 = 17 + \frac{5}{▮}$ 10	E $74.6 = 74 + \frac{6}{▮}$ 10	H $3.7 = 3 + \frac{▮}{10}$ 7
C $0.6 = \frac{▮}{10}$ 6	F $18.1 = 18 + \frac{▮}{10}$ 1	I $37.4 = 37 + \frac{4}{▮}$ 10

3. Give the correct decimal for each sum.

A $7 + \frac{4}{10}$ 7.4	D $66 + \frac{1}{10}$ 66.1	G $53 + \frac{8}{10}$ 53.8
B $23 + \frac{8}{10}$ 23.8	E $1 + \frac{1}{10}$ 1.1	H $60 + \frac{7}{10}$ 60.7
C $80 + \frac{9}{10}$ 80.9	F $237 + \frac{9}{10}$ 237.9	I $48 + \frac{2}{10}$ 48.2

★ 4. Give the missing numerator and then give the decimal for the sum n.

A $4 + \frac{1}{2} = 4 + \frac{▮}{10} = n$ 5, 4.5	E $36 + \frac{2}{5} = 36 + \frac{▮}{10} = n$ 4, 36.4
B $6 + \frac{1}{5} = 6 + \frac{▮}{10} = n$ 2, 6.2	F $126 + \frac{1}{2} = 126 + \frac{▮}{10} = n$ 5, 126.5
C $23 + \frac{1}{2} = 23 + \frac{▮}{10} = n$ 5, 23.5	G $74 + \frac{3}{5} = 74 + \frac{▮}{10} = n$ 6, 74.6
D $10 + \frac{4}{5} = 10 + \frac{▮}{10} = n$ 8, 10.8	H $81 + \frac{1}{5} = 81 + \frac{▮}{10} = n$ 2, 81.2

More practice, page A-26, Set 48

Resources for Active Learning

Experiences in Mathematical Ideas, Vol. 1, Unit 5, Experience 5; Unit 6, Experience 3, NCTM.

Mathematics in Modules, F5, Addison-Wesley.

Workbook, page 87

Using the Exercises

The exercises on page 253 reinforce the concept that the decimal notation here is simply another way of representing fractional numbers. Notice that the children work exclusively with tenths, so this concept is their major concern. When children have finished the exercises, it would be helpful to have them write their answers on the chalkboard and read them aloud. Starred exercise 4 is optional, but a discussion of it would help to emphasize the main points of this lesson.

Assignments (page 253)

Minimum: 1–3. Average: 1, 3–4D. Maximum: 1–4.

PAGES 254–255

Objective

Given decimals involving tenths, hundredths, or thousandths, the child will be able to read and write them.

Preparation

To prepare for this lesson, review the use of tenths, as developed in the previous lesson. For example, write lists of fractions and decimals using tenths and have the children match them.

$\frac{5}{10}$	7.5
$2\frac{3}{10}$	0.7
$7\frac{1}{2}$	7.6
$7\frac{3}{5}$	2.3
$\frac{7}{10}$	0.5

How can we read and write decimals?

Discussing the Ideas

1. Study the decimal for $9876 + \frac{4}{10} + \frac{3}{100} + \frac{2}{1000}$. Then give the missing word and number in each exercise that follows.

Thousands | Hundreds | Tens | Ones | Tenths | Hundredths | Thousandths

9 8 7 6 . 4 3 2

The 4 in the **tenths'** place means $\frac{4}{10}$.
The 3 in the **hundredths'** place means $\frac{3}{100}$.
The 2 in the **thousandths'** place means $\frac{2}{1000}$.

A 28.45: The 2 in the _?_ place means |||||. tens', 20
B 28.45: The 4 in the _?_ place means |||||. tenths', $\frac{4}{10}$
C 531.64: The 5 in the _?_ place means |||||. hundreds', 500
D 531.64: The 4 in the _?_ place means |||||. hundredths', $\frac{4}{100}$
E 2876.354: The 2 in the _?_ place means |||||. thousands', 2000
F 2876.354: The 4 in the _?_ place means |||||. thousandths', $\frac{4}{1000}$
G 5.04: The 4 in the _?_ place means |||||. hundredths', $\frac{4}{100}$
H 26.008: The 8 in the _?_ place means |||||. thousandths', $\frac{8}{1000}$
I 0.326: The 2 in the _?_ place means |||||. hundredths', $\frac{2}{100}$
J 0.605: The 5 in the _?_ place means |||||. thousandths', $\frac{5}{1000}$

2. Study examples A and B below. Then give the missing numerator in each exercise.

A $8.25 = 8 + \frac{2}{10} + \frac{5}{100} = 8 + \frac{20}{100} + \frac{5}{100} = 8\frac{25}{100}$

We read "eight and twenty-five hundredths" for 8.25.

B $8.396 = 8 + \frac{3}{10} + \frac{9}{100} + \frac{6}{1000} = 8 + \frac{300}{1000} + \frac{90}{1000} + \frac{6}{1000} = 8\frac{396}{1000}$

We read "eight and three hundred ninety-six thousandths" for 8.396.

A $7.36 = 7 + \frac{3}{10} + \frac{6}{100} = 7 + \frac{\overset{30}{|||||}}{100} + \frac{6}{100} = 7\frac{\overset{36}{|||||}}{100}$
B $2.94 = 2 + \frac{9}{10} + \frac{4}{100} = 2 + \frac{|||||\,90}{100} + \frac{|||||\,4}{100} = 2\frac{|||||\,94}{100}$

3. Read each decimal in question 1 of the Discussion.
Sample: For 28.45 read, "Twenty-eight and forty-five hundredths".

Discussion

Explain to the children that the diagram in discussion exercise 1 shows how we extend our ordinary place-value system to write sums such as

$$9876 + \frac{4}{10} + \frac{3}{100} + \frac{2}{1000}.$$

Be sure to draw parallels for the children between the tens' place and the tenths' place, the hundreds' place, and the hundredths' place, the thousands' place and the thousandths' place.

Using an extension of our place-value system, the children will learn to write $9876 + \frac{4}{10} + \frac{3}{100} + \frac{2}{1000}$ as 9876.432. You may find it helpful to use other examples to illustrate this point further.

Exercise 2 provides the basis for our manner of reading decimals. Help children see that a sum such as $\frac{2}{10} + \frac{5}{100}$ can be arrived at quickly and they can write $\frac{25}{100}$ without much deliberation. Also point out how knowing the place values enables them to read decimals. The place of the last digit to the right gives its name to the decimal being read. Thus, 32.574 is read, "thirty-two and five hundred seventy-four *thousandths*." The decimal point is read as "and," and the decimal fraction is read as an ordinary base-ten numeral labelled according to the *thousandths'* place. Exercise 3 is intended to provide needed practice in reading decimals orally.

Using the Ideas

1. Copy each exercise on your paper and give the missing numerator.

A $8.6 = 8\frac{\square}{10}$ 6	E $20.7 = 20\frac{\square}{10}$ 7	I $15.04 = 15\frac{\square}{100}$ 4
B $7.9 = 7\frac{\square}{10}$ 9	F $56.8 = 56\frac{\square}{10}$ 8	J $9.07 = 9\frac{\square}{100}$ 7
C $0.4 = \frac{\square}{10}$ 4	G $17.6 = 17\frac{\square}{10}$ 6	K $1.007 = 1\frac{\square}{1000}$ 7
D $0.05 = \frac{\square}{100}$ 5	H $156.4 = 156\frac{\square}{10}$ 4	L $0.008 = \frac{\square}{1000}$ 8

2. Write each fractional number as in the examples. See Answers, T.E. page 255.

Example 1: $13.28 = 13 + \frac{2}{10} + \frac{8}{100}$

Example 2: $4.725 = 4 + \frac{7}{10} + \frac{2}{100} + \frac{5}{1000}$

A 4.62	E 8.62	I 7.81	M 926.4	Q 76.8	U 43.4
B 7.83	F 86.2	J 7.812	N 92.64	R 7.68	V 43.04
C 9.25	G 7.32	K 7.846	O 9.264	S 0.768	W 4.304
D 3.14	H 73.2	L 70.84	P 9.064	T 0.076	X 4.004

3. Give the correct decimal for each sum.

A $7 + \frac{2}{10}$ 7.2	H $6 + \frac{7}{10} + \frac{8}{100} + \frac{1}{1000}$ 6.781	O $7 + \frac{8}{1000}$ 7.008
B $7 + \frac{2}{10} + \frac{6}{100}$ 7.26	I $8 + \frac{2}{10} + \frac{3}{100} + \frac{6}{1000}$ 8.236	P $4 + \frac{1}{10} + \frac{5}{1000}$ 4.105
C $7 + \frac{2}{10} + \frac{6}{100} + \frac{7}{1000}$ 7.267	J $8 + \frac{2}{10} + \frac{0}{100} + \frac{6}{1000}$ 8.206	Q $765 + \frac{2}{10}$ 765.2
D $3 + \frac{5}{10}$ 3.5	K $8 + \frac{2}{10} + \frac{6}{1000}$ 8.206	R $76 + \frac{5}{10} + \frac{2}{100}$ 76.52
E $3 + \frac{5}{10} + \frac{1}{100}$ 3.51	L $8 + \frac{0}{10} + \frac{0}{100} + \frac{6}{1000}$ 8.006	S $7 + \frac{6}{10} + \frac{5}{100} + \frac{2}{1000}$ 7.652
F $3 + \frac{5}{10} + \frac{1}{100} + \frac{9}{1000}$ 3.519	M $8 + \frac{6}{1000}$ 8.006	T $7 + \frac{5}{100} + \frac{2}{1000}$ 7.052
G $6 + \frac{7}{10} + \frac{8}{100}$ 6.78	N $9 + \frac{6}{100}$ 9.06	U $7 + \frac{5}{10} + \frac{2}{1000}$ 7.502

★ **4.** Give a decimal for each sum.

A $\frac{2}{10} + \frac{3}{100} + \frac{6}{1000} + \frac{7}{10\,000}$ .2367

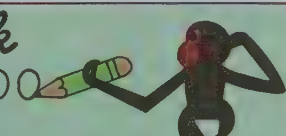
B $\frac{5}{10} + \frac{3}{100} + \frac{8}{10\,000} + \frac{9}{100\,000}$ .53089

C $100\,000 + \frac{1}{100\,000}$ 100 000.00001

D $1000 + \frac{1}{1000} + \frac{1}{1\,000\,000}$ 1000.001001

think

.000



Write a decimal for $\frac{1}{25}$. .04

Write a fraction for 0.000001. $\frac{1}{1\,000\,000}$

More practice, page A-26, Set 49

Mathematics

Part of the purpose of this lesson is to justify reading a decimal such as 8.25 as eight and twenty-five hundredths, after we have defined such a decimal to mean $8 + \frac{2}{10} + \frac{5}{100}$. Properly, the thing to show is that $\frac{2}{10} + \frac{5}{100}$ is, in fact, equal to $\frac{25}{100}$:

$$\frac{2}{10} + \frac{5}{100} = \frac{20}{100} + \frac{5}{100} = \frac{25}{100}$$

This clearly shows that 8.25 could be read as eight and twenty-five hundredths. This should be demonstrated rather than simply taken for granted.

Answers, exercise 2, page 255

A $4.62 = 4 + \frac{6}{10} + \frac{2}{100}$
B $7.83 = 7 + \frac{8}{10} + \frac{3}{100}$
C $9.25 = 9 + \frac{2}{10} + \frac{5}{100}$
D $3.14 = 3 + \frac{1}{10} + \frac{4}{100}$
E $8.62 = 8 + \frac{6}{10} + \frac{2}{100}$
F $86.2 = 86 + \frac{2}{10}$
G $7.32 = 7 + \frac{3}{10} + \frac{2}{100}$
H $73.2 = 73 + \frac{2}{10}$
I $7.81 = 7 + \frac{8}{10} + \frac{1}{100}$
J $7.812 = 7 + \frac{8}{10} + \frac{1}{100} + \frac{2}{1000}$
K $7.846 = 7 + \frac{8}{10} + \frac{4}{100} + \frac{6}{1000}$
L $70.84 = 70 + \frac{8}{10} + \frac{4}{100}$
M $926.4 = 926 + \frac{4}{10}$
N $92.64 = 92 + \frac{6}{10} + \frac{4}{100}$
O $9.264 = 9 + \frac{2}{10} + \frac{6}{100} + \frac{4}{1000}$
P $9.064 = 9 + \frac{0}{10} + \frac{6}{100} + \frac{4}{1000}$
Q $76.8 = 76 + \frac{8}{10}$
R $7.68 = 7 + \frac{6}{10} + \frac{8}{100}$
S $0.768 = \frac{7}{10} + \frac{6}{100} + \frac{8}{1000}$
T $0.076 = \frac{7}{100} + \frac{6}{1000}$
U $43.4 = 43 + \frac{4}{10}$
V $43.04 = 43 + \frac{0}{10} + \frac{4}{100}$
W $4.304 = 4 + \frac{3}{10} + \frac{0}{100} + \frac{4}{1000}$
X $4.004 = 4 + \frac{4}{1000}$

Using the Exercises

Have the children do the exercises on page 255 independently. When they have finished, check the exercises together and then give the children an opportunity for further practice in reading decimals orally, perhaps using those in exercise 2.

Many children will find the *Think* problem challenging. To write a decimal for $\frac{1}{25}$, the children would be expected to consider the set of equivalent fractions for $\frac{1}{25}$. That is, by observing the fractions $\frac{2}{50}$, $\frac{3}{75}$, and $\frac{4}{100}$, they should conclude that the decimal for $\frac{1}{25}$ is .04. To write a fraction for .000001, the children need merely observe that the 1 is in the millionths' place, and hence, the fraction is $\frac{1}{1\,000\,000}$.

Assignments (page 255)

Minimum: 1–2L, 3A–G.
Average: 1–2P, 3A–N.
Maximum: 1–4, alternate parts.

Follow-up

Ask children to find examples of various uses of decimals. The boys may enjoy working with the baseball averages of their favorite players.

Resources for Active Learning

Developmental Math Cards, 1¹10, Addison-Wesley.
Mathematics in Modules, F6, Addison-Wesley.

Duplicator Masters, page 54
Workbook, page 88
Skill Masters, page 54

PAGES 256–257

Objectives

Given 2 numbers written in decimal notation, the child will be able to write an inequality statement showing which represents the greater number.

The child will be able to find sums or differences by considering the equivalent fractions for each given decimal.

Preparation

Materials

slips of paper approximately 7 by 12 centimetres, or paper suitable to be cut as the slips illustrated in the text

Because of the nature of the investigation, you might prefer that children begin it immediately. However, you might also write several decimal fractions on the chalkboard and give children practice in reading them aloud.

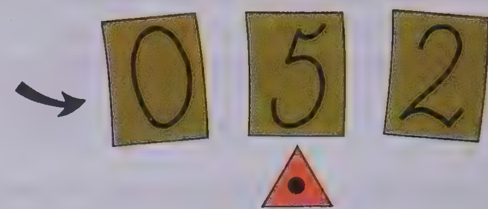
Investigation

Although children would be able to investigate the ideas of this lesson by simply using paper and pencil, working with the slips of paper should provide a more concrete appreciation for the number of possible arrangements of the digits. Remind children to keep a record of each decimal numeral they form. They must remember that the decimal point should be between two of the digits; thus, numerals such as .025, 205., or .250 are not acceptable.

● *Can we think of decimals in the same way we think of fractions?*

Investigating the Ideas

Cut out 4 slips of paper and label them like this.



? Using all your slips, how many decimals can you form, if the decimal point is always between two digits? | Record your decimals.

See Investigation and Discussion.

Examples:

Discussing the Ideas

1. A Which of your decimals represent numbers greater than 1? See Discussion.
   B Which are less than 1? See Discussion.
   C Can you match any of your decimals with these numbers?
   $2\frac{1}{2}$, $5\frac{2}{10}$, 25, $50\frac{1}{5}$
   $2\frac{1}{2} = 2.50$; $5\frac{2}{10} = 5.20$; $25 = 25.0$; $50\frac{1}{5} = 50.2$

2. For each pair, tell which is greater.
   A 25.0 or 2.50 C 25.0 or 52.0 E 0.052 or 0.025
   B 25.0 or 5.20 D 0.052 or 0.52 F 52.0 or 50.2

3. A Solve the equation: $\frac{2}{10} + \frac{5}{10} = n$ $\frac{7}{10}$
   B What numeral should go on the blank slip? 7

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Discussion

Have children write their numerals on the chalkboard:

0.52	5.02	2.50
05.2	50.2	25.0
0.25	5.20	2.05
02.5	52.0	20.5

As you work through discussion exercises 1 and 2, it would be helpful to have children explain the place-value meaning of several digits. Stress, for example, that 5.02 means $5 + \frac{0}{10} + \frac{2}{100}$, but moving the decimal one place can change this to 50.2, or $50 + \frac{2}{10}$. Also, use exercise 1C to stress that these decimals represent fractional numbers which may be written as fractions as well as decimals. Show how $2\frac{1}{2} = 2\frac{5}{10} = 2\frac{50}{100} = 2.50$; $5\frac{2}{10} = 5\frac{20}{100} = 5.20$; $25 = 25\frac{0}{10} = 25.0$; $50\frac{1}{5} = 50\frac{2}{10} = 50.2$. You might find it helpful to express several of the decimals in exercise 2 in fractional form, to help children compare them. Since $25\frac{0}{10} > 2\frac{50}{100}$, then clearly $25.0 > 2.50$.

Exercise 3 continues to develop the basic fact that fractions with a denominator of ten may easily be written as decimal numerals. Since the children know how to add fractions, $\frac{2}{10} + \frac{5}{10} = \frac{7}{10}$ clearly leads to $0.2 + 0.5 = 0.7$.

Using the Ideas

1. Give the correct sign (<, =, or >) for each ▒.

A 75.6 ▒ 75.3 >	E 53.2 ▒ 52.9 >	I 6.231 ▒ 6.232 <
B 84.6 ▒ 84.9 <	F 8.75 ▒ 8.76 <	J 6.451 ▒ 6.351 >
C 67.2 ▒ 68.2 <	G 8.74 ▒ 8.64 >	K 7.213 ▒ 6.987 >
D 61.6 ▒ 62.1 <	H 3.05 ▒ 3.04 >	L 5.607 ▒ 5.617 <

2. Find the sums and differences.

A $\frac{6}{10}-\frac{3}{10}=n$ $\frac{3}{10}$ B $\frac{2}{10}+\frac{5}{10}=n$ $\frac{7}{10}$ C $\frac{23}{100}+\frac{34}{100}=n$ $\frac{57}{100}$ D $\frac{15}{100}+\frac{63}{100}=n$ $\frac{78}{100}$

$0.6-0.3=m$ 0.3 $0.2+0.5=m$ 0.7 $0.23+0.34=m$ 0.57 $0.15+0.63=m$ 0.78

E $\frac{25}{100}-\frac{15}{100}=n$ $\frac{10}{100}$ F $\frac{38}{100}+\frac{15}{100}=n$ $\frac{53}{100}$ G $\frac{37}{100}-\frac{19}{100}=n$ $\frac{18}{100}$ H $\frac{56}{100}+\frac{24}{100}=n$ $\frac{80}{100}$

$0.25-0.15=m$ 0.10 $0.38+0.15=m$ 0.53 $0.37-0.19=m$ 0.18 $0.56+0.24=m$ 0.80

I $\frac{3}{10}+\frac{5}{10}=\frac{8}{10}$ $0.3+0.5=0.8$ J $\frac{7}{10}-\frac{2}{10}=\frac{5}{10}$ $0.7-0.2=0.5$ K $\frac{16}{100}+\frac{42}{100}=\frac{58}{100}$ $0.16+0.42=0.58$ L $\frac{82}{100}-\frac{26}{100}=\frac{56}{100}$ $0.82-0.26=0.56$

M $\frac{29}{100}+\frac{17}{100}=\frac{46}{100}$ $0.29+0.17=0.46$ N $\frac{58}{100}-\frac{28}{100}=\frac{30}{100}$ $0.58-0.28=0.30$ O $\frac{27}{100}+\frac{7}{10}=\frac{97}{100}$ $0.27+0.70=0.97$ P $\frac{5}{10}+\frac{32}{100}=\frac{82}{100}$ $0.50+0.32=0.82$

3. In each exercise, copy the first equation and give the missing numerators. Then give the correct decimal for the sum in the second equation.

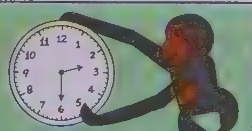
A $\frac{6}{10}+\frac{7}{10}=\frac{10}{10}+\frac{3}{10}=1+\frac{3}{10}$
$0.6+0.7=n$ 1.3

B $\frac{5}{10}+\frac{9}{10}=\frac{10}{10}+\frac{4}{10}=1+\frac{4}{10}$
$0.5+0.9=n$ 1.4

C $\frac{8}{10}+\frac{6}{10}=\frac{10}{10}+\frac{4}{10}=1+\frac{4}{10}$
$0.8+0.6=n$ 1.4

D $\frac{4}{10}+\frac{7}{10}=\frac{10}{10}+\frac{1}{10}=1+\frac{1}{10}$
$0.4+0.7=n$ 1.1

think



When will the two hands of the clock next come together?

A Between 3:12 and 3:15
B Between 3:15 and 3:18
C Between 3:18 and 3:21

See Solution, T.E. Page 257

More practice, page A-27, Set 50

Using the Exercises
Depending on the children's need, work through parts of exercise 2 before assigning page 257 as independent work. In particular, point out the alignment of the decimal point in the vertical notation of parts I to P of exercise 2. If place value is understood, children should realize that the decimal points must be aligned so that all place values are aligned and tenths will be added to tenths, hundredths to hundredths, and so on.

If you check the inequalities in exercise 1 orally, children will have another opportunity to practice reading decimals.

Assignments (page 257)
Minimum: 1, 2A–H. Average: 1–2.
Maximum: 1–3.

Solution, Think, page 257
Observe with the children that the hands will come together after 3 o'clock. That is, the minute hand will have to go up to 12 and then come part of the way down from 12 before the hands could possibly meet. Therefore, it will be between 3 and 4 o'clock when the hands meet again. It could not be 3:12 or 3:15, because the hour hand will have to be between 3 and 4. Therefore, we are left with the choices of 3:15 to 3:18 or 3:18 to 3:21. Now, notice with the children that the hour hand will be somewhere in the upper half of the space between 3 and 4, because the minute hand is not past 6. Therefore, the correct answer would have to be **B,** since this answer takes in the upper half of the space between 3 and 4.

Follow-up
A game such as the following will help children become more familiar with the place values in decimal notation.

Form two teams and make large digit cards so that each team has a set of cards on which are printed the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and a decimal point. Ideally, each of the eleven members of a team must place himself in the position which his digit card occupies in a designated numeral. When a numeral is named by the teacher (or by a child selected by the teacher), the team members must position themselves so that the leader facing them, or other members in the class, can read the number. The first team which places themselves properly scores a point. The team with the highest score, after several numbers have been given, is the winning team. Sample numbers for the leader to call are:

3.76 27.564 80.425
491.36 321.98

Variations of this game may include preparing more than one card for each digit.

Workbook, page 89

Objective

Given addition or subtraction problems written in decimal notation, the child will be able to find the sums or differences.

Preparation

No preparation is essential for this investigation, but if you choose to have an introductory session, write several pairs of decimals on the chalkboard and have children compare them and then read them as inequalities.

Investigation

Direct the children to do this short investigation independently. Remind them to check the addition as well as to place the decimal points. Watch for typical errors such as

$$\begin{array}{r} .7 \\ +.6 \\ \hline .13 \end{array}$$

Ask a child who makes such a mistake to write the problem in fraction notation ($\frac{7}{10} + \frac{6}{10} = \frac{13}{10}$) and discuss how he would write $\frac{13}{10}$ as a decimal. You might find it helpful to stress that $\frac{13}{10}$ is greater than one, but 0.13, or $^{13}/_{100}$, is not.

Let's add and subtract using decimals.

Investigating the Ideas

Marty was usually a careful mathematics student. But on this paper she missed every answer because she forgot all of the decimal points.

Marty

(1) $\begin{array}{r} 2.3 \\ +4.5 \\ \hline 68 \end{array}$ X 6.8 (2) $\begin{array}{r} 0.7 \\ +0.6 \\ \hline 13 \end{array}$ X 1.3 (3) $\begin{array}{r} 4.8 \\ +3.7 \\ \hline 85 \end{array}$ X 8.5

(4) $\begin{array}{r} 6.9 \\ -2.7 \\ \hline 42 \end{array}$ X 4.2 (5) $\begin{array}{r} 1.4 \\ -0.6 \\ \hline 0.8 \end{array}$ X (6) $\begin{array}{r} 7.2 \\ -1.5 \\ \hline 57 \end{array}$ X 5.7

? Can you copy Marty's problems and correct them for her? Be careful!

Discussing the Ideas

Explain each step in the two examples. See Discussion.

1. Step 1	Step 2	Step 3
Add hundredths. $\begin{array}{r} 7.84 \\ +6.79 \\ \hline 3 \end{array}$	Add tenths; bring down decimal point. $\begin{array}{r} 7.84 \\ +6.79 \\ \hline .63 \end{array}$	Add ones. $\begin{array}{r} 7.84 \\ +6.79 \\ \hline 14.63 \end{array}$
$\frac{4}{100} + \frac{9}{100} = \frac{13}{100}$	$\frac{1}{10} + \frac{8}{10} + \frac{7}{10} = 1\frac{6}{10}$	$1 + 7 + 6 = 14$

2. Step 1	Step 2	Step 3
Regroup ones and tenths. $\begin{array}{r} \overset{5}{\cancel{6}}.\overset{1}{2} \\ -3.8 \\ \hline \end{array}$	Subtract tenths; bring down decimal point. $\begin{array}{r} \overset{5}{\cancel{6}}.\overset{1}{2} \\ -3.8 \\ \hline .4 \end{array}$	Subtract ones. $\begin{array}{r} \overset{5}{\cancel{6}}.\overset{1}{2} \\ -3.8 \\ \hline 2.4 \end{array}$
$6 + \frac{2}{10} = 5 + \frac{12}{10}$	$\frac{12}{10} - \frac{8}{10} = \frac{4}{10}$	$5 - 3 = 2$

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Discussion

As you work through the explanations of each problem in the discussion section, stress the equation shown at the bottom of each step of the problem. The understanding of decimal computation is developed through a study of fractions. Once understanding is achieved, the computation is strictly mechanical. The method for adding and subtracting decimals is the same as for whole numbers, except for decimal point rules (which again can be understood by using fractions).

The examples in the discussion section of the text are treated briefly because the process is mechanical and the rules are simple. Have children work a problem step by step, individually, as each example is being discussed. Work through other examples as necessary.

Stress the importance of aligning the decimal point correctly. Note with the children that in this way we simply assure ourselves that we are always working with fractions having the same denominator. Use fractions to explain how 2.45 may be thought of as 2.450 ($2\frac{45}{100} = 2\frac{450}{1000}$). Do not overemphasize such exercises, however, as children will rarely be required to solve them.

Using the Ideas

1. Each example is worked correctly except for the decimal point. Copy the answers and place the decimal point correctly in each sum or difference.

A $7.6 + 8.9$ = 165 — 16.5	B $3.69 + 1.54$ = 523 — 5.23	C $0.632 + 0.819$ = 1451 — 1.451
D $5.1 - 1.7$ = 34 — 3.4	E $0.62 - 0.43$ = 19 — 0.19	F $7.06 - 2.19$ = 487 — 4.87

2. Find the sums.

A $8.38 + 6.75$ — 15.13	B $0.927 + 0.846$ — 1.773	C $32.8 + 65.4$ — 98.2	D $9.762 + 8.431$ — 18.193	E $64.35 + 74.69$ — 139.04
F $72.80 + 97.54$ — 170.34	G $8.346 + 7.52$ — 15.866	H $92.6 + 87.59$ — 180.19	I $600.4 + 738.7$ — 1339.1	J $92.65 + 34.71$ — 127.36
K $87.4 + 65.2 + 93.1$ — 245.7	L $64.3 + 2.74 + 84.5$ — 151.54	M $52.74 + 6.5 + 23.88$ — 83.12		

N $4.58 + 7.6 + 25.8$ 37.98
O $0.832 + 5.26 + 39.1$ 45.192
P $9.642 + 376 + 84.75$ 470.392

3. Find the differences. Check each of your answers by addition.

A $6.4 - 2.8$ — 3.6	B $2.3 - 0.9$ — 1.4	C $8.6 - 1.9$ — 6.7	D $0.92 - 0.65$ — 0.27	E $0.83 - 0.26$ — 0.57
F $0.76 - 0.09$ — 0.67	G $6.82 - 0.63$ — 6.19			
J $7.41 - 5.07$ — 2.34	K $6.95 - 2.9$ — 4.05			
L $8.07 - 1.58$ — 6.49	M $0.930 - 0.307$ — 0.623			
N $7.064 - 1.255$ — 5.809	O $7.602 - 0.009$ — 7.593			

H $27.9 - 8.6$ 19.3 I $0.832 - 0.57$ 0.262

think

This is April for a certain year.

APRIL						
S	M	T	W	T	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

May has 31 days and June has 30 days. In this year, on what day of the week is the last day of May? of June?
Friday Sunday

More practice, page A-27, Set 51

Follow-up
Depending on the needs of the children, you might prepare and distribute a worksheet similar to the following for children to reinforce and practice the concepts and skills of the present and previous lessons.

> Complete each inequality with the correct symbol (<, =, >)
>
> A 3.26 ○ 3.4 D 0.568 ○ 0.67
> B 4.98 ○ 49.8 E 81.0 ○ 81
> C 2.7 ○ 2.70 F 52.0 ○ 5.29
>
> Copy the decimals in column form and add.
>
> A 1.2 + 0.8 + 3.42
> B 75.68 + 0.432 + 2.909
> C 0.986 + 0.05 + 0.687
> D 2.757 + 5.4 + 7.83 + 43.7
>
> Subtract.
>
> A $5.0 - 0.45$ B $1.653 - 0.72$ C $31.58 - 2.09$
> D $4.45 - 2.0$ E $2.0 - 0.326$ F $0.463 - 0.108$

Resources for Active Learning
Discovery, Section II. Unit 5/5, Encyclopaedia Britannica Educational Corp.
Developmental Math Cards: J[1]13, Addison-Wesley. [Using an abacus]

Duplicator Masters, page 55
Workbook, page 90
Skill Masters, page 55

Using the Exercises
Have children do the exercises on page 259 independently. When they have finished, check their papers and allow time for discussion. The stress again should be on proper placement of the decimal point. Note this particularly in exercises 2N–P, in which the child must align the decimals in vertical notation.

For the *Think* problem, if children consider the last day of April as zero, then all multiples of seven will fall on Tuesday; thus, they need only count from Tuesday the 28th to land on Friday the 31st. Others may choose simpler solutions.

Assignments (page 259)
Minimum: 1, 2A–J, 3A–E.
Average: 1, 2, 3A–E.
Maximum: 1–3.

PAGES 260–261

Objective

Given addition and subtraction problems using decimal numerals and monetary notation, the child will be able to relate money notation to decimal notation and solve the problems.

Preparation

To stimulate interest in this lesson, ask children how expensive it is to own and use a car. Lead them to discuss the different expenses necessary to keep a car running and in good condition. Finally, focus on the need for fuel and the use and expense of gasoline, and direct the children's attention to the investigation.

Investigation

Note that, in the data provided for this investigation, both the litres and the cost make use of decimals. If any child tries to add or subtract combinations of litres and dollars, help him think through his work to discover his mistake. If any children should happen to write a multiplication or division problem, discuss with them how they might solve it, since they have not yet studied any methods for multiplying and dividing either decimals or fractions. If capable children want to try to solve such a problem, suggest ideas mentioned in the follow-up section of this lesson. If children remain within the scope of the investigation question, they should finish this investigation quickly.

● *Let's solve some decimal problems.*

Investigating the Ideas

Jan Jones kept a record of the number of litres and the cost of the gasoline she used in her car in one week.

	Litres	Cost
Mon.	7.8	$2.57
Wed.	13.5	$4.44
Sat.	16.6	$5.46

? Can you write and solve one addition and one subtraction problem from the chart?

Sample problem: How much did Jan Jones spend on gas during the week?

Problems will vary. See Investigation. $2.57 + $4.44 + $5.46 = $12.47

Discussing the Ideas

1. Explain how decimals are used in symbols for money, such as $7.54. To distinguish between number of dollars and number of cents
2. What are some other uses of decimals? See Discussion.
3. Give the missing amounts and explain your answers.
   - A Since 6.59 + 8.23 = 14.82, we know that $6.59 and $8.23 is ||||. $14.82
   - B Since 3.65 + 6.25 = 9.90, we know that $3.65 and $6.25 is ||||. $9.90
   - C Since 4.87 + 7.38 = 12.25, we know that $4.87 and $7.38 is ||||. $12.25
   - D Since 5.95 − 3.49 = 2.46, we know that $5.95 less $3.49 is ||||. $2.46
   - E Since 10.00 − 6.98 = 3.02, we know that $10.00 less $6.98 is ||||. $3.02

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Discussion

As you discuss exercise 1, ask children to express the value of various coins in decimal-money notation. Then ask them to change these numerals to fractions. For example, a dime, $0.10, is 1/10 of a dollar; a nickel, $0.05, is 5/100 of a dollar; a penny, $0.01, is 1/100 of a dollar; a quarter, $0.25, is 25/100 or 1/4 of a dollar; a fifty-cent piece, $0.50, is 50/100 or 1/2 of a dollar.

During your discussion of exercise 2, the children might mention the use of decimals to show kilometres on an odometer, weights or volumes on grocery packages, precise temperatures, baseball batting averages and earned run averages, results of track and field events, and the like.

Exercise 3 develops the important point that addition and subtraction with money notation may be worked out as with decimal notation. Point out that $6.59 may be thought of as six and fifty-nine hundredths dollars.

Using the Ideas

1. Find the total amounts.

A	B	C	D
\$8.53	\$0.96	\$10.90	\$34.59
4.27	4.34	5.38	86.79
\$12.80	\$5.30	\$16.28	\$121.38

2. Find the difference in the amounts.

A	B	C	D
\$9.67	\$6.75	\$10.00	\$50.00
4.83	0.86	3.67	46.72
\$4.84	\$5.89	\$6.33	\$3.28

3. A gasoline pump shows these amounts after 9.3 litres of gasoline have been put into a gas tank.

1 3 0 Dollars   9 3 Litres

A After 0.9 of a litre more goes into the tank, how many litres will the pump show? 10.2

B If gasoline costs \$0.14 per litre, how much will it cost to fill the tank with 10 litres? \$1.40

4. The picture shows the odometer reading before and after a trip. How many kilometres were travelled? 85.7

5 9 4 6 8   6 0 3 2 5

5. A litre of pure water weighs 1 kilogram. A litre of gasoline weighs about 0.62 kilograms. Which weighs more? How much more? 0.38 kg
Water

6. Oil is lighter than water but heavier than gasoline.
A litre of oil weighs about 0.81 kg.

A How much more is the weight of a litre of water than a litre of oil? 0.19 kg

B How much more is the weight of a litre of oil than a litre of gasoline? 0.19 kg

C By how much does a litre of water, a litre of oil, and a litre of gasoline increase the weight of a car? 2.43 kg

★7. Mrs. Gomez spent \$2.75 on fresh fruits and vegetables, \$7.58 on meat, and \$9.24 on other items. How much discount did she get? 195

The store gives 1/10¢ discount for each 10-cent purchase.

$\frac{1¢}{10}$

More practice, page A-28, Set 52

Using the Exercises
Have the children do the exercises on page 261 independently. Since problems 3–7 may seem difficult to some, encourage children to share their ideas about how to solve them. You may want to work through problem 7 with the children. Help them see that first they must find the total amount spent by Mrs. Gautier. Then, they must think how many 10-cent purchases are contained in \$19.57. Since they are dealing with money, they should not round their answer.

$$10\overline{)1957} \quad 195 \text{ R7}$$

Assignments (page 261)
Minimum: 1–3. Average: 1–6.
Maximum: 1–7.

Follow-up
To challenge capable children who are interested in figuring out how much Mr. Jones paid for one litre of gasoline on each day, encourage them to use their estimating and rounding skills. For example, 7.8 litres is almost 8, so they may think $8 \times ? = \$2.57$ and divide or use trial and error to arrive at an estimate. Any estimate between 34 and 32 is near enough to the best estimate, which would be 33.

Resources for Active Learning
Math Activity Cards, D21, Macmillan.

Workbook, page 91

PAGES 262–263

Objective

Given measurements in the metric system (metres, decimetres, centimetres, and millimetres), the child will be able to relate his understanding of decimals to equivalents between these units.

Preparation

Materials

metre sticks (1 for every 2 or 3 children, if possible); centimetre rulers (1 per child)

To prepare for this lesson, you might briefly review the place value for tenths, hundredths, and thousandths. Present examples such as these:

$$11.111 = 10 + 1 + \tfrac{1}{10} + \tfrac{1}{100} + \tfrac{1}{1000}$$
$$7.346 = 7 + \tfrac{3}{10} + \tfrac{4}{100} + \tfrac{6}{1000}$$

Point out how each place to the right of the decimal gets smaller: $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$. The relation of these ideas to the metric system will be explored in the investigation.

Investigation

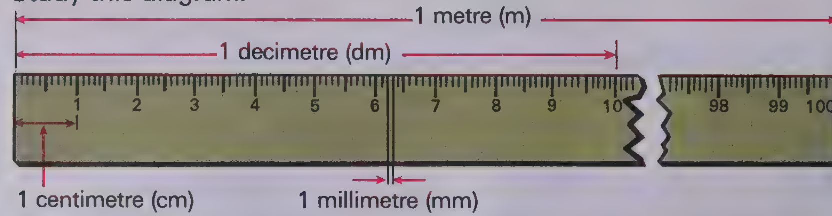
This investigation can best be handled by children in small groups. It is important that children help each other make the measurements. Most of the measurements may be made with centimetre rulers, but metre sticks should be used for the measurement of the children's heights. Remind children that they should use the diagram to help them figure out how to record their measurements. Move around the room helping them interpret the diagram.

The decimetre unit might be new to some children; others may need guidance in reading and recording millimetres.

● *How are decimals and metric units related?*

Investigating the Ideas

Study this diagram.



How many of these measurements can you find?

A Your height in metres.
B The length of your shoe in decimetres.
C The length and width of your mathematics book in centimetres.
D The thickness of your pencil in millimetres.

See Investigation.

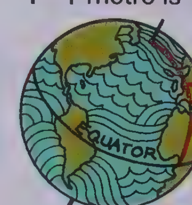
Discussing the Ideas

1. Janet found that her height was 147 centimetres. How could Janet write a decimal that would give her height in metres? 1.47

2. Use the diagram above to help you give the missing numbers.

A 1 metre is ▒ cm. 100	D 1 centimetre is ▒ mm. 10
B 1 decimetre is ▒ cm. 10	E 1 decimetre is ▒ mm. 100
C 1 metre is ▒ dm. 10	F 1 metre is ▒ mm. 1000

3. A metre is one ten millionth of the distance from the North Pole to the equator.



A About how many metres is it from the North Pole to the South Pole? 20 000 000

B About how many metres is it all the way around the world? 40 000 000

Discussion

A transparent plastic centimetre ruler for use with the overhead projector would be helpful during this discussion. Stress that, like our decimal number system, the metric system is based on ten. Explain the symbols used in exercise 2: cm = centimetre; dm = decimetre; m = metre; mm = millimetre. It would also be helpful to discuss the following relationships:

10 dm = 1 m	10 mm = 1 cm
10 cm = 1 dm	100 cm = 1 m

If possible, use a globe while discussing exercise 3. If children wish to check their findings in reference books, you might observe that the figure used in the problem is approximate rather than exact.

Using the Ideas

1. Give the missing fractions.
   - A 1 dm is ▮ of a metre. $\frac{1}{10}$
   - B 1 cm is ▮ of a metre. $\frac{1}{100}$
   - C 1 mm is ▮ of a metre. $\frac{1}{1000}$

2. Give the correct decimal for each exercise.
   - A $\frac{1}{10}$ m = ▮ metre
     Answer: 0.1
   - B 1 dm = ▮ metre 0.1
   - C $\frac{1}{100}$ m = ▮ metre 0.01
   - D 1 cm = ▮ metre 0.01
   - E $\frac{1}{1000}$ m = ▮ metre 0.001
   - F 1 mm = ▮ metre 0.001
   - G $\frac{25}{100}$ m = ▮ metre 0.25
   - H 25 cm = ▮ metre 0.25

Ellen spent half her money at the drug store. She then spent half of what was left at the candy store.

1. What fractional part of her money has she spent? $\frac{3}{4}$
2. If she has 20¢ left, how much did she start with? 80¢

3. Give the missing numbers.
   - A 0.001 metre = ▮ millimetre 1
   - B 0.01 metre = ▮ centimetre 1
   - C 0.1 metre = ▮ decimetre 1
   - D 0.007 metre = ▮ millimetre 7
   - E 0.06 metre = ▮ centimetre 6
   - F 0.8 metre = ▮ decimetre 8

4. Study the example. Then copy the sentences on your paper and give the missing numbers.
   Example: 6.254 m = 6 m, 2 dm, 5 cm, and 4 mm
   - A 7.834 m = ▮ m, ▮ dm, ▮ cm, and ▮ mm 7, 8, 3, 4
   - B 8.203 m = ▮ m, ▮ dm, ▮ cm, and ▮ mm 8, 2, 0, 3
   - C 9.640 m = ▮ m, ▮ dm, ▮ cm, and ▮ mm 9, 6, 4, 0
   - D 7.023 m = ▮ m, ▮ dm, ▮ cm, and ▮ mm 7, 0, 2, 3

★ 5. Find the totals so the number of millimetres, centimetres, and decimetres is less than 10.

A	B
2 m 5 dm 1 cm 8 mm	6 m 7 dm 9 cm 4 mm
4 m 7 dm 2 cm 6 mm	4 m 4 dm 8 cm 7 mm
7 m 2 dm 4 cm 4 mm	11 m 2 dm 8 cm 1 mm

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Resources for Active Learning

Experiences in Mathematical Ideas, Vol. 1, "The Concept and the Numeral," Unit 6, Experience 1, NCTM.

Measure and Find Out, Book 2, "The Metric System," Activities 1/1-1/9, Scott Foresman. (Available from Gage Educational Publishing)

Using the Exercises

Although for capable children you might assign the exercises on page 263 as independent work, you may wish to work through parts of exercises 1, 2, and 3 with the children. For example, in order to help children understand exercise 1, remind them that 10 dm = 1 m (so 1 dm = $\frac{1}{10}$ m); 100 cm = 1 m (so 1 cm = $\frac{1}{100}$ m); and since 10 mm = 1 cm, 1000 mm = 1 m (so 1 mm = $\frac{1}{1000}$ m).

For those who try exercise 5, point out the similarity of these problems to a problem in the decimal form 2.518 + 4.726. Check the exercises carefully with the children and allow ample time for further discussion.

Assignments (page 263)

Minimum: 1–3. Average: 1–4. Maximum: 1–5.

PAGES 264–265

Objective

The child will demonstrate his ability to work with the concepts presented in this chapter.

Preparation

To prepare for this lesson, review with the children any topics with which they have had particular difficulty in the chapter. It might be most helpful to review the reading of decimals and to work through a few problems which can be used to bring out many basic concepts, such as the following:

$$7.325 + 32.4 + 8.102 = ?$$
$$2.01 - 0.895 = ?$$

1. Copy each exercise and give the missing numerators.

A $6.79 = 6 + \frac{7}{10} + \frac{9}{100}$ C $9.263 = 9 + \frac{2}{10} + \frac{6}{100} + \frac{3}{1000}$

B $8.07 = 8 + \frac{0}{10} + \frac{7}{100}$ D $8.407 = 8 + \frac{4}{10} + \frac{7}{1000}$

2. Give the correct decimal for each sum.

A $2 + \frac{3}{10} + \frac{6}{100}$ 2.36 C $1 + \frac{4}{10} + \frac{6}{100} + \frac{7}{1000}$ 1.467 E $18 + \frac{6}{10} + \frac{0}{100} + \frac{7}{1000}$ 18.607

B $75 + \frac{7}{10} + \frac{9}{100}$ 75.79 D $\frac{8}{10} + \frac{7}{100} + \frac{6}{1000}$ .876 F $18 + \frac{6}{10} + \frac{7}{1000}$ 18.607

3. Give the fraction suggested by each decimal.

A 0.7 $\frac{7}{10}$ B 0.07 $\frac{7}{100}$ C 0.007 $\frac{7}{1000}$ D 0.06 $\frac{6}{100}$ E 0.76 $\frac{76}{100}$ F 0.076 $\frac{76}{1000}$

4. Give a mixed numeral for each decimal. Use 10, 100, or 1000 for your denominators.

A 17.6 $17\frac{6}{10}$ B 38.23 $38\frac{23}{100}$ C 29.07 $29\frac{7}{100}$ D 6.124 $6\frac{124}{1000}$ E 18.062 $18\frac{62}{100}$

5. Give the correct sign (<, =, >) for each ■.

A 7.8 ■ 7.7 > D 0.67 ■ 0.68 < G 0.832 ■ 8.30 <

B 6.4 ■ 6.40 = E 8.32 ■ 8.30 > H 832 ■ 840 <

C 0.32 ■ 3.2 < F 83.2 ■ 8.40 > I 0.005 ■ 0.05 <

6. Find the sums and differences.

A $6.4 + 8.7$ 15.1 B $9.2 + 27.5$ 36.7 C $27.5 - 9.2$ 18.3 D $35.6 + 8.92$ 44.52 E $72.4 - 26.7$ 45.7 F $0.83 + 0.969$ 1.799

G 5.34 + 23.7 + 58.60 87.64

H 0.586 + 4.9 + 23.64 29.126

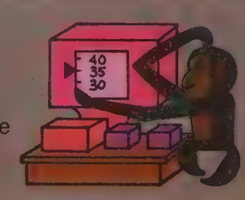
I 8.64 + 39.5 + 0.807 48.947

★**7.** Give a decimal for each fraction.

A $\frac{1}{2}$ .5 B $\frac{1}{5}$ .2 C $\frac{4}{5}$ .8 D $\frac{1}{4}$ .25

★**8.** Give the lowest-terms fraction for 0.750. $\frac{3}{4}$

think



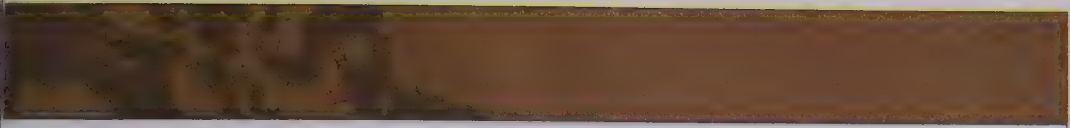
The large block weighs 3 kg more than the 2 small blocks together. What does each block weigh?

264

Discussion

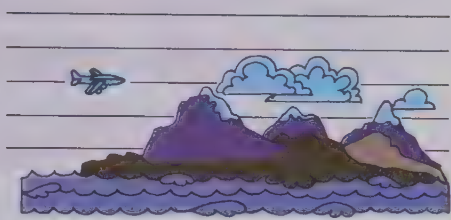
Page 264 may be used as an evaluation instrument or as a review page. Whether you discuss the exercises before the children have done them or afterward, continue to stress the idea that decimals represent fractional numbers, which may be written as fractions or as decimals. Use examples of expanded notation to exhibit this relationship.

Encourage children to use a trial and error method for solving the *Think* problem. Discuss it only after those interested have had an opportunity to work on it. Often it is best to wait a day or two before discussing these problems so that children can think about them over an extended period of time.

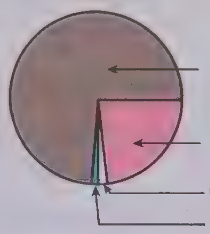


Land, Sea, and Air

The chart below shows how North America's land area and coastline is divided among 10 countries.



1. Canada and the United States make up what part of North America's land area? 0.87
2. If Mexico's land area is added to that of the U.S. and Canada, what part remains? 0.04
3. What part of North America's coastline do the 9 countries other than Canada share? 0.27
4. What part of North America's coastline do El Salvador, Guatemala, and the Honduras have? None
5. Air surrounds us in a thin layer above the land and sea. Air is made up mostly of nitrogen and oxygen. The circle graph shows what part is nitrogen and what part is oxygen.
   - A Nitrogen and oxygen together make up what part of the air? 0.99
   - B What part of the air is made up by other gases? 0.001



REGION	AREA	COASTLINE	REGION	AREA	COASTLINE
British Honduras	—	—	Honduras	0.01	—
Canada	0.45	0.73	Mexico	0.09	0.08
Costa Rica	—	0.01	Nicaragua	0.01	0.01
El Salvador	—	—	Panama	—	0.02
Guatemala	0.01	—	United States	0.42	0.14

*No entry means that this part of the whole is less than 0.01.

265

Using the Exercises

Before assigning page 265, it might be helpful to introduce a map of North America and discuss it with the children. Point out the location of the different countries, especially those in Central America.

Also point out that $\frac{78}{100}$ of the air is nitrogen and about $\frac{21}{100}$ is oxygen. Give children an opportunity to discuss this material. Then, have them do the exercises. When they have finished, allow time for further discussion and for checking papers.

Workbook, page 92

PAGES 266–267

Objective

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

Review any topic in the exercises which the children have found troublesome. For example, review changing fractions to fractions with common denominators and work through a few examples of comparing or adding fractions with unlike denominators.

Keeping in Touch with Computing Measurement Geometry Fractional numbers

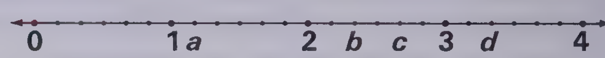
1. Find the product, difference, and quotients.

A 5905 − 1767 = 4138 B 376 × 27 = 10 152 C $7\overline{)36\,574}$ 5224R6 D $65\overline{)3445}$ 53 E $87\overline{)4567}$ 52R43

2. Choose the point on the number line for the fractional number.

A $\{\frac{5}{2}, \frac{10}{4}, \frac{15}{6}, \frac{20}{8}, \ldots\}$ c

B $\{\frac{7}{3}, \frac{14}{6}, \frac{21}{9}, \frac{28}{12}, \ldots\}$ b



3. Mark true (T) or false (F) for each exercise.

A $\frac{3}{4} = \frac{6}{8}$ T D $\frac{0}{5} > \frac{0}{2}$ F G $\frac{5}{5} < \frac{8}{8}$ F J $\frac{1}{4} < \frac{4}{10}$ T M $\frac{12}{4} = 3$ T

B $\frac{1}{4} = \frac{2}{6}$ F E $3\frac{1}{7} < 4$ T H $3\frac{1}{2} = \frac{7}{2}$ T K $\frac{1}{5} < \frac{1}{6}$ F N $\frac{15}{3} > 5$ F

C $\frac{3}{2} = 1\frac{1}{2}$ T F $\frac{5}{2} = \frac{10}{2}$ F I $\frac{3}{4} = \frac{4}{5}$ F L $\frac{3}{7} = \frac{4}{10}$ F O $4\frac{1}{8} > 4$ T

4. Find the sums and differences.

A $\frac{3}{4} + \frac{4}{5}$ $\frac{31}{20}$ or $1\frac{11}{20}$ C $\frac{12}{7} - \frac{6}{7}$ $\frac{6}{7}$ E $\frac{4}{5} + \frac{3}{10}$ $\frac{9}{6}$ or $1\frac{1}{2}$ G $\frac{4}{5} + \frac{1}{10}$ $\frac{17}{18}$

B $2\frac{1}{6} + 3$ $5\frac{1}{6}$ D $5\frac{3}{4} - 2\frac{1}{4}$ $3\frac{1}{2}$ F $\frac{1}{4} - \frac{1}{5}$ $\frac{1}{12}$ H $6\frac{1}{5} + 8\frac{3}{4}$ $14\frac{19}{20}$

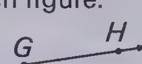
5. Find the totals. Write the answer so that you have the greatest possible number of the larger unit.

A	B	C
6 days 12 h	24 wk 9 days	7 h 51 min
5 days 18 h	8 wk 12 days	24 h 36 min
12 days 6h	35 weeks	32h 27 min

6. Give a symbol for each figure.

A 

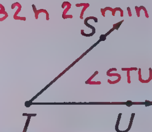
$\overline{AB}$

B 

$\overrightarrow{GH}$

C 

$\overleftrightarrow{RS}$

D 

∠STU

You are invited to explore

ACTIVITY CARD 13 Page 339

266

Discussion

Before assigning page 266, you may want to work through one of the problems in exercise 5, to help the children recall how measurements given in smaller units can sometimes be converted to measurements in larger units. You might also find it necessary to review the geometric symbols used in exercise 6 or refer children to Chapter 4, pages 69 and 75, to review how they used the symbols previously.

When children finish the exercises, carefully check their work, discussing with them any areas of difficulty. Provide additional help for any children who had difficulty with exercises 1 through 4; the skills treated here are essential to future work.

AIRPLANE SPEEDS

Speed with no wind: 500 km/h

Speed with head wind: 460 km/h — 40 km/h

40 km/h — Speed with tail wind: 540 km/h

Each of the three airplanes would travel at the rate of 500 kilometres per hour if there were no wind. The airplane that has a 40-kilometre-per-hour head wind travels only 460 kilometres in one hour. The airplane that has a 40-kilometre-per-hour tail wind travels 540 kilometres in one hour.

1. Give the distance that each plane travels in one hour.
   A 570 km D 655 km
   B 642 km E 253 km
   C 320 km

Plane	Tail wind	Speed without wind	Head wind
A	—	600	30
B	47	595	—
C	—	320	—
D	75	580	—
E	—	310	57

2. If an airplane travels 620 kilometres in one hour and would travel 580 kilometres per hour if there were no wind, is there a head wind or a tail wind? How fast is the wind?
   Tail wind, 40 km/h
3. Give the missing numbers in this chart.
   A 3432 km C 1567 km/h
   B 573 km/h D 17 h

	Kilometres per hour	Hours	Kilometres
A	286	12	▥
B	▥	14	8022
C	▥	5	7835
★D	586	▥	9962

★4. An airplane flies 600 km/h with no wind. If it flies $\frac{1}{2}$ hour with no wind, then flies 2 hours with a 50-km/h head wind, and finally flies $1\frac{1}{2}$ hours with a 20 km/h tail wind, how far does it fly?
   2330 km

267

Using the Exercises
Help the children analyze the top of page 267. Discuss the effects of a head wind and a tail wind. For example, compare the distances each plane would cover if they all flew for two hours. After flying two hours with no wind the airplane would have travelled 1000 kilometres. With the head wind, it would go only 920 kilometres in two hours. With the tail wind, it would travel 1080 kilometres in two hours. When the children finish the exercises, check the work and discuss any problems of particular interest.

Follow-up
Page 267 might motivate children to do some research regarding airplane speeds. They might write to local airports inquiring about speed of various airplanes and the frequency and speed of winds which would affect the airplanes' speed. This information might be charted for display or used to make assignment cards similar to the exercises on page 267. Such a research activity might be valuable for the children even if no specific mathmatics problems are treated.

CHAPTER 13 | Geometry and Measurement II

Pages 268–287

General Objectives

To provide experience in space geometry

To provide experience in constructing models and drawing pictures of space figures

To review the concept of volume

To introduce the concept of surface area

To introduce perspective in viewing space figures

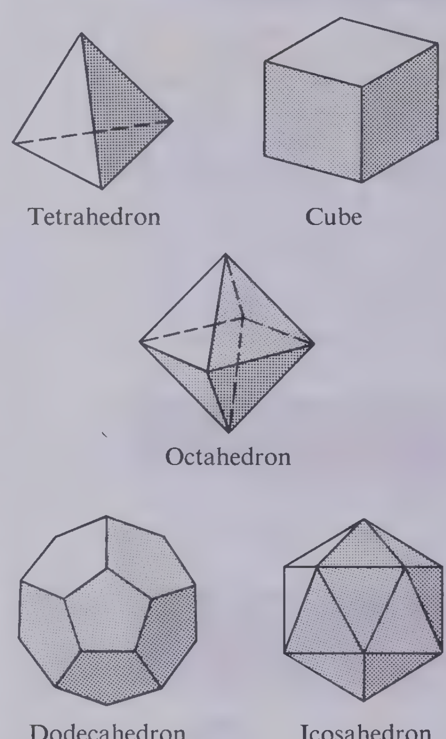
The opening lessons of this chapter introduce some basic space figures and explore various space relationships. The next two lessons give children an opportunity to construct models of space figures and to draw pictures of them. After the concept of volume is reviewed, surface area concepts of rectangular solids are developed. Following the exploration of surface area, the relation between volume and surface area is studied. Next, three-dimensional objects are viewed in different perspectives, and finally the faces, edges, and vertices of some interesting space figures are compared. The chapter concludes with both a chapter and a cumulative review which should help you assess the children's understanding of the geometry and measurement topics covered thus far.

Mathematics

Most children will enjoy making models of space figures like those discussed in this chapter, and some may be motivated to explorations beyond the limits of the text.

A *polyhedron* is a special kind of space figure whose faces are plane geometric regions. If all of the faces of a polyhedron are regular polygons and all of its dihedral angles are congruent, the figure is a regular polyhedron. There are exactly five regular polyhedra. (See the figures above, right.)

These five polyhedra are known as the Platonic solids. Their name derives from the Greek philosopher Plato, who is reported to have discovered them about 400 B.C. (It is known, however, that the Egyptians, long before Plato, were familiar with all of the polyhedra, with the possible exception of the dodecahedron.)



The Five Regular Polyhedra

Another interesting set of polyhedra are the Archimedean solids. These polyhedra are sometimes called semi-regular polyhedra because they have a variety of regular polygons as faces and each vertex is the same, that is, all of the dihedral angles are congruent. There are exactly 13 Archimedean polyhedra.

Some of the most beautiful and striking pholyhedra are the non-convex star polyhedra, also called the stellated polyhedra. Patterns for these polyhedra may be found in many reference books and can be constructed by anyone who has some skill and patience.

Teaching the Chapter

Materials

Bricks or similar rectangular prisms
Centimetre rulers
Construction paper
Cubes (at least 12 per child)
Frozen-juice cans
Geometric solids (Commercially made sets would be best. Most sets contain at least the following: cubes, triangular and rectangular prisms, cone, sphere, triangular and rectangular pyramids, cylinder, truncated cone, hemisphere. See also, *Duplicator Masters,* pages 68–72.)
Glue
Graph paper, 1 cm grid
Scissors
Small boxes (1 per child)
Tape
Tracing paper

Vocabulary

cone
cube
cubic unit
cylinder
dimensions
edges
faces
height
hemisphere
hexagonal prism
length
octagonal prism
octagonal pyramid
pentagonal prism
pentagonal pyramid
plane figure
rectangular prism
rectangular pyramid
regular dodecahedron
regular icosahedron
regular octahedron
space figure
sphere
square prism
square pyramid
surface area
torus

triangular prism
triangular pyramid
truncated cone
vertices
volume
width

The vocabulary list for this chapter is quite long. Many of the words listed may already be familiar to the children since they have occurred in measurement lessons in earlier parts of the text or in geometry and measurement lessons in previous books. The children should not be required to memorize the names of space figures since it will be some time before these words are used again. The names are given here simply to provide a convenient means of talking about specific space figures, and should be used primarily for that purpose. (If the names reappear in subsequent review material, you can have the children look them up, if necessary.) The other words in the vocabulary list are vital to the study of geometry, and an attempt should be made to see that the children not only remember them but also understand their meaning.

Lesson Schedule

Plan to spend from two to two and a half weeks on this chapter. Of course, you will want to adjust your schedule according to the needs, abilities, and interests of your children. Some of your children may find this one of the most stimulating chapters in the text, so you may want to allot some extra time to capitalize on this interest.

Evaluation of Progress

In evaluating the children's achievement for this chapter, try to avoid overemphasizing the mechanical skills involved in construction. Rather, stress understanding of a few general ideas.

Though it may be quite difficult to evaluate the children's understanding of the space concepts, take special care to evaluate the children on a day-to-day basis.

We stress again the fact that one of the chief objectives of this chapter is to give the children a general feeling for the study of geometry rather than to teach specific facts, notation, or names.

Pages 286 and 287 will help you assess children's understanding of space figures and associated measurement concepts, and will provide general information about their retention of previously introduced concepts and skills.

Resources for Active Learning

GENERAL ACTIVITIES

Experiments in Mathematics, Stage 1, pp. 26–29; Stage 2, pp. 44–45, Houghton Mifflin [Patterns for cube, pyramid, other polyhedra] (Available from Thomas Nelson and Sons)

Freedom to Learn, "Geometry—Properties of Shapes," pp. 149–152, Addison-Wesley

Math Activity Cards, D13; D36, Macmillan

Maths Mini-lab, Cards 107–109, Selective Educational Equipment [Using Geo Blocks]

Nuffield Project: *Shape and Size 3,* "Polyhedra," pp. 55–59, Wiley

MANIPULATIVE DEVICES

Cuisenaire Cubes, Squares, and Rods (Cuisenaire Co.)

Geo Blocks (McGraw-Hill Ryerson; Selective Educational Equipment)

Geo-D-Stix (Childcraft; Creative Playthings; Cuisenaire Co.)

Geometric Figures and Solids (school supplier)

Geometric Models Construction Kit (Milton Bradley)

Lake and Island Board (Math Media; Responsive Environments Corp.)

LaPine Sage Kit (LaPine Scientific)

Moby Lynx (Kendry)

Space Spider (Childcraft; LaPine Scientific; Nasco)

Straw Polyhedra Kit (Creative Publications)

"Thing-Sticks" (Edmund Scientific)

True Equal-Volume Geometric Set (Edmund Scientific)

COMMERCIAL GAMES

Geometry Matching (Selective Educational Equipment)

Polyhedron Rummy (Scott Foresman)

Soma Cube and block puzzles (Creative Publications; Cuisenaire Co.; Edmund Scientific)

Two-Piece Pyramid Puzzle (World Wide Games)

PAGES 268–269

Objective

Given names of basic space figures, such as rectangular prism, cylinder, sphere, pyramid, and cone, the child will be able to recognize objects which suggest these figures.

Preparation

Materials

geometric solids (at least one set, containing prisms, cone, sphere, pyramids, cylinder, cube, and truncated cone)

There are a variety of ways you might prepare for this lesson. For example, although the preparation time is usually short, you might stimulate interest in the whole chapter by taking the children on a short walk and having them observe and list several large objects. These would then be used during the discussion. Or, you might pass around geometric solids and have the children feel and study them.

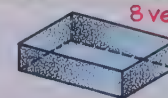
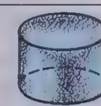
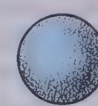
Investigation

Read through the chart with the children. Then encourage them to think of each of the three figures and to list objects which match them. It would be appropriate to have the children work in small groups so that they might stimulate each others' imaginations and think of a variety of objects. Observe the children closely, taking care to give them sufficient time to name many objects and also to introduce the discussion before interest wanes.

13 Geometry and Measurement II

● *What are some basic space figures?*

Investigating the Ideas

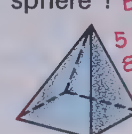
We **see** these physical objects.	We **think** about these space figures.	We **say** these names.
book box block ice cube building 	 8 vertices 12 edges 6 faces	rectangular prism
can barrel glass chalk checker tank 	 0 vertices 0 edges 2 faces	cylinder
ball balloon orange marble Earth 	 0 vertices 0 edges 0 faces	sphere

? How many other physical objects that remind you of these space figures can you name? See sample answers above.

Discussing the Ideas

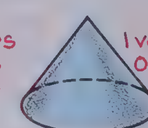
1. What simple name is often used for a
   A rectangular prism? Box B sphere? Ball

2. What are some physical objects that remind you of these space figures? See Discussion.



5 vertices
8 edges
5 faces

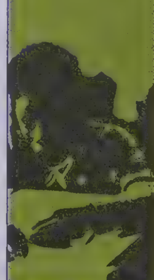
rectangular pyramid



1 vertex
0 edges
1 face

cone

3. Can you describe some space figures that are different from any of the ones shown on this page?
   Answers will vary. Sample possibilities: Donut shape (torus), tent shape (triangular prism)



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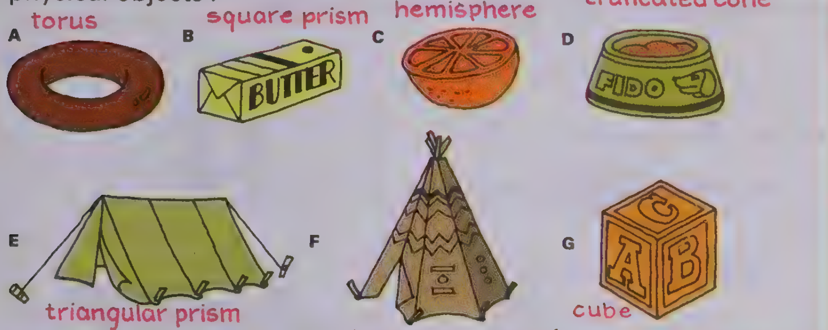
Discussion

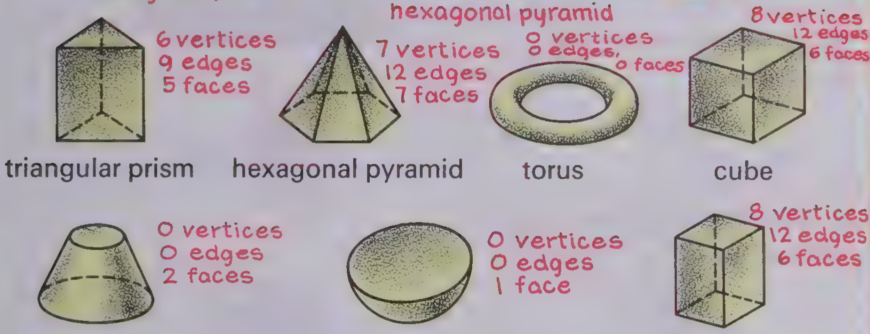
Have several children share their list of objects for each figure. Use the geometric solids to show each figure as it is being discussed. Similarly, use the demonstration solids as you discuss the exercises. Children might find it humorous that a simple box has such an imposing name as "rectangular prism," and that a "sphere" is commonly known as a ball. Also, note that a cube is a particular kind of rectangular prism. In exercise 2, as examples of pyramids, children might suggest a church steeple, a hunk of cheese cut in pyramid shape, a pile of oranges arranged in a pyramid shape, the Egyptian pyramids, and so on. As examples of cones, they might suggest ice cream cones, funnels, mountain peaks, or a top. Throughout the discussion, try to cite examples which are visible in the classroom, such as a circular wastebasket which suggests a truncated cone. The children may come up with some rather far-fetched suggestions, and you should allow them considerable latitude.

During the entire discussion, stress the fact that the figures in question are space figures. They do not lie in one plane, as does a square or circle; they are three-dimensional.

Using the Ideas

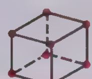
1. Which space figure below is suggested by each of these physical objects?





See Using the Exercises.

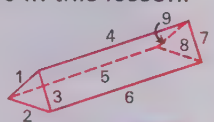
2. A cube has 8 **vertices.**



   A Which other figures have 8 vertices? Square prism, rectangular prism

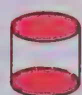
   B Give the number of vertices for each figure in this lesson. See answers beside figures above.

3. A triangular prism has 9 **edges.**



   Give the number of edges for each figure. See answers beside figures above.

4. A cylinder has 2 flat **faces.** Give the number of flat faces each figure has. See answers beside figures above.



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Using the Exercises

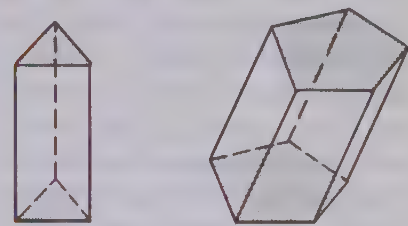
On page 269, after reading through the new terms in exercise 1, have the children do the exercises. Note that the triangular prism and the square prism are named according to the shape of their bases, not the shape of their sides (which are rectangular).

In exercises 2, 3, and 4, the children may have some difficulty in deciding the number of faces, edges, and vertices for some of the figures. Thus, the torus has neither vertices nor edges and has no flat faces. The truncated cone has two flat faces that are circular regions, but it has no vertices or edges since edges are usually thought to be segments. The hemisphere can be thought of as having one flat circular face but no edges or vertices. Allow sufficient time for discussion of these ideas after the children have completed the exercises.

Assignments (page 269)
Minimum: 1–4, oral. Average: 1 oral; 2–4. Maximum: 1–4.

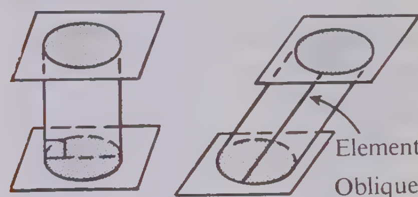
Mathematics

A *prism* is a three-dimensional figure whose bases are congruent polygons in parallel planes and whose faces are parallelograms.



Triangular prism　　Pentagonal prism

A *cylinder* is the figure formed by two congruent curves in parallel planes and the parallel line segments connecting corresponding points of the curves.



Right circular cylinder　　Oblique cylinder

A *sphere* is the set of all points in space at a fixed distance from some given point.

A *pyramid* is a three-dimensional figure with a polygonal base and triangular lateral faces. A tetrahedron is a triangular pyramid.

Triangular pyramid (tetrahedron)　　Square pyramid

A *cone* is the space figure formed by a closed plane curve and all line segments drawn from a point not in the plane of the curve to points of the curve.

Follow-up

Exercises 2, 3, and 4 on page 269 might be extended into a follow-up activity. For example, have children chart the name of the object and the number of edges, vertices, and faces. They might also write their own description of each object and see whether their classmates can recognize them. The set of geometric solids should be available for handling and study during such an activity.

PAGES 270–271

Objective

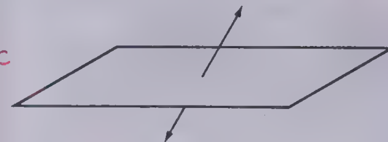
Given pictures of geometric space figures and of physical objects, the child will be able to match the space figure with the corresponding physical object.

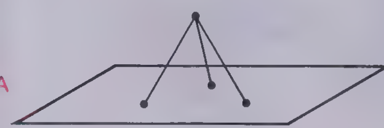
Preparation

To prepare for this lesson, it would be helpful to review the ideas of plane and line. Ask the children to look around the room for objects which make them think of a line, or a plane. Stress how things such as the chalk tray or a window frame only suggest a line—the idea of a line is an idea of something which goes on and on in one dimension. Similarly, a wall or the floor suggest the idea of a plane which extends indefinitely in two dimensions.

Let's explore space relationships.

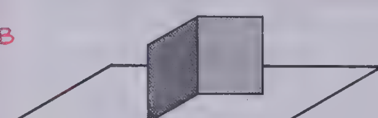
For each space figure on the left, give the letter of the physical object which best reminds you of the figure.

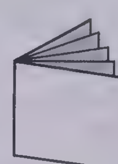
1. C

A line intersecting a plane

2. A

3 segments from a point to a plane

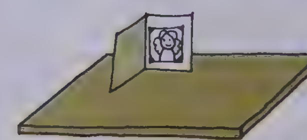
3. E

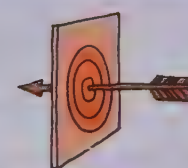
A plane through a line on another plane

4. B

3 planes through a single point

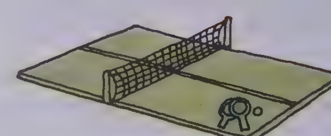
5. D

Several planes through one line

A


B


C


D


E


270

Discussion

Before assigning the exercises on page 270, make sure that the children can interpret the space figure pictures in the left column. Explain that each of these pictures represents a three-dimensional, or space, figure. The children are to match one of the pictured physical objects to one of the space figures. Remind them that figures such as squares, circles, and triangles are plane figures but that the figures in this lesson are space figures.

When the children have finished the exercises on both pages and as you are checking the exercises, ask children to mention other physical objects which the space figures might represent. During this discussion, you will want to emphasize continually the fact that the geometric objects are ideal things that we think about and that the physical objects simply remind us of these ideal geometric concepts.

Tell which picture best reminds you of the space figure that is described.

1. Two lines in space that do not meet and that are not in one plane D

2. Four planes F

3. Many lines in space, all going in the same direction A

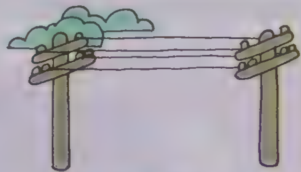
4. Two lines in one plane that do not cross B

5. A circle touching a plane at one point C

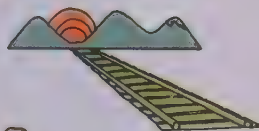
6. A plane, a circle, a sphere, and a truncated cone G

7. A sphere on a plane E

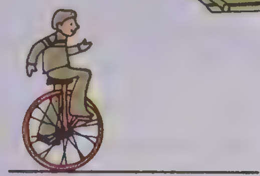
A



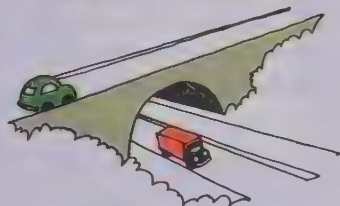
B



C



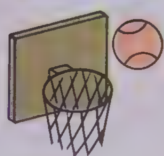
D



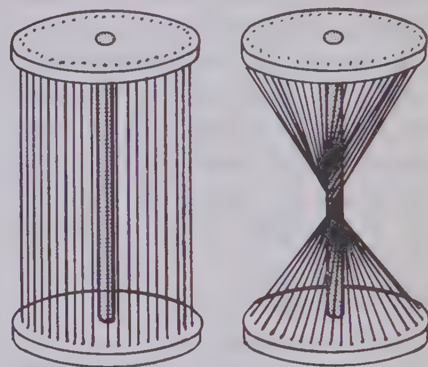
E

F

G



Follow-up
Deciding what boundaries the space surfaces have is highly abstract. Yet providing some sample space figures for the children to handle may give them some intuitive background for developing insight into the characteristics of space figures. Those made of clear plastic are especially good. An elastic-string model of a cylinder that can be twisted into a pair of cones also gives children valuable insights into these two figures of space geometry. A beam or plane of light projected through the cylinder will show some of the interesting waves and cross-sections.



Resources for Active Learning

Experiments in Mathematics, Stage 1, "Regular Polyhedra," pp. 24-25, Houghton Mifflin (Available from Thomas Nelson & Sons)

Franklin Series: *Patterns and Puzzles*, "Cutting and Folding Geometric Forms," pp. 49-55, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Franklin Series; *Pencil and Paper Geometry*, pp. 85-98, Lyons and Carnahan. [Making space figures] (Available from McGraw-Hill Ryerson)

Maths Mini-lab, Card 106, Selective Educational Equipment. [Making space figures]

Workbook, page 93

Assignments (page 271)
Minimum: 1–7, oral. Average: 1–7.
Maximum: 1–7.

PAGES 272–273

Objectives

Given appropriate materials, the child will be able to construct a pyramid, octahedron, pentagonal prism, and regular dodecahedron.

The child will be able to construct regular space figures from given models of faces for the figures.

Preparation

Materials

tracing paper; construction paper (*Duplicator Masters*, pages 68–71)

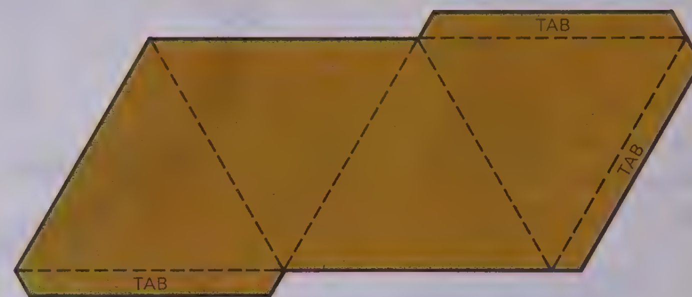
Use models to review with the children the various shapes and names of the more common space figures. Be sure to include a pyramid and a cube in this review. If available, also display a regular octahedron, a pentagonal prism, and a regular dodecahedron, since these are the shapes children will be constructing as a part of this lesson.

Investigation

Distribute tracing paper to the children and explain that they should trace both the heavy lines and the dotted lines but cut only along the heavy lines. The dotted lines indicate folding. The tracing paper may be used to fold and construct the figure but if it is very thin you might suggest that they use this as a template to draw the outline of the pattern on a piece of construction paper. Then they can cut and fold the construction paper to make the figure. If you choose, you might even duplicate the pattern for the triangular pyramid on stiff paper and simply direct the children to cut and fold (*Duplicator Masters*, page 69).

Can you make space figure models?

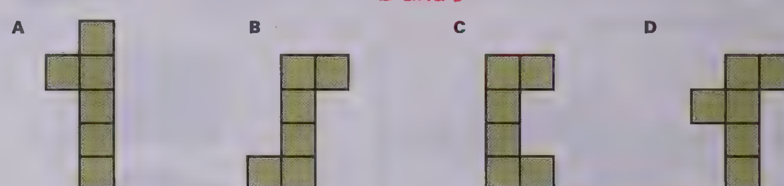
Investigating the Ideas



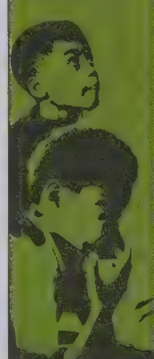
? Can you trace the pattern above, cut it along the solid lines, and fold on the dotted lines to form a space figure? See Investigation.

Discussing the Ideas

1. A What is the name of the space figure you made in the Investigation? Triangular pyramid
   B How many vertices does your figure have? 4
   C How many faces? 4 How many edges? 6
2. Which of the patterns below do you think would form a **cube** if cut out and folded? B and D



3. How many faces, edges, and vertices does this triangular prism have?
   5 faces, 9 edges, 6 vertices



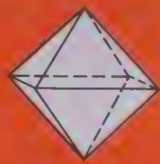
272

Discussion

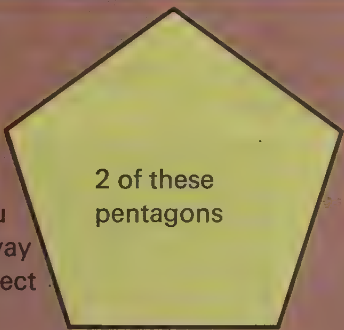
As you discuss exercise 1, point out that since each face of the figure is a triangle, we call the pyramid a *triangular pyramid*. For exercise 2, you might suggest that the children try out the patterns of the cube to check their answer. Point out the rectangular and triangular faces of the prism discussed in exercise 3. Throughout your discussion of these exercises, stress how the various shapes of the faces change the shape of the space figure.

Trace the figures and make the space figures shown.
See Using the Exercises.

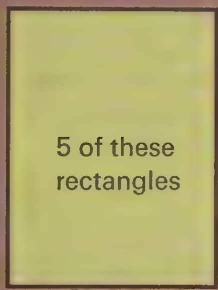
1 Can you connect 8 triangles like those in the Investigation to make this regular octahedron?



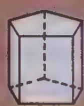
2 Can you find a way to connect



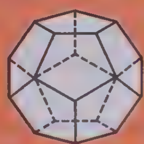
and



to make a pentagonal prism? →



★**3** Can you connect 12 pentagons like those in exercise 2 to make this regular dodecahedron?



Using the Exercises
The exercises on page 273 give the children an opportunity to construct other space figures. As suggested in the investigation, you might duplicate the shapes needed for the faces of these figures on stiff paper. You would need the same triangle used in the investigation, and the pentagon and rectangle illustrated on page 273. If you have the children use tracing paper instead of the duplicated figures, they will need heavier paper for constructing the space figures. They should cut out the shapes they have traced and then outline them on the construction paper.

Assignments (page 273)
Minimum: 1. Average: 1–2.
Maximum: 1–3.

PAGES 274–275

Objective

Given specific directions, the child will be able to draw a cube, a rectangular pyramid, and other space figures.

Preparation

Materials

cubes and pyramids (rectangular or triangular)

Use the demonstration solids to display the various space figures children will be drawing. Review the names of each figure, particularly those which are treated in this lesson: cube, rectangular prism, rectangular pyramid, triangular prism, triangular pyramid, and cylinder.

Investigation

If possible, place a cube and a rectangular pyramid in a position prominently visible to the children. This investigation is more an activity than a true exploration. Children should follow directions carefully. They may draw a couple of pictures for each figure, varying the size. Move around the room giving guidance when necessary, but do not do any of the actual drawing for any child. Point out how faces that are really squares or rectangles appear as rhombuses or parallelograms in the drawings of the figures.

● *Can you draw space figures?*

Investigating the Ideas

? Can you follow these directions and draw a larger picture of each of these space figures? See Investigation.

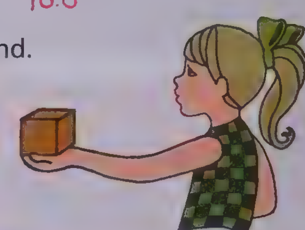
Cube

Draw a square for the "front" face. | Draw the corners of a square for the back face. | Make "hidden" edges dotted. Make other edges solid.

Rectangular Pyramid

Draw a parallelogram, half solid, half dotted. | Mark the "top" vertex. | Make "hidden" edges dotted. Make other edges solid.

Discussing the Ideas



1. Suppose you hold a cube in your hand.
   - A What is the greatest number of edges that could be hidden from your view? 8
   - B What is the smallest number? 3

2. Could you hold a pyramid so that you can see all of its edges at once? Explain.
   See Discussion.

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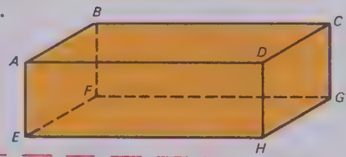
Discussion

If possible, have each child hold a cube in his hand and examine it. In viewing the cube, no matter how it is twisted, at least 3 edges will be hidden. If it is held flat on the palm and just one face is viewed, eight edges are hidden; only the four edges around the face are visible.

Suggest that children use both triangular and rectangular pyramids and that they view them from varying positions. They should discover that, if they view a pyramid from directly above, they can see all of its edges at once. Again, point out that the triangular and rectangular faces often appear to have different shapes when viewed from different perspectives. The shape does not change, only our perception of the shape changes.

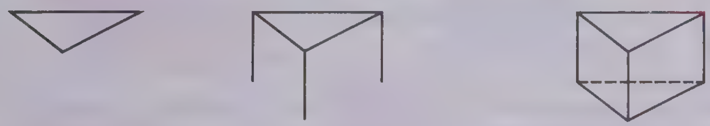
Using the Ideas

1. The front face of the rectangular prism shown here is named *ADHE*.
   A Name all the other faces.
   B Name the edges.
   C Which faces are hidden?
   D Which edges are hidden?



A ABFE, ABCD, BCGF, DCGH, EFGH; B $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{AD}$, $\overline{AE}$, $\overline{BF}$, $\overline{CG}$, $\overline{DH}$, $\overline{EF}$, $\overline{FG}$, $\overline{GH}$, $\overline{EH}$; C ABFE, BCGF, EFGH; D $\overline{EF}$, $\overline{BF}$, $\overline{FG}$

2. Follow the steps below and draw a **triangular prism.**



Draw a triangle. Draw 3 parallel, congruent segments. Make hidden edges dotted. Make other edges solid.

think

Draw and cut out a circle. Fold the paper so that the circle is divided into 12 equal parts. Label your circle like this:

If you start at 1 and draw to the dot 5 spaces away each time until you get back to 1, what design will you make? Can you make other designs like this?

3. Draw a triangular pyramid. Example 2 in the Investigation may help you.

4. Draw a rectangular prism. Problem 1 above may help you. See figure for problem 1.

5. Draw a cylinder.

★ 6. Pick out an object, such as a doghouse or a piano, and draw a picture of it. Be sure to include dotted lines to show hidden edges.

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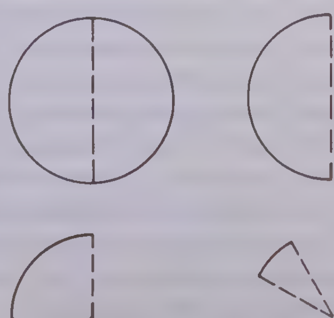
Using the Exercises
The exercises on page 275 continue the suggestions for drawing various space figures. You may wish to use exercise 1 as a basis for discussion. However, it is intended that children concentrate on their drawings. Do not expect the children to draw perfect figures, but help them realize how the dotted lines and perspective are used to picture three-dimensional, space figures in a two-dimensional drawing.

Children might have difficulty properly labelling the circle in the *Think* problem. Encourage them to figure out how to do it themselves, but if necessary show some how to fold the circle into fourths and then into twelfths.



Assignments (page 275)
Minimum: 1–2. Average: 1–4.
Maximum: 1–6.

Workbook, page 94

PAGES 276–277

Objective

Given the dimensions of a rectangular prism, the child will be able to find its volume.

Preparation

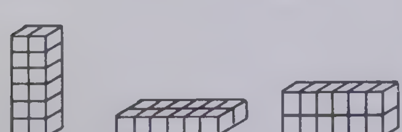
Materials

12 cubes for each 1 or 2 children (Sugar cubes could be used.)

It would be appropriate to begin immediately with this investigation. Two-centimetre cubes would be best, but if you do not have enough so that every 1 or 2 children have 12, use a substitute. Note: Check the materials list for the next lesson; you may wish to have children bring in their own small boxes.

Investigation

Have children work independently or in pairs. Remind them that they should record their work, and let them discuss and decide among themselves the best way to do this. However, you might want to point out certain disadvantages of such recording methods as drawing pictures; for most children, this would take an excessive amount of time. Others may decide to record the prisms by listing the length, width, and height. Although this is perhaps the best method, it might easily lead the children to consider more shapes than there actually are. For example, the prism with length 2, width 1, and height 6 has the same shape as a prism with length 6, width 2, and height 1.



$2 \times 1 \times 6$ $6 \times 2 \times 1$ $6 \times 1 \times 2$

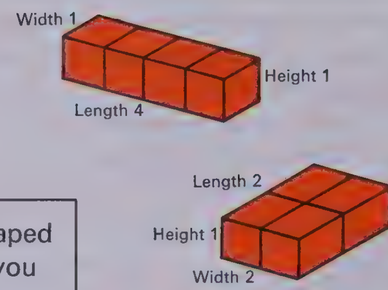
All of these prisms have the same shape, though height, length, and width may be considered to be different. The four different shapes will be some arrangement of the following sets of dimensions.

12, 1, 1	3, 4, 1
6, 2, 1	2, 2, 3

Let's find the volume of a space figure.

Investigating the Ideas

Here are the only two different-shaped boxes (rectangular prisms) that can be made from 4 cubes.

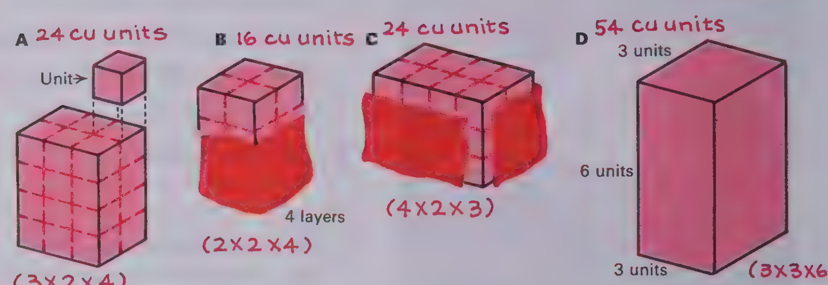


How many different-shaped rectangular prisms can you make with 12 cubes?

See Investigation.

Discussing the Ideas

1. The **dimensions** of a rectangular prism are its **length**, **width**, and **height**. What are the dimensions of each rectangular prism you found in the Investigation? See Investigation.
2. What would be the dimensions of a cube built from eight cubes? 2×2×2
3. We count cubes (**cubic units**) to find the **volume** of space figures. Suppose you have made these figures with cubes. How would you find the volume of each figure? See Discussion.



4. What are the dimensions of each figure in exercise 3? See above. Can you figure out a shortcut for finding the volume of a rectangular prism if you know the dimensions of the prism? See Discussion.

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Discussion

As you discuss exercise 1, have the children list dimensions of the prisms they made. Demonstrate how a different arrangement of the same set of dimensions does not change the shape of the prism, as the example in the investigation shows. Exercise 2 directs children's attention to the distinguishing characteristics of a cube: it is a rectangular prism that has equal height, length, and width.

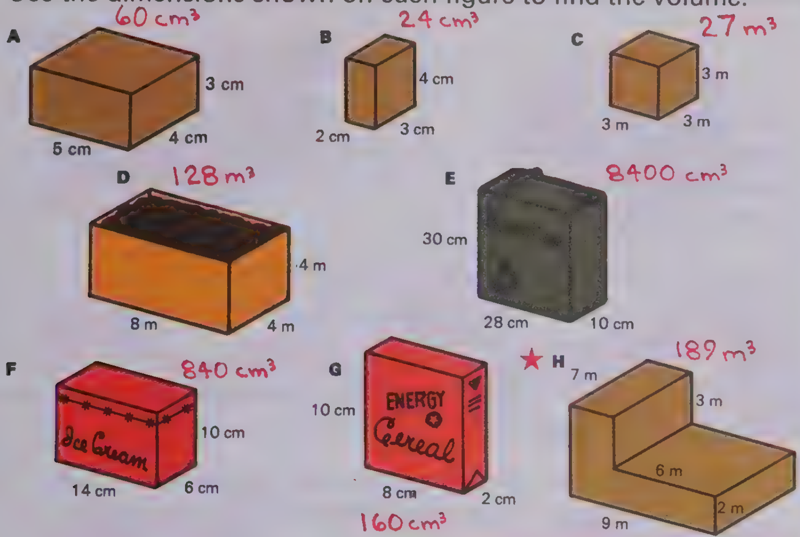
During the discussion of exercise 1, some children may have noticed that the product of each set of dimensions is 12. Exercise 3 progressively develops the idea of determining volume, from counting to finding the product of the dimensions. Finally, in exercise 4, the child is encouraged to formalize the general formula for finding volume. It would be helpful to display a table to show the dimensions of each figure in exercise 3.

	l	w	h	V
A				
B				
⋮				

If necessary, have children make other prisms with the cubes until they realize that $V = l \times w \times h$.

Using the Ideas

1. Use the dimensions shown on each figure to find the volume.



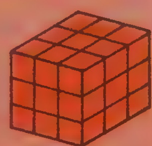
2. A Which holds more, the ice-cream carton or the box of cereal? What is the difference? F; 680 cm³

   B A litre occupies about 1000 cubic centimetres. Does the gasoline can in exercise 1 E hold more or less than 9 litres? Less

   C Does the wood box hold more or less than the L-shaped carton? What is the difference? Less; 61 m³

think

Suppose you used 27 blocks to form a cube. You painted its faces orange. Then you took it apart.



1. How many of the 27 blocks would have 4 or more orange faces? just 3 orange faces? just 2 orange faces? just 1 orange face? no orange faces?

2. Can you answer these questions for a 4-by-4-by-4 cube?

See Answers, T.E. page 277.

Follow-up

You might use the *Think* problem as a follow-up activity. Encourage children to make guesses before building the suggested cube. Chalk may be used to mark the faces of the cubes if you want to use something less permanent than paint. Any children who experience difficulty with this lesson may benefit from finding the volume of prisms by counting how many cubes they can fit into empty rectangular containers. Others may make estimates of the volume of a box and then verify their guess by counting cubes or measuring the dimensions and applying the formula $V = l \times w \times h$.

Resources for Active Learning

Applied Mathematics Cards, Group 2/5, Schofield and Sims. (Available from Mafex Associates, Willowdale)

Mathematics in Modules, M13, Addison-Wesley.

Mathex: Measurement No. 10, "Volume," pp. 18-21, Encyclopaedia Britannica Publications Ltd.

Maths Mini-Lab, Card 97; 120, Selective Educational Equipment.

Measure and Find out, Book 1, Activity 4, Scott Foresman. (Available from Gage Educational Publishing)

Workbook, page 95

Using the Exercises

Have children do the exercises on page 277 independently. If they understood the development of the formula for finding volume, they should be able to apply the formula and find the volume, on the basis of the dimensions for each figure in exercise 1.

Answers, Think, page 277

1. 4 or more orange faces – 0
   3 orange faces – 8
   2 orange faces – 12
   1 orange face – 6
   No orange faces – 1
2. 4 or more orange faces – 0
   3 orange faces – 8
   2 orange faces – 24
   1 orange face – 24
   No orange faces – 8

Assignments (page 277)

Minimum: 1A–E. Average: 1A–G.
Maximum: 1–2.

PAGES 278–279

Objective

Given a rectangular prism, the child will be able to find its surface area.

Preparation

Materials

small boxes (1 per child; graph paper (centimetre grid preferred); glue or tape; scissors

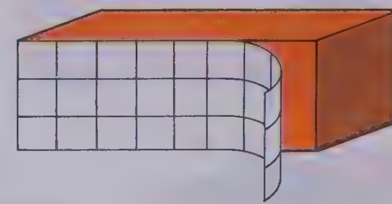
Before children begin the investigation, you might have them examine their graph paper and explain what unit each square represents. If possible, use centimetre graph paper so that children can count in whole units and need not account for too great an error if the graph paper does not come out even.

Investigation

Direct children to fit the graph paper onto their boxes, beginning at one corner and matching the squares of the graph paper exactly along two edges. When the graph paper overlaps, they should carefully trim off the surplus. Others may place their box on the graph paper and trace around the box to outline the paper they need. When the squares do not match exactly with the edges of the box, encourage the child to count halves of squares and estimate the surface area.

● *What is the surface area of space figures?*

Investigating the Ideas

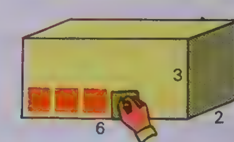


Use a small box. Cut out pieces of graph paper to cover the entire surface of the box.

? Can you count how many square units it takes to cover the box? See Investigation.

Discussing the Ideas

1. The number of square units it takes to "cover" the surface of a space figure is called the **surface area** of the figure. What is the surface area of a 1-unit cube? Why? 6 sq units, because the cube has 6 faces, each with a surface area of 1 sq unit.
2. Suppose you use an inked stamper that prints a 1-unit square each time you press it against this box.



   A If the squares don't overlap, how many times will you have to stamp to cover the box completely? 72
   B What is the surface area of the box? 72 sq units
3. Can you figure out another way to find the surface area of the box in exercise 2? See Discussion.
4. Can you make up a problem which could be solved by finding the surface area of an object? Sample problem: How many square units of gift wrapping are needed to cover a package 4×6×5?

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Discussion

As you discuss exercise 1, help children relate the number of square units they found it took to cover their box to the explanation of what is meant by *surface area*. Display a single cube and explain that a unit cube has the volume of one cubic unit. Thus, a cube that is one cubic centimetre is one centimetre high, one centimetre wide, and one centimetre long. Help children realize that the area of each face of such a cube is one square centimetre. To find the surface area for the unit cube, they simply total the number of faces of the cube. As you discuss exercise 2, stress that, to find the surface area of any rectangular prism, you can count the number of square units on each face and find the total for all the faces. Emphasize that surface area is a measure of area, using square units; it is not a measure of volume.

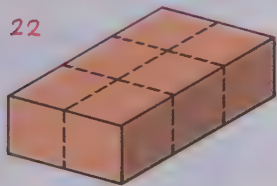
Use exercise 3 to point out that the surface area may be found without actually counting square units. The area of each face may be found by multiplying its length times its width. Again, the surface area will be found by adding the areas of each of the faces.

Using the Ideas

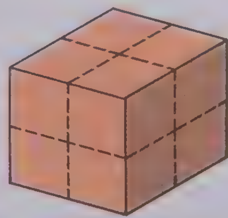
The square centimetre  is the unit for these exercises.

1. Find the surface area of each figure.

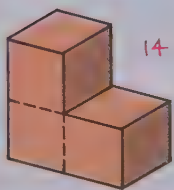
A 22



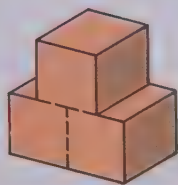
B 24



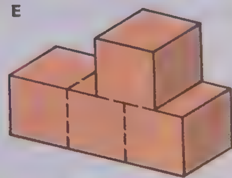
C 14



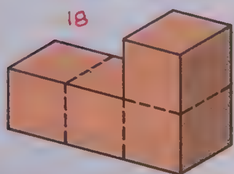
D 14



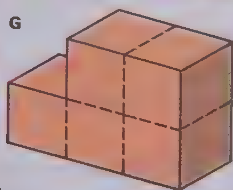
E 18



F 18

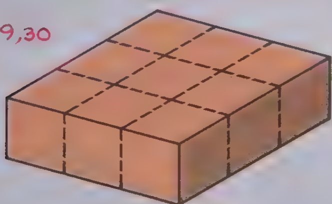


G 20

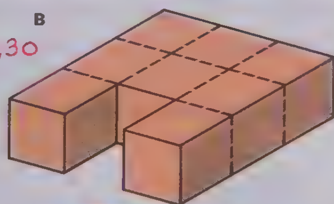


2. Give the volume and surface area of each figure.

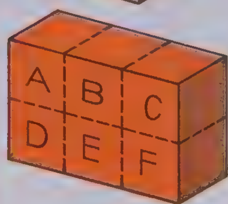
A 9,30



B 8,30



★ **3.** The volume of this figure is 6. The surface area is 22. Give the letter of a cube that you could remove so the volume would be 5 and the surface area would still be 22. B or E



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Using the Exercises

Before assigning the exercises on page 279, make sure children realize that the area of all the "hidden" faces must also be counted, including the faces on which the prisms rest. That is, a cube has the surface area of six whether it is held, is suspended in midair, or is lying on a table with one face hidden.

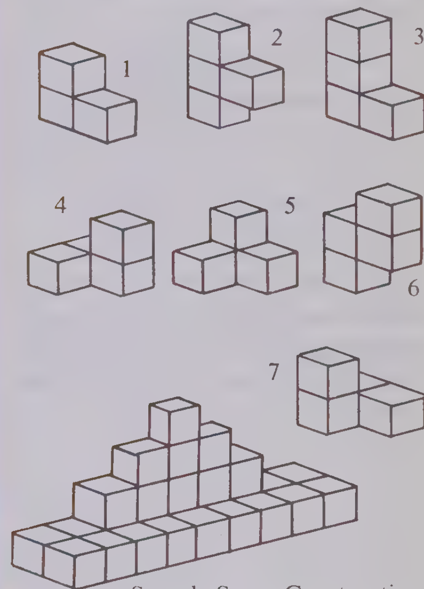
In starred exercise 3, the point is that, with a constant surface area of 22, we can arrive at different volumes by exposing different parts of the blocks. Suggest that children try various methods to get different surface areas by stacking six blocks in various ways.

Assignments (page 279)
Minimum: 1A–E. Average: 1–2. Maximum: 1–3.

Follow-up

Many children may enjoy rearranging the pieces of a cube puzzle like the Soma cube devised by the Danish writer, Piet Hein. The seven pieces of the puzzle can be made by gluing 27 small cubes together as shown in the illustration. Hein proved that these pieces can be put together in such a way as to make a 3-by-3-by-3 cube. Encourage the children, then, to invent and name other shapes using all the pieces.

The Seven Soma Pieces



Sample Soma Construction

Resources for Active Learning

Math Activity Cards, "The Painted Cube," D17, Macmillan.
Maths Mini-lab, Card 127, Selective Educational Equipment.

Duplicator Masters, page 56

Objective

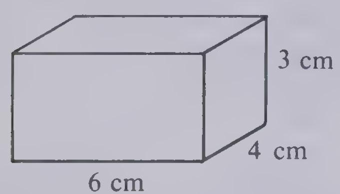
Given the length, height, and width of rectangular prisms, the child will be able to find the volume and surface area of each prism.

Preparation

Materials

construction paper; scissors; tape; centimetre rulers

It would be helpful to briefly review how to find both volume and surface area. For example, describe a box whose dimensions are 4 cm, 6 cm, and 3 cm.



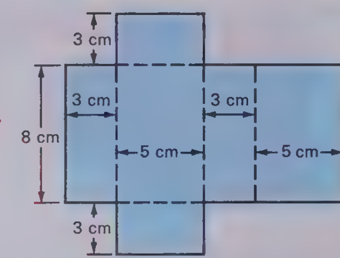
Then work with the children to find volume ($6 \times 4 \times 3 = 72$ cubic centimetres) and surface area ($2 \times [(3 \times 4) + (6 \times 4) + (3 \times 6)] = 108$ cm^2).

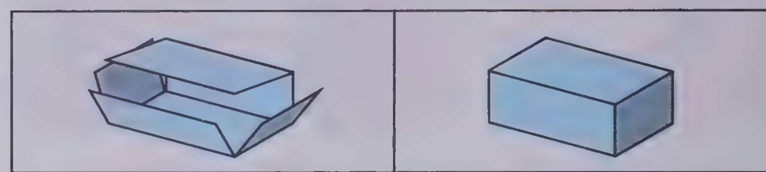
How do volume and surface area compare?

1. A Draw a figure like this. It is made of rectangles. The lengths and widths are shown on the drawing.

B What is the area of the figure? 158 cm^2

C Cut out the figure and fold on the dotted lines as shown below. Use cellophane tape to fasten the sides together.

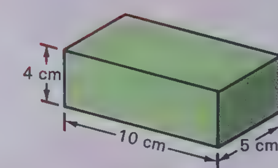




D What is the **surface area** of the figure? 158 cm^2

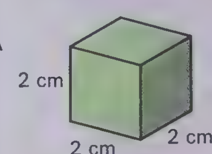
E What is the **volume** of the figure? 120 cm^3

2. Think about the rectangles that form the surface of a figure with the length, width, and height as shown. Find the surface area of this figure. 220 cm^2



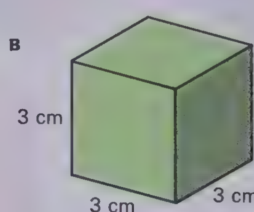
3. Find the volume and surface area of each figure.

A



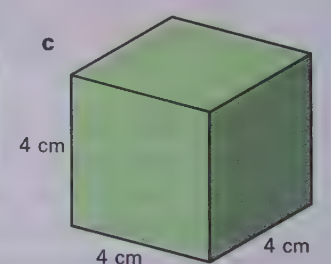
V = 8 cm^3
SA = 24 cm^2

B



V = 27 cm^3
SA = 54 cm^2

C



V = 64 cm^3
SA = 96 cm^2

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Discussion

These independent study pages provide both activities and computational exercises. Exercise 1 is designed to give children a feeling for the relation of surface area to space figures. This is accomplished by having the children determine the area of a plane figure and then use this plane figure to construct a space figure. Emphasis is placed on the fact that the surface area for the space figure is the same as the area of the plane figure with which the children started.

Starred exercises 5, 6, 7, and 8 may be used as class activities or as optional activities for interested children. In exercise 6, they should see that to have a volume of $1\frac{1}{2}$, the figure shown must have dimensions $1 \times 1 \times 1\frac{1}{2}$. The lower cube then has 5 square units of surface exposed, while the half cube has 1 whole unit and 4 half units, or 3 square units of surface. Thus, the total surface area is 8.

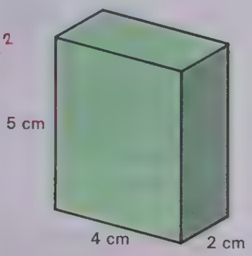
The children may wish to construct at least a partial model for exercise 7. (The unit rod for some sets of number blocks is a cube 1 centimetre on an edge.) The top and bottom cubes will each have 5 surfaces exposed for a total of 10 square units, while the remaining 98 will have only 4 surfaces

Resources for Active Learning
Developmental Math Cards, I^{2}4; J^{3}5, Addison-Wesley.

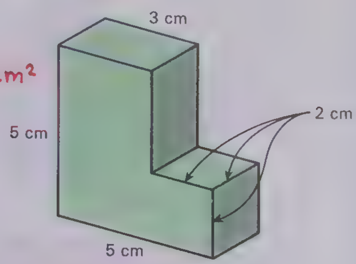
Workbook, page 96

4. Find the **surface area** of each figure.

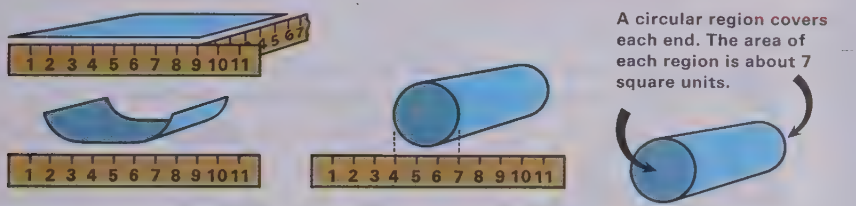
A 76 cm^2



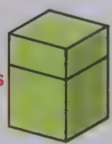
B 78 cm^2



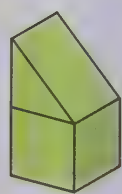
★ 5. Use the information given in the pictures to find the **surface area** of a cylinder that is constructed as shown. 68 sq units



★ 6. The volume of this figure is $1\frac{1}{2}$.
What is the surface area? 8 sq units



★ 7. Give the surface area of the tallest possible tower made of 100 cubic centimetres. 402 cm^2



★ 8. The volume of this figure is $1\frac{1}{2}$.
Is the surface area of the figure
A more than 8? B less than 8? C equal to 8?

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exposed. Thus, total surface area is $(2 \times 5) + (98 \times 4) = 10 + 392$, or 402 square centimetres. Some children will probably realize that the surface area of the figure shown in exercise 8 is more than 8 since the diagonal surface is greater than 1.

Assignments (pages 280–281)
Minimum: 1–3. Average: 1–4.
Maximum: 1–8.

PAGES 282–283

Objective

Given pictorial representations of three-dimensional objects, the child will be able to picture the top, side, and front views of the objects.

Preparation

Materials (optional)

frozen-juice can; brick; geometric solids

To prepare for this lesson, you might draw on the chalkboard an illustration such as the following and ask children what they think it is.

Then tell them you will draw the same object from a different view, and draw the figure below.

Those who thought your first figure was an inner tube or a donut could both be right. Then tell the children that the first figure might also have been a drawing of an object that, from a different view, looks like this:

They may be surprised to see that the "donut" could also have been a picture of a sombrero, viewed from the bottom. Use this or any similar illustration to stimulate interest in viewing objects from different positions.

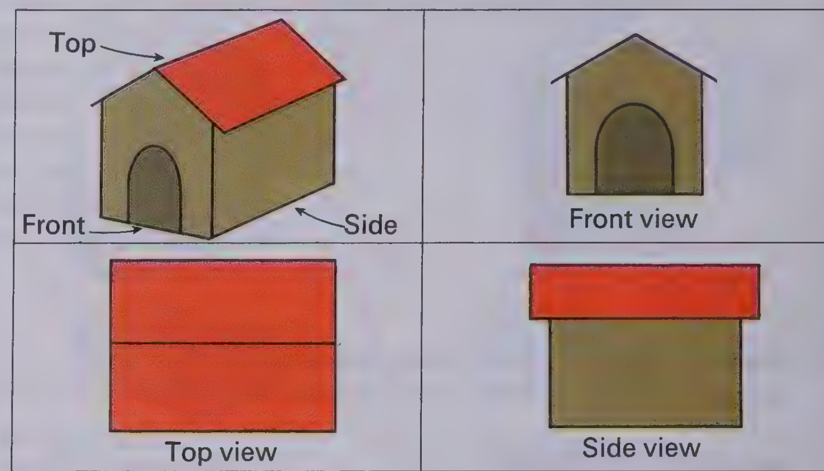
Investigation

Although children should be able to imagine how the objects look from different views, you might want them to actually view them from the different positions. If possible, have some frozen juice cans, or two-holed bricks, and encourage children to use them in drawing the various views. Remind them that for each view they should look directly from the front or directly from the top, so that their images will not need many lines of perspective.

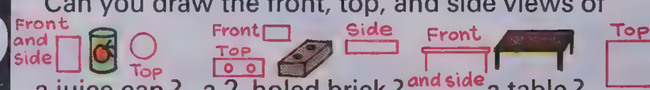
● *Let's explore different points of view.*

Investigating the Ideas

Here are a **front view**, a **top view**, and a **side view** of a doghouse.



Can you draw the front, top, and side views of



a juice can? a 2-holed brick? a table? Drawings will vary.

Discussing the Ideas

1. A Which objects above have front and side views that are the same? Can, table
   B Which objects have side and front views that are both rectangles? Can, brick

2. When do you think drawings of front, top, and side views might be used? For designs, blueprints, and scale drawings, in which all dimensions of an object must be shown in proper scale.

3. Can you describe objects with two views the same? Cylinders, pyramids, etc. all three views the same? Spheres, cubes, etc.

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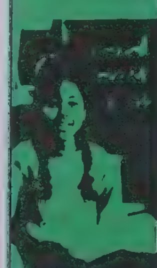


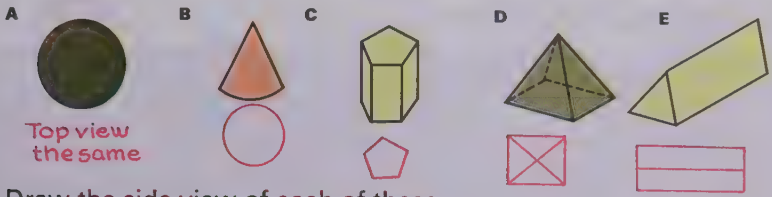
Discussion

This lesson is intended to be treated with a light touch. Children should have fun picturing the same object in different ways. As you work through the discussion exercises, simply stress the difference that a particular viewpoint makes when drawing an object. Plans for the construction of an object often present the three views of the object, as in blueprints.

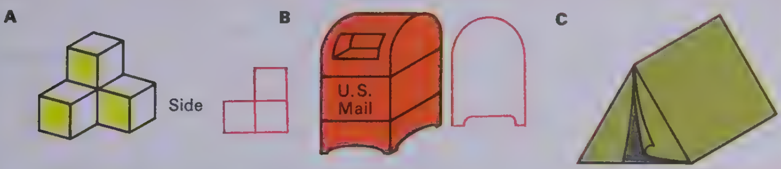
Their earlier study of rectangular prisms and cubes should enable the children to name some objects for exercise 3.

Using the Ideas

1. Draw the top view of each of these figures.



2. Draw the side view of each of these.



3. Draw the front views of the figures in exercises 1B, 1C, 1D, 2A, 2B. lc ld 2A 2B

Sample answers: Cylinder, cone

4. A Describe a figure with front and side views the same.

B Describe a figure (different from A) with top and front views the same.

Sample answers: Cube, sphere

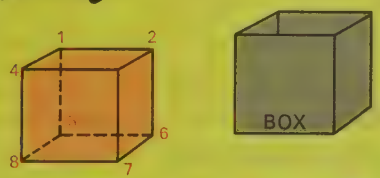
C Describe a figure (different from A and B) with all 3 views the same.

Sample answers: Sphere, cube

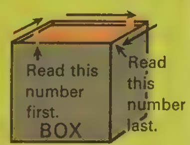
★5. Choose an object from your home or school (different from those mentioned in this lesson). Make accurate drawings of the front, side, and top views.

Drawings will vary.

think



Suppose each vertex of this cube has a number. Here is one way the cube can be placed into this box. We call it **Position** 1234. How many other positions can you find? See Solution, T.E. page 283



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Resources for Active Learning
Math Activity Cards, "Point of View," C46, Macmillan.

Solution, Think, page 283
Since the cube has six faces, with four vertices per face, there are in all 24 ways the cube can be put into the box. Besides Position 1234, these 23 positions are possible:

2341	1265
3412	2651
4123	6512
2367	5126
3672	1485
6723	4851
7236	8514
3487	5148
4873	5678
8734	6785
7348	7856
	8567

Workbook, page 97

Using the Exercises
Encourage the children to work through the exercises on page 283 independently. However, most children would benefit from sharing and discussing the results of their work. Have available as many as possible of the geometric solids that are pictured in exercise 1. Although 1A, 1B, 1C, 1D, and 2A meet the requirements for exercise 4A, 2A should be chosen for exercise 4B and 1A should be the choice for 4C.

Encourage many children to try the *Think* problem. Some may want to use a labelled cube in a box to check their results.

Exercise 5 is starred, not because of its difficulty, but because it might be most suitably used as a follow-up activity for those interested.

Assignments (page 283)
Minimum: 1–2. Average: 1–4.
Maximum: 1–5.

PAGES 284–285

Objective

Given a geometric solid, the child will be able to identify and count its faces, vertices, and edges.

Preparation

Materials

geometric solids

Use the preparation time to further familiarize the children with various geometric solids. Display and name as many as you have, and combine some to form others. For example, you may place two rectangular pyramids base to base to form an octahedron. The more children can see and handle these various shapes, the better will be their ability to interpret the pictorial representations.

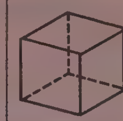
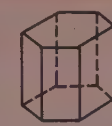
Investigation

It is not essential that children handle the actual solids to complete their charts. The illustrations should enable them to provide the missing data. However, if some wish to refer to any solids you have available, do not discourage them. You might duplicate copies of the chart for children to fill in rather than have them spend time copying it.

Let's explore faces, vertices, and edges.

Investigating the Ideas

Study the figures in the table.

Figure				
Name	cube	hexagonal prism	rectangular pyramid	regular octahedron
Number of faces	? 6	8	? 5	? 8
Number of vertices	? 8	? 12	5	? 6
Number of edges	? 12	? 18	? 8	12

? Can you copy and complete the blue part of the table? See above.

Discussing the Ideas

1. Do you see any interesting patterns or relationships in your completed table? See Discussion.

2. A regular icosahedron has 20 faces and 12 vertices. Without a model or picture, can you find out how many edges it has? (Hint: For each figure in your table, add the number of faces to the number of vertices. Is this sum close to the number of edges? How close?) See Discussion.

Discussion

Although children may discover other relationships, the pattern referred to in discussion exercise 1 is seen by combining the number of faces and vertices and subtracting two to find the number of edges. Ask children to follow the hint given in exercise 2. In each case, the sum of the number of faces and vertices is two greater than the number of edges. Thus, if a regular icosahedron (children might need help in reading this term) has 20 faces and 12 vertices, it should have 30 edges: $(20 + 12) - 2$.

Using the Ideas

1\.

A How many vertices, faces, and edges does the triangular pyramid have? 4 vertices, 4 faces, 6 edges

B Trace and complete the drawing of the square pyramid. How many vertices, faces, and edges does the square pyramid have? 5 vertices, 5 faces, 8 edges

C Do you think each pyramid has the same number of faces as vertices? Trace and complete the other drawings to check your guess. Yes; see above.

D Write down the number of vertices and faces for each pyramid. Without counting, give the number of edges for each pyramid. See above.

2\.

A A triangle has 3 sides. A triangular prism has 9 edges. (Check it.) A rectangle has 4 sides. A rectangular prism has 12 edges. (Check it.) A pentagon has 5 sides. A pentagonal prism has __?__ edges. 15

B How many faces, vertices, and edges do hexagonal and octagonal prisms have? Trace and complete the drawings to check your answers.
Hexagonal: 8 faces, 12 vertices, 18 edges
Octagonal: 10 faces, 16 vertices, 24 edges

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Resources for Active Learning

Developmental Math Cards, I-38, Addison-Wesley.

Modern Math Games. . ., "Geometric Solids," pp. 37–38, Fearon. (Available from Clarke, Irwin)

Using the Exercises

You might have children extend the charts filled in for the investigation and use them to record the results of exercise 1. In exercise 2, children should discover that the number of edges of a prism is three times the number of sides in the base. You might also point out the difference between the rectangular prism here and the square prism shown on page 269. The name of a prism is determined by its base. Besides exploring various relationships between faces, edges, and vertices, these exercises give children further practice in tracing and drawing three-dimensional objects.

Assignments (page 285)
Minimum: 1. Average: 1–2.
Maximum: 1–2.

PAGES 286–287

Objectives

The child will demonstrate his ability to work with the concepts presented in this chapter.

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

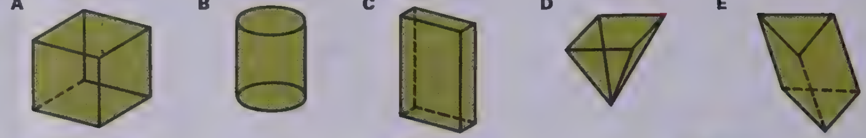
Preparation

If you have identified a concept which children found difficult, review it at this time. You may prefer to review operations with fractions such as children will encounter on page 287. Remind them of the importance of common denominators and ways to find them.

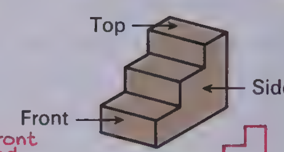
Reviewing the Ideas

1. Which of these space figures is drawn correctly? C

A B C D E

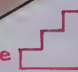


2. Draw a triangular pyramid. 



3. Draw the **front view**, **top view**, and **side view** of these "stairs."

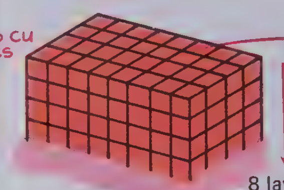

Front and top


Side

4. Find the volume of the figure indicated in each exercise.

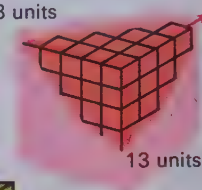
A

256 cu units



B

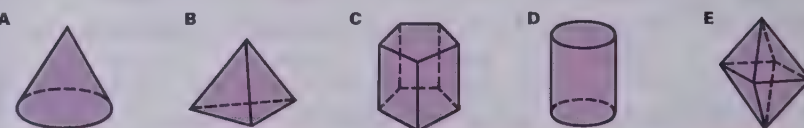
624 cu units



5. Find the surface area of the box.
448 sq units

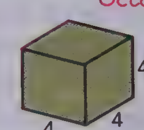
6. Give the name of each space figure below.

A B C D E



Cone Triangular pyramid Pentagonal prism Cylinder Regular Octahedron

7. What are the volume and the surface area of a cube that measures 4 cm on each edge?
$V = 64\ cm^3$ $SA = 96\ cm^2$



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Discussion

The most important exercises to stress on page 286 are those related to finding volume and surface area. Whether you use this page as an evaluation instrument or as a class review, emphasize the meaning of volume and surface area during your discussion. Do not expect children to draw exact figures in exercise 2; it is hoped that exercises such as 1 and 2 will enable the children to interpret two-dimensional drawings of three-dimensional objects and recognize the difficulties of portraying physical objects in two dimensions.

Keeping in Touch with

Computing
Measurement
Geometry
Fractional numbers

1. Find the sums and differences.

A $13\frac{5}{12} + 9\frac{7}{8}$ = $23\frac{7}{24}$

B $15\frac{7}{8} - 8\frac{5}{12}$ = $7\frac{11}{24}$

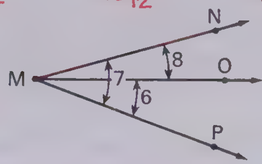
C $58\frac{5}{12} + 39\frac{1}{6}$ = $97\frac{7}{12}$

D $6\frac{7}{8} - 2\frac{1}{6}$ = $4\frac{17}{24}$

E $3\frac{7}{12} + 9\frac{5}{6}$ = $13\frac{5}{12}$

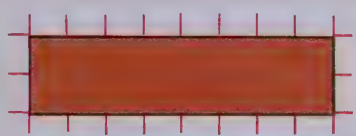
F $16\frac{13}{15} - 2\frac{2}{5}$ = $14\frac{7}{15}$

2. A Give the number name for $\angle NMO$. $\angle 8$
   B What is the number name for $\angle NMP$? $\angle 7$
   C What is the number name for $\angle OMP$? $\angle 6$

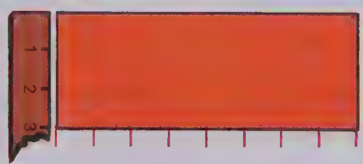


3. Use the unit shown to find the area of each rectangle.

A 16 sq units



B 24 sq units



4. Give the numbers for the gray screens.

Products ÷ Quotients ×

Products				Quotients
8	4	×÷	2	2
27	9	×÷	3	3
32	8	×÷	4	2
6	6	×÷	1	6
49	7	×÷	7	1

think

Suppose ★ is an operation so that
$a \star b = (a \times b) + b$.
Example: $3 \star 4 = (3 \times 4) + 4 = 16$

1. Solve the equations:
   A $5 \star 0 = n$ 0   C $5 \star 1 = n$ 6
   B $0 \star 5 = n$ 5   D $1 \star 5 = n$ 10
2. Choose numbers a, b, and c to show that the grouping principle does not hold for ★
   $(a \star b) \star c = a \star (b \star c)$
   See below.

You are invited to explore

ACTIVITY CARD 14
Page 340

Think, part 2: $(3 \star 4) \star 5 \overset{?}{=} 3 \star (4 \star 5)$ $16 \star 5 \overset{?}{=} 3 \star 25$ $85 \neq 100$
(Sample solution) $[(3 \times 4) + 4] \star 5 \overset{?}{=} 3 \star [(4 \times 5) + 5]$ $(16 \times 5) + 5 \overset{?}{=} (3 \times 25) + 25$

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Resources for Active Learning
Developmental Math Cards, J²20, Addison-Wesley.

Workbook, page 98 (Use with text page 286.)

Using the Exercises
Page 287 reviews various concepts treated previously in the text. Although the following chapter deals with fractions, it is not essential for children to have mastered addition and subtraction of fractional numbers before studying Chapter 14, since its principal concern is multiplication and division. However, provide help to any who have difficulty with exercise 1.

Note that for children who finish page 287 early, there is a *Think* problem, which they should find stimulating. You may find this an interesting problem to discuss with the entire class, once children have had an opportunity to solve the problem on their own. Especially, point out that the examples in part 1 of the problem show that the commutative (order) principle does not hold for operation ★, and that using numbers for a, b, and c in part 2 shows that the associative (grouping) principle does not hold for this operation, either.

Pages 288–301

General Objectives

To introduce multiplication of fractional numbers
To provide work with regions as a model for illustrating the product of fractional numbers
To provide work with the number line as a model for illustrating the product of two fractional numbers
To establish the basic principles for multiplication of fractional numbers
To provide a logical development of the general rule for finding the product of any two fractional numbers
To introduce division of fractional numbers
To provide word-problem experiences

The first lesson in this chapter provides work with regions to illustrate finding the product of two fractional numbers that are represented by unit fractions. The second lesson of the chapter provides number-line experiences that elucidate the concept of finding the product of a whole number and a fractional number represented by a unit fraction. Following this, the basic principles for multiplication of fractional numbers are introduced. Then, a carefully planned sequence of exercises presents the general rule for finding the product of any two fractional numbers. Division is introduced through work with fractional parts of the familiar centimetre strips. The chapter ends with the usual chapter and cumulative reviews, including practice with word problems.

Mathematics

Many different approaches can be used to arrive at the standard definition for the product of any two fractional numbers $\frac{a}{b}$ and $\frac{c}{d}$:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

The approach we show below directly parallels the development in the children's text. The following outline shows the steps involved.

A. Establish the following two facts regarding whole numbers and unit fractions.

$$\frac{1}{a} \times \frac{1}{b} = \frac{1}{a \times b} \quad \text{and} \quad a \times \frac{1}{b} = \frac{a}{b}$$

B. Assume the commutative and associative principles hold for multiplication of fractional numbers.

C. Apply the principles to derive the product of any two fractional numbers.

$$\begin{aligned}\frac{a}{b} \times \frac{c}{d} &= \left(a \times \frac{1}{b}\right) \times \left(c \times \frac{1}{d}\right)\\ &= (a \times c) \times \left(\frac{1}{b} \times \frac{1}{d}\right)\\ &= (a \times c) \times \left(\frac{1}{b \times d}\right)\\ &= \frac{a \times c}{b \times d}\end{aligned}$$

The two facts in part A are presented in the children's text through the use of physical models (regions and number lines). The basic principles are then presented as a parallel to the principles for addition of fractional numbers, and, finally, the steps outlined in part C are developed gradually through exercise sets.

Teaching the Chapter

Materials

Colored crayons
Colored strips
Demonstration number line
Overhead projector (if available) and transparencies
Paper suitable for folding
Ruler
Scissors

While there are only a few materials listed for this chapter, use of the strips and of paper folding will be extremely helpful to the children. A demonstration number line will also be beneficial in several lessons. The overhead projector is quite useful for showing shaded areas under a number line or in a region. Note that no new vocabulary is introduced in this chapter. All the words vital to this development have been introduced in previous work.

Lesson Schedule

This chapter is designed to be covered in about a week and a half. Since it appears near the end of the text, however, you will undoubtedly want to adjust your schedule to meet not only the needs and abilities of your children but also your year-end schedule. Keep in mind that this material is presented again in Book 6 of this series.

Evaluation of Progress

You should keep in mind the fact that multiplication and division of fractional numbers will occupy a major portion of the work in sixth grade, so thorough coverage of this material is not expected in Grade 5. Omission of topics or incomplete comprehension of the ideas presented here will not prove to be a great handicap to the children when they use Book 6.

Since mastery of all the material in this chapter is not expected, a major portion of your evaluation should be based on a daily check of children's work and progress.

One important thing to notice as you work with this chapter is the way in which the ideas unfold and culminate in the development of the general rule for finding the product of two fractional numbers. That is, beginning in the first two lessons, we develop, for the children, a technique for finding the

product of two fractional numbers that can be represented by unit fractions, and a way to find the product of a whole number and a fractional number that can be represented as a unit fraction. Then basic principles are presented, and a carefully designed sequence leads to examples of the general rule, as on page 292. If children discover the rule themselves, they will be more likely to remember it when they attack the problem again in Book 6. As you proceed through the chapter, you should assist the children in discovering this rule, but try to avoid telling them too much. Many of the children will see that they can find the product of two fractional numbers simply by finding the product of the numerators and the product of the denominators. Those who do not make this discovery will have another opportunity to attack this problem in Book 6.

Pages 298–301 provide review material to help you evaluate the children's understanding of the ideas presented in this chapter as well as their retention of ideas introduced previously.

Resources for Active Learning

General Activities

A Cloudburst, Vol. 2, Nos. 3313, 3323, 3324, 3333–3338

Discovery, Section II, Unit 12/6, Encyclopaedia Britannica Educational Corp.

Franklin Series: *Making and Using Graphs and Nomographs,* "Nomographs. . .," pp. 87–88, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Mathex: Numeration No. 7, "Number Sentence Game," p. 47 (pupil pages 47–51), Encyclopaedia Britannica Publications Ltd.

Commercial Games

Action Fraction Games (CCM School Materials; Lakeshore; Math Media)

Imout – fractions (Imout)

Spinner Number Games – fractions (Heath)

TUF (Creative Publications; Cuisenaire Co; TUF)

Objective

Given regions which demonstrate multiplication of two fractional numbers using unit fractions, the child will be able to name the product.

Preparation

Materials

paper suitable for folding, sheets no smaller than 10-by-15 centimetres (at least 4 per child)

To prepare for this lesson, it would be helpful to review how the lowest-terms fraction is commonly used to represent the fractional number which corresponds to a set of equivalent fractions. You might do this simply by writing various lowest-terms fractions on the chalkboard and asking children to name some fractions equivalent to each. Or, you might write some sets of equivalent fractions and have children pick out or figure out which lowest-terms fraction is commonly used to represent the number which matches the whole set.

$$\{\tfrac{12}{18}, \tfrac{2}{3}, \tfrac{10}{15}, \tfrac{4}{6}, \tfrac{16}{24}, \ldots\}$$

$$\{\tfrac{7}{10}, \tfrac{14}{20}, \tfrac{21}{30}, \ldots\}$$

$$\{\tfrac{15}{18}, \tfrac{10}{12}, \tfrac{5}{6}, \ldots\}$$

$$\{\tfrac{6}{15}, \tfrac{8}{20}, \tfrac{4}{10}, \ldots\}$$

Investigation

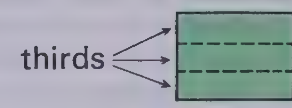
Direct the children to study the illustrated model and discuss with two or three classmates how they would fold their paper to match the illustration. Then they should try to fold and color paper to show each of the given fractional situations. As you move around the room, ask children if they can rename each section that they color. To help those who have difficulty, ask them how many sections their piece of paper has, and compare the total number of sections to the number of colored. For example, when they have folded $\frac{1}{2}$ of $\frac{1}{4}$, they should have 8 folded sections in the paper, one of which should be colored, so $\frac{1}{8}$ is colored.

14 Multiplication and Division of Fractional Numbers

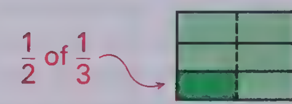
● *Let's explore regions and multiplication.*

Investigating the Ideas

This paper was folded into thirds.



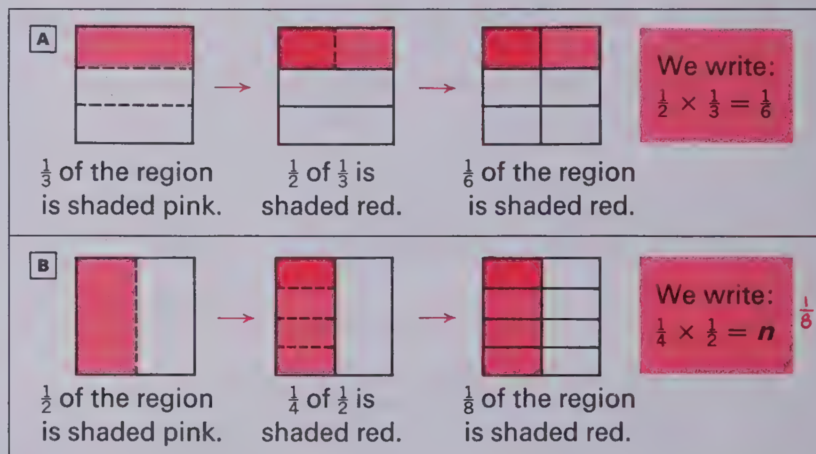
After one more folding, the paper shows half of one third.



? Can you fold and color a paper to show each of these?
$\frac{1}{2}$ of $\frac{1}{2}$ $\frac{1}{2}$ of $\frac{1}{4}$ $\frac{1}{2}$ of $\frac{1}{8}$ See Investigation.

Discussing the Ideas

Study example A. Then explain example B and solve the equation.



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Discussion

Use examples from the investigation to show children how their results may be interpreted in terms of multiplication. For example, use the explanation given in the investigation section to show that $\frac{1}{2}$ of $\frac{1}{4}$ is the same as $\frac{1}{8}$. Children might want to symbolize this as $\frac{1}{2}$ of $\frac{1}{4}$ $= \frac{1}{8}$, but explain that "of" is not a mathematical symbol and substitute $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$. If children question this substitution of the times sign for the word "of," you might give the following explanation:

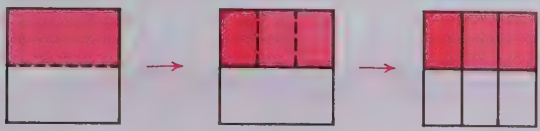
"For 2 times a number we think of taking that number two times. For 1 times a number, we think of taking that number once. For $\frac{1}{2}$ times a number we think of taking that number one-half times, that is of taking only one half of it. Thus, for $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$, we think of taking the amount that is one fourth of something only one half times; in other words, we take only one-half of the one fourth."

To see whether children follow this thinking, ask them which is greater: $\frac{1}{2}$, or $\frac{1}{2} \times \frac{1}{4}$; $\frac{1}{4}$, or $\frac{1}{4} \times \frac{1}{8}$. Use the examples in the text to continue this explanation. It is important that children realize that the product $\frac{1}{6}$, is less than either of the factors $\frac{1}{2}$ or $\frac{1}{3}$.

Help children discover the rule

Using the Ideas

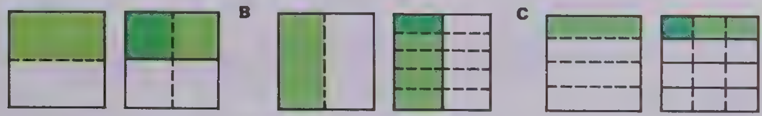
1. Give the fraction of the region that is shaded red. Then copy the equation and give the product n.



$\frac{1}{3} \times \frac{1}{2} = n$ $\frac{1}{6}$

$\frac{1}{2}$ of the region is shaded pink. — $\frac{1}{3}$ of $\frac{1}{2}$ is shaded red. — $\frac{1}{6}$ ▒ of the region is shaded red.

2. Study the figures. Then give the product.



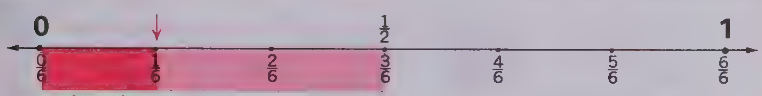
A: $\frac{1}{2}$ — $\frac{1}{2}$ of $\frac{1}{2}$ — $\frac{1}{2} \times \frac{1}{2} = n$ $\frac{1}{4}$

B: $\frac{1}{2}$ — $\frac{1}{5}$ of $\frac{1}{2}$ — $\frac{1}{5} \times \frac{1}{2} = n$ $\frac{1}{10}$

C: $\frac{1}{4}$ — $\frac{1}{3}$ of $\frac{1}{4}$ — $\frac{1}{3} \times \frac{1}{4} = n$ $\frac{1}{12}$

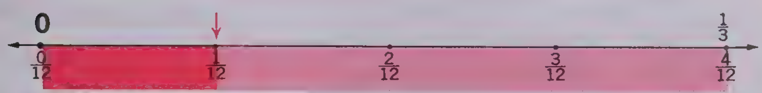
3. Study the number-line picture. Then copy the equation and give the product.

A



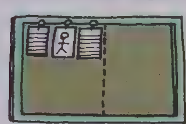
$\frac{1}{3}$ of $\frac{1}{2}$ — We write: $\frac{1}{3} \times \frac{1}{2} = n$ $\frac{1}{6}$

B



$\frac{1}{4}$ of $\frac{1}{3}$ — We write: $\frac{1}{4} \times \frac{1}{3} = n$ $\frac{1}{12}$

4. The class used $\frac{1}{2}$ of the bulletin board for a science display. Sue's papers took $\frac{1}{3}$ of the space. What part of the whole board did Sue's papers cover? Write and solve an equation. $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$



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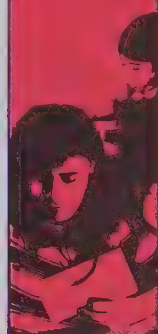


Mathematics

A unit fraction is a fraction which has a numerator of 1. Although the children are not introduced to the term *unit fraction* in this lesson, the purpose of the lesson is to offer a plausible explanation for our assumption regarding the product of two unit fractions.

If a and b are nonzero whole numbers, then $\frac{1}{a} \times \frac{1}{b} = \frac{1}{a \times b}$.

This fundamental relation, used with the basic principles for fractional numbers, can be used to justify the familiar algorithm or rule that we use to find the product of any two fractional numbers.

Workbook, page 99

that, for multiplication of unit fractions, the numerator of the product is one and the new denominator of the product is the product of the denominators.

Using the Exercises

Either before or after the children do the exercises on page 289, point out the sequential development used in the illustrations. For example, in exercise 3A children should first think of $\frac{1}{2}$ of the number line, then observe that $\frac{1}{3}$ of $\frac{1}{2}$ is shaded, and finally that this amount is the same as $\frac{1}{6}$ of the unit.

Note that in all of these exercises only unit fractions are used. Similarly, limit any additional examples you use to unit fractions.

Assignments (page 289)

Minimum: 1–3. Average: 1–4.
Maximum: 1–4.

PAGES 290–291

Objective

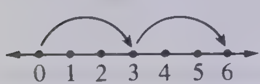
Given one factor which is a unit fraction and another factor which is a whole number, the child will be able to find their product, particularly by using the number line.

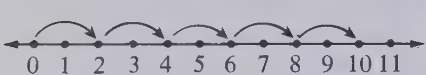
Preparation

Materials

demonstration number line

A review of the use of the number line to show multiplication of whole numbers would help prepare the children for this lesson. For example, illustrate $2 \times 3 = 6$, or $5 \times 2 = 10$.





You might also show a number line partitioned into fractional parts to remind children that the number line is simply a tool which we can use to illustrate many arithmetical operations.

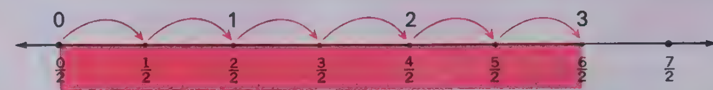
Investigation

Direct the children to study number lines A and B and then compare, with two or three classmates, their product for number line B. Although sharing ideas for finding the products in the investigation might be helpful, each child should draw his own number line for each of these multiplication problems. However, point out to the children that they must draw each number line to show the fractional parts needed for each multiplication. Note that the markings on the number line do not have to correspond to a centimetre unit, though centimetre markings would be suitable for 7 jumps of ¼, and half-centimetre markings would be suitable for 9 jumps of ½. As you move around the room, help those having difficulty to partition their number lines properly.

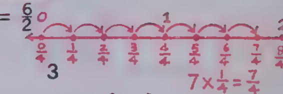
● *How can you find the product of a whole number and a fraction?*

Investigating the Ideas

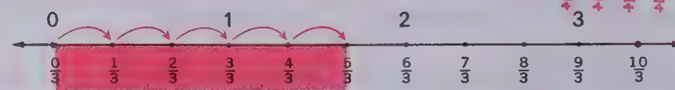
A



6 jumps of $\frac{1}{2}$ We write: $6 \times \frac{1}{2} = \frac{6}{2}$

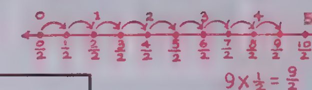


B



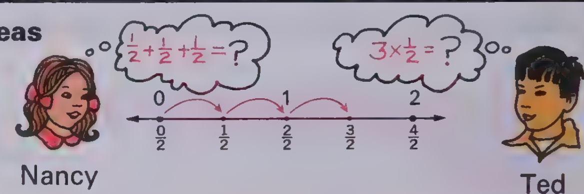
5 jumps of $\frac{1}{3}$ We write: $5 \times \frac{1}{3} = ▥ \frac{5}{3}$

What is the product for number line B? $\frac{5}{3}$



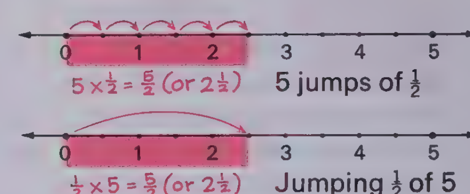
? Can you draw number lines and write the multiplication equations for these jumps?
7 jumps of $\frac{1}{4}$ 9 jumps of $\frac{1}{2}$

Discussing the Ideas



1. A Can you explain how Nancy is thinking about the number-line picture? Repeated addition
   B How is Ted thinking about it? Multiplication
   C Do you think Nancy and Ted will get the same answer? Yes

2. What multiplication equation does each number line suggest?



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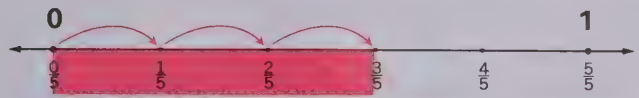
Discussion

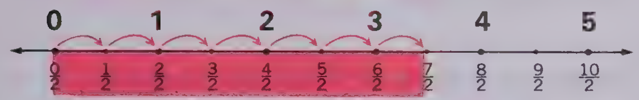
Have volunteers exhibit the number lines and equations for each of the jumps in the investigation. Use exercise 1 as a basis for discussing how multiplication of a fraction by a whole number can be thought of as repeated addition of that fraction. The examples in exercise 2 demonstrate the commutative property of multiplication. Help the children realize that the product of both $5 \times \frac{1}{2}$ and $\frac{1}{2} \times 5$ is the same, namely $\frac{5}{2}$ or $2\frac{1}{2}$. As you work through these exercises, elicit from the children the observation that, in multiplication of the type $5 \times \frac{1}{2}$ or $\frac{1}{2} \times 5$, we simply use the whole number as the numerator and we use the denominator of the fraction as the denominator in our answer. It would be helpful to review multiplication of unit fractions, helping children see that with examples such as $\frac{1}{2} \times \frac{1}{3}$, we simply use 1 as the numerator and we use the product of the denominators as the denominator of the answer.

Using the Ideas

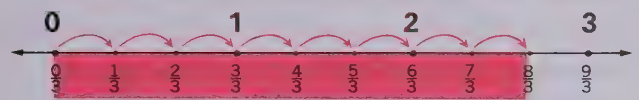
1. Study the number-line picture. Then copy the equation and give the product.

A 

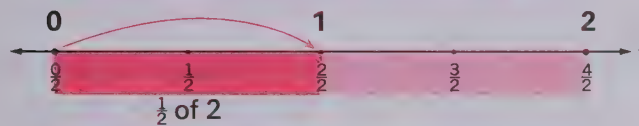
$3 \times \frac{1}{5} = n$ $\frac{3}{5}$

B 

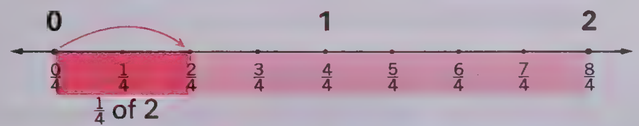
$7 \times \frac{1}{2} = n$ $\frac{7}{2}$ (or $3\frac{1}{2}$)

C 

$8 \times \frac{1}{3} = n$ $\frac{8}{3}$ (or $2\frac{2}{3}$)

D 

$\frac{1}{2} \times 2 = n$ $\frac{2}{2}$ (or 1)

E 

$\frac{1}{4} \times 2 = n$ $\frac{2}{4}$ (or $\frac{1}{2}$)

2. Solve the equations.

$5 \times \frac{1}{3} = \frac{5}{3}$ $\qquad \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

A $3 \times \frac{1}{2} = n$ $\frac{3}{2}$ $\qquad$ **G** $\frac{1}{3} \times \frac{1}{4} = n$ $\frac{1}{12}$

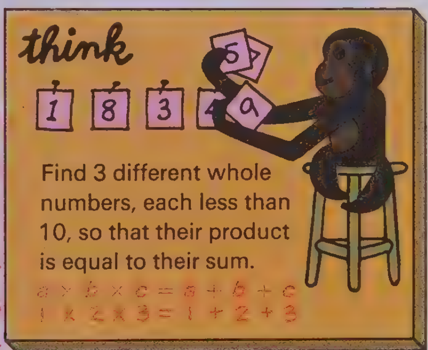
B $7 \times \frac{1}{2} = n$ $\frac{7}{2}$ $\qquad$ **H** $\frac{1}{2} \times \frac{1}{5} = n$ $\frac{1}{10}$

C $5 \times \frac{1}{6} = n$ $\frac{5}{6}$ $\qquad$ **I** $\frac{1}{3} \times \frac{1}{3} = n$ $\frac{1}{9}$

D $9 \times \frac{1}{4} = n$ $\frac{9}{4}$ $\qquad$ **J** $\frac{1}{6} \times \frac{1}{2} = n$ $\frac{1}{12}$

E $\frac{1}{2} \times 8 = n$ $\frac{8}{2}$ $\qquad$ **K** $\frac{1}{4} \times \frac{1}{5} = n$ $\frac{1}{20}$

F $\frac{1}{7} \times 12 = n$ $\frac{12}{7}$ $\qquad$ **L** $\frac{1}{7} \times \frac{1}{3} = n$ $\frac{1}{21}$



Mathematics

This lesson presents a second basic assumption about fractional numbers represented by unit fractions:

$$a \times \frac{1}{b} = \frac{a}{b}, \text{ where } a \text{ and } b \text{ are whole numbers and } b \neq 0.$$

This relation is made plausible to the child by jumps on the number line and related to repeated addition of the fractional number given by the unit fraction.

Follow-up

An activity that will prepare the child for better understanding of the multiplication algorithm to be developed in the next lesson is the game, "Factor Detectives." To carry out this activity, one child writes a fraction on the chalkboard or on a sheet of paper. A second child must write the fraction as the product of a whole number and a unit fraction or as the product of two unit fractions. For example, if a student writes $\frac{3}{4}$, the second student would write $3 \times \frac{1}{4}$; $\frac{1}{12}$ could be written as $\frac{1}{3} \times \frac{1}{4}$ or $1 \times \frac{1}{12}$.

Resources for Active Learning

Developmental Math Cards, J13, Addison-Wesley.

Workbook page 100

Using the Exercises

As you assign the exercises on page 291, remind children to study both the jumps on the number line and the shaded regions before they find the product, or as a check of their answer. In exercise 2, work with the two types of equations studied so far is provided. If necessary, discuss again the rules for these types of equations, as mentioned in the discussion section.

The children will probably experiment to find that only 1, 2, and 3 will work in the equation given in the *Think* problem.

Assignments (page 291)
Minimum: 1–2F. Average: 1–2.
Maximum: 1–2.

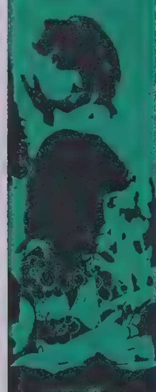
PAGES 292–293

Objective

Given two fractional numbers, the child will be able to find their product by applying his understanding of basic principles.

Preparation

You might spend five or ten minutes reviewing the types of products studied in the previous lessons (such as $\frac{1}{3} \times \frac{1}{4}$ and $5 \times \frac{1}{6}$), or use the follow-up suggested in the last lesson. It would also be helpful to review the basic principles of multiplication as they apply to whole numbers. For example, write $3 \times 6 = 6 \times 3$ and ask children if they remember which principle this demonstrates. Similarly, write $(7 \times 3) \times 4 = 7 \times (3 \times 4)$ to review the associative principle. The more familiar children are with these principles, the better prepared they will be to apply them to fractional numbers.



Can the basic principles be used to find products?

Discussing the Ideas

1. Read and complete each of the **basic principles** for multiplication of fractional numbers.

   A THE 1 PRINCIPLE
   When you choose a fractional number and multiply by 1, the product is ____?____. the fractional number

   B THE COMMUTATIVE PRINCIPLE (ORDER PRINCIPLE)
   When you multiply two fractional numbers, the order of the factors does not affect the ____?____. product

   C THE ASSOCIATIVE PRINCIPLE (GROUPING PRINCIPLE)
   When you multiply fractional numbers, you can change the grouping and get the same ____?____. product

2. Give the missing word.
   According to the order and grouping principles for multiplication of fractional numbers, you can rearrange the ____?____ in any way factors and get the same product.

3. Study the diagram and give the missing number.

   Find the product of these factors.

   $$\frac{2}{3} \times \frac{3}{4} = \left(2 \times \frac{1}{3}\right) \times \left(3 \times \frac{1}{4}\right) = 6 \times \frac{1}{12} = \boxed{} \quad \frac{6}{12}$$

   Find the product of these factors.

4. Can you give a rule for finding the product of two fractional numbers? See Discussion.

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Discussion

As you discuss exercise 1, ask children to give examples of each principle applied to fractional numbers. For example, the equations $1 \times \frac{1}{4} = \frac{1}{4}$, $\frac{1}{2} \times \frac{1}{4} = \frac{1}{4} \times \frac{1}{2}$ and $(\frac{1}{3} \times \frac{1}{2}) \times \frac{1}{4} = \frac{1}{3} \times (\frac{1}{2} \times \frac{1}{4})$ would demonstrate the identity principle, the commutative property, and the associative property, respectively. Remind children that by combining the commutative and associative principles for whole numbers they were able to rearrange factors, such as $4 \times 6 \times 25 = 4 \times 25 \times 6 = 100 \times 6 = 600$, or $5 \times 7 \times 20 \times 2 = 5 \times 20 \times 7 \times 2 = 100 \times 14 = 1400$. Similarly, now with fractional numbers they will be able to rearrange factors without changing the product. It is important for children to understand the combined use of these principles because we will not show the commutative and associative principles used individually. Rather, we combine these two principles to get the one general principle stated in exercise 2 and applied in exercise 3. The development up to exercise 4 logically leads to the quick rule of finding the product of the numerators to find the new numerator, and finding the product of the denominators to find the new denominator.

Using the Ideas

1. Give the products.

A $5 \times \frac{1}{3}$ $\frac{5}{3}$ D $5 \times \frac{1}{4}$ $\frac{5}{4}$ G $\frac{1}{4} \times \frac{1}{8}$ $\frac{1}{32}$ J $\frac{1}{5} \times \frac{1}{4}$ $\frac{1}{20}$

B $3 \times \frac{1}{2}$ $\frac{3}{2}$ E $\frac{1}{3} \times \frac{1}{4}$ $\frac{1}{12}$ H $\frac{1}{5} \times \frac{1}{6}$ $\frac{1}{30}$ K $\frac{1}{6} \times \frac{1}{3}$ $\frac{1}{18}$

C $4 \times \frac{1}{3}$ $\frac{4}{3}$ F $\frac{1}{5} \times \frac{1}{3}$ $\frac{1}{15}$ I $\frac{1}{8} \times \frac{1}{3}$ $\frac{1}{24}$ L $\frac{1}{5} \times \frac{1}{7}$ $\frac{1}{35}$

2. Give the products.

A $\frac{3}{4}\times\frac{2}{3}=(3\times2)\times(\frac{1}{4}\times\frac{1}{3})=n$ $\frac{6}{12}$

B $\frac{4}{3}\times\frac{5}{2}=(4\times5)\times(\frac{1}{3}\times\frac{1}{2})=n$ $\frac{20}{6}$

C $\frac{4}{5}\times\frac{2}{3}=(4\times2)\times(\frac{1}{5}\times\frac{1}{3})=n$ $\frac{8}{15}$

D $\frac{3}{8}\times\frac{5}{4}=(3\times5)\times(\frac{1}{8}\times\frac{1}{4})=n$ $\frac{15}{32}$

E $\frac{7}{2}\times\frac{2}{3}=(7\times2)\times(\frac{1}{2}\times\frac{1}{3})=n$ $\frac{14}{6}$

F $\frac{5}{6}\times\frac{3}{2}=(5\times3)\times(\frac{1}{6}\times\frac{1}{2})=n$ $\frac{15}{12}$

G $\frac{3}{4}\times\frac{7}{2}=(3\times7)\times(\frac{1}{4}\times\frac{1}{2})=n$ $\frac{21}{8}$

H $\frac{5}{6}\times\frac{5}{4}=(5\times5)\times(\frac{1}{6}\times\frac{1}{4})=n$ $\frac{25}{24}$

I $\frac{4}{3}\times\frac{3}{7}=(4\times3)\times(\frac{1}{3}\times\frac{1}{7})=n$ $\frac{12}{21}$

J $\frac{5}{8}\times\frac{4}{3}=(5\times4)\times(\frac{1}{8}\times\frac{1}{3})=n$ $\frac{20}{24}$

K $\frac{6}{4}\times\frac{3}{5}=(6\times3)\times(\frac{1}{4}\times\frac{1}{5})=n$ $\frac{18}{20}$

L $\frac{4}{3}\times\frac{5}{3}=(4\times5)\times(\frac{1}{3}\times\frac{1}{3})=n$ $\frac{20}{9}$

3. Give the products.

A $\frac{3}{4} \times \frac{2}{5}$ $\frac{6}{20}$ H $\frac{5}{6} \times \frac{3}{8}$ $\frac{15}{48}$

B $\frac{5}{3} \times \frac{3}{2}$ $\frac{15}{6}$ I $\frac{8}{3} \times \frac{3}{4}$ $\frac{24}{12}$

C $\frac{2}{7} \times \frac{5}{2}$ $\frac{10}{14}$ J $\frac{3}{4} \times \frac{5}{8}$ $\frac{15}{32}$

D $\frac{4}{5} \times \frac{7}{3}$ $\frac{28}{15}$ K $\frac{5}{6} \times \frac{7}{4}$ $\frac{35}{24}$

E $\frac{3}{8} \times \frac{2}{5}$ $\frac{6}{40}$ L $\frac{4}{7} \times \frac{7}{4}$ $\frac{28}{28}$

F $\frac{5}{4} \times \frac{3}{5}$ $\frac{15}{20}$ M $\frac{5}{5} \times \frac{3}{5}$ $\frac{15}{25}$

G $\frac{2}{3} \times \frac{2}{3}$ $\frac{4}{9}$ N $\frac{0}{7} \times \frac{3}{8}$ $\frac{0}{56}$

think

Give a number for n that will make this inequality true.

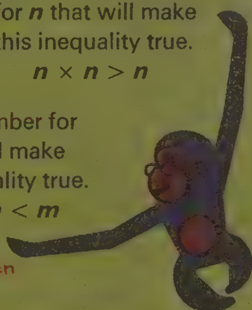
$n \times n > n$

Any whole number other than 0 and 1

Give a number for m that will make this inequality true.

$m \times m < m$

Any fractional number between 0 and 1



More practice, page A-28, Set 53

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Mathematics

This lesson completes the development of the multiplication algorithm for fractional numbers. As was discussed in the mathematics section introducing the chapter, our assumptions concerning unit fractions, and application of the basic principles of associativity and commutativity for multiplication of fractional numbers, enable us to show that

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}.$$

The example in exercise 3, page 292, shows the proof for the special case of $\frac{2}{3}$ and $\frac{3}{4}$. A general proof for any two fractional numbers would be made in much the same manner.

Duplicator Masters, page 57
Workbook, page 101

Using the Exercises

If necessary, use parts of exercise 2 on page 293 to help children better understand the method of multiplying fractional numbers. Note that after study of examples like those in exercise 2, the children may well think for exercise 3A, "$3 \times 2 = 6$, and $\frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$; therefore, the answer is $\frac{6}{20}$," rather than thinking in the traditional way, "$3 \times 2 = 6$ and we put that over 4×5." Note that the only difference is that they think $\frac{1}{4}$ of $\frac{1}{5}$ rather than 4×5. This type of thinking is highly desirable; however, either way they wish to think is satisfactory.

Assignments (page 293)

Minimum: 1A–F, 2A–F, 3A–G.
Average: 1–3F. Maximum: 1–3.

Objective

Given one of the strips as the unit, the child will use other strips to solve simple division problems involving fractional numbers.

Preparation

Materials

brown, purple, light green, red, and white strips

It would be helpful to review with the children how a strip can be thought of as a fractional part of another strip chosen as the unit. For example, if the red strip is the unit, how can we think of the white strip? Or, if the blue strip is the unit, how can we think of the light green strip? Finally, point out the unit used in the investigation: if the purple strip is the unit, we can think of the white strip as $\frac{1}{4}$, the red strip as $\frac{1}{2}$, and the light green strip as $\frac{3}{4}$.

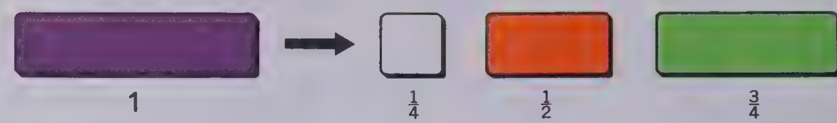
Investigation

Since the purple strip is considered to be the unit in the investigation, the purple train represents the whole number 3. Thus, children will actually be working with a situation which involves dividing 3 by $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$, respectively. To help children realize this while they are working, move around the room and ask questions such as, "Since the white strip is considered $\frac{1}{4}$, how many fourths do you have in 1? in 2? in 3?" or "How many light green strips match the 3 purple strips? So, how many three-fourths do you have in 3?" Questions such as these will help children relate what they are doing to the concepts developed in the discussion.

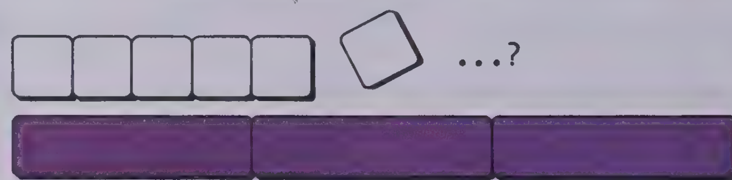
Can we divide by a fractional number?

Investigating the Ideas

Use the purple strip as the unit.

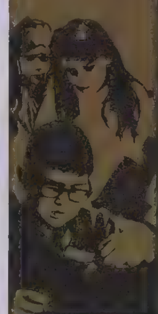


How many white strips will "match" the purple train? 12



? Can you make a red or light green train that matches the purple train? See Investigation.

6 red strips = 3 purple strips
4 lt. green strips = 3 purple strips



Discussing the Ideas

1. A How many white strips did it take to match the 3 purple strips? 12
   B How many fourths are in 3? 12
   C Solve: $3 \div \frac{1}{4} = n$ 12

2. A How many halves are in 3? 6
   B Solve: $3 \div \frac{1}{2} = n$ 6

3. A How many three-fourths are in 3? 4
   B Solve: $3 \div \frac{3}{4} = n$ 4

4. Can you use your strips to find how many fourths are in three halves? 6 (6 white strips = 3 red strips)

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Discussion

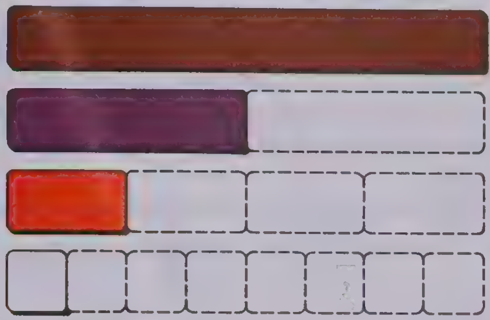
This lesson relates division by a fractional number less than one to the physical situation of partitioning a whole number or a unit. Thus, in the investigation, children were able to find how many times the $\frac{1}{4}$-strip fit into 3, and how many times the $\frac{1}{2}$-strip, and the $\frac{3}{4}$-strip, fit into 3. In each case they were dividing 3 by fractional number $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$.

The discussion exercises develop this concept and relate it to the written division equation. Thus, each equation should be solved by reference to the corresponding matching made in the investigation.

You may need to help children decide on a unit for exercise 4. If they keep the purple strip as 1, then 3 red strips would represent one half and children should find that 6 white or quarter strips will match the three halves. Relate their work to the division equation $\frac{3}{2} \div \frac{1}{4} = 6$. This same equation may be represented by other sets of strips, such as the brown, purple, and red, if the brown strip is the unit.

Using the Ideas

1. Give the missing fractions. When the brown strip is 1,



   A the purple strip is ▮. $\frac{1}{2}$
   B the red strip is ▮. $\frac{1}{4}$
   C the white strip is ▮. $\frac{1}{8}$

2. A How many halves in 1 ? 2
   B Solve: $1 \div \frac{1}{2} = n$ 2

3. A How many fourths in 1 ? 4
   B Solve: $1 \div \frac{1}{4} = n$ 4

4. A How many eighths in 1 ? 8
   B Solve: $1 \div \frac{1}{8} = n$ 8

5. A How many fourths in $\frac{1}{2}$? 2
   B Solve: $\frac{1}{2} \div \frac{1}{4} = n$ 2

6. A How many eighths in $\frac{1}{2}$? 4
   B $\frac{1}{2} \div \frac{1}{8} = n$ 4

7. A How many eighths in $\frac{1}{4}$? 2
   B $\frac{1}{4} \div \frac{1}{8} = n$ 2

★ 8. Study the figure. Then solve the equations.

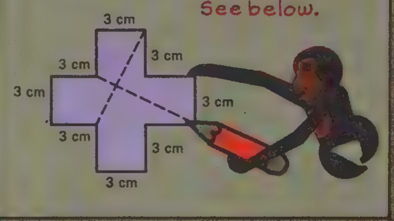
You find this quotient $n \times \frac{1}{2} = \frac{3}{8}$

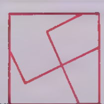
$\frac{3}{8} \div \frac{1}{2} = n$ when you find this factor.

A $\frac{1}{2} \times \frac{1}{4} = a$ $\frac{1}{8}$ → $\frac{1}{8} \div \frac{1}{4} = b$ $\frac{1}{2}$
B $\frac{1}{2} \times \frac{1}{4} = a$ $\frac{1}{8}$ → $\frac{1}{8} \div \frac{1}{2} = b$ $\frac{1}{4}$
C $\frac{3}{8} \times \frac{2}{3} = a$ $\frac{6}{24}$ → $\frac{6}{24} \div \frac{2}{3} = b$ $\frac{3}{8}$
D $\frac{1}{8} \times \frac{5}{6} = a$ $\frac{5}{48}$ → $\frac{5}{48} \div \frac{1}{8} = b$ $\frac{5}{6}$
E $\frac{7}{10} \times \frac{1}{2} = a$ $\frac{7}{20}$ → $\frac{7}{20} \div \frac{1}{2} = b$ $\frac{7}{10}$

think

Draw a figure like the one shown below. Cut it out. Cut along the dotted lines. Now fit the four pieces together to make a square. See below.





More practice, page A-29, Set 54

Mathematics
In this book, we do not attempt to develop, in any detail, the division algorithm for fractional numbers. In Book 6, however, division of fractional numbers will be developed fully and it will be shown that

$$\text{if } \frac{c}{d} \neq 0, \frac{a}{b} \div \frac{c}{d} = \frac{a \times d}{b \times c}$$

At this time, the emphasis is placed upon the meaning of division of fractional numbers and the interpretation of some simple examples of the division operation by means of the centimetre strips.

Duplicator Masters, page 58
Workbook, page 102

Using the Exercises
Have the children do the exercises on page 295. However, make sure they consider the purple, red, and white strips as the correct fractional parts before they continue with exercises 2 through 7.

Exercise 8 is starred, but it would be helpful in discussing the relation between multiplication and division of fractional numbers. The important thing to observe with the children is that division of fractional numbers has exactly the same relationship to multiplication of fractional numbers as has division of whole numbers to multiplication of whole numbers. We still find quotients by thinking about missing factors.

Assignments (page 295)
Minimum: 1–5. Average: 1–7.
Maximum: 1–8.

PAGES 296–297

Objective

Given function rule problems and short story problems, the child will be able to solve them by applying his understanding of fractional numbers.

Preparation

To prepare for this lesson you might play a game of "What's My Rule." For example, write this row of numbers on the chalkboard:

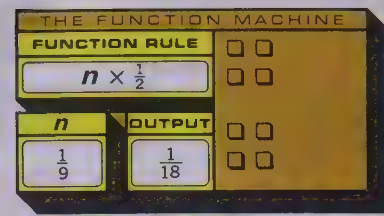
$3 \quad 5 \quad 4 \quad \frac{1}{2} \quad 2 \quad \frac{1}{3} \quad \frac{1}{4} \quad \frac{3}{5} \quad \frac{2}{3}$

Then begin to fill another row according to a particular rule such as $n \times \frac{1}{3}$.

1	$1\frac{2}{3}$	$1\frac{1}{3}$	$\frac{1}{6}$	?	?	?	?	?
3	5	4	$\frac{1}{2}$	2	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{3}{5}$	$\frac{2}{3}$

Ask the children who think they know the rule to complete the row. When a row is complete, ask a child to name the rule. Then continue to fill in other rows according to varying but simple rules. This same type of activity may be done by writing tables similar to those on page 296 on the chalkboard.

The Function Machine



We can use fractional numbers in the function machine. When the input number is $\frac{1}{9}$ and the function rule is $n \times \frac{1}{2}$, the output number is $\frac{1}{18}$.

Think about the function machine and give what you think should go in each gray space.

1. Function Rule: $n \times \frac{1}{3}$

	n	Output
A	5	$\frac{5}{3}$ or $1\frac{2}{3}$
B	3	$\frac{3}{3}$ or 1
C	$\frac{1}{6}$	$\frac{1}{18}$
D	$\frac{2}{5}$	$\frac{2}{15}$
E	0	$\frac{0}{3}$ or 0
F	$\frac{4}{7}$	$\frac{4}{21}$

2. Function Rule: $n \times \frac{1}{4}$

	n	Output
A	3	$\frac{3}{4}$
B	$\frac{1}{6}$	$\frac{1}{24}$
C	$\frac{3}{5}$	$\frac{3}{20}$
D	4	$\frac{4}{4}$ or 1
E	$\frac{1}{5}$	$\frac{1}{20}$
F	$\frac{4}{5}$	$\frac{4}{20}$

3. Function Rule: $\frac{2}{3} \times n$

	n	Output
A	5	$\frac{10}{3}$ or $3\frac{1}{3}$
B	$\frac{1}{3}$	$\frac{2}{9}$
C	$\frac{1}{4}$	$\frac{2}{12}$ or $\frac{1}{6}$
D	3	$\frac{6}{3}$ or 2
E	$\frac{3}{5}$	$\frac{6}{15}$ or $\frac{2}{5}$
F	$\frac{4}{5}$	$\frac{8}{15}$

4. Function Rule: $\frac{3}{4} + n$

	n	Output
A	$\frac{1}{2}$	$\frac{5}{4}$ or $1\frac{1}{4}$
B	$\frac{3}{4}$	$\frac{6}{4}$ or $1\frac{1}{2}$
C	$\frac{1}{8}$	$\frac{7}{8}$
D	$\frac{1}{3}$	$\frac{13}{12}$ or $1\frac{1}{12}$
E	$\frac{2}{3}$	$\frac{17}{12}$ or $1\frac{5}{12}$
F	$\frac{3}{4}$	$1\frac{1}{2}$

★5. Function Rule: A $n \times \frac{1}{2}$

	n	Output
	$\frac{1}{2}$	$\frac{1}{4}$
	3	$\frac{3}{2}$
	8	4
B	$\frac{5}{6}$	$\frac{5}{12}$
C	$\frac{3}{7}$	$\frac{3}{14}$
D	$\frac{1}{4}$	$\frac{1}{8}$

★6. Function Rule: A $n + \frac{1}{2}$

	n	Output
	$\frac{1}{2}$	1
	$\frac{3}{4}$	$\frac{5}{4}$
	1	$1\frac{1}{2}$
B	$\frac{2}{3}$	$\frac{7}{6}$ or $1\frac{1}{6}$
C	$\frac{5}{4}$	$\frac{7}{4}$ or $1\frac{3}{4}$
D	$1\frac{1}{5}$	$\frac{17}{10}$ or $1\frac{7}{10}$

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Discussion

Observe with the children that the function machine may be used with fractional numbers as well as whole numbers. Point out that the tables include problems with whole numbers as well as some with fractional numbers only. Also, stress that the starred problems are a special challenge: they must first discover the rule and then apply it to complete the table.

Solving Short Stories

1 $\frac{3}{5}$ km. $\frac{1}{2}$ that far. How far? $\frac{3}{16}$ km

2 $\frac{3}{4}$ of a pie. Ate $\frac{1}{3}$ of that. How much? $\frac{3}{12}$ or $\frac{1}{4}$

3 Walked $1\frac{1}{4}$ blocks. Ran $\frac{2}{3}$ block. How far in all? $1\frac{11}{12}$ blocks

4 Used stamps: $\frac{1}{10}$ cent. How much are 50 stamps worth? 5¢

5 Ran 5 races. Each was $\frac{3}{10}$ km. Ran how far? $\frac{15}{10}$ or $1\frac{1}{2}$ km

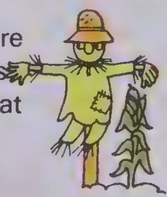
6 Square: Each side is $\frac{3}{10}$ metre. What is the perimeter? $\frac{12}{10}$ or $1\frac{1}{5}$ m

7 $\frac{2}{10}$ litre of milk for 1 cake. 4 cakes. How much milk is needed? $\frac{8}{10}$ or $\frac{4}{5}$ l

8 One day: $\frac{1}{7}$ of a week. What part of a week is $\frac{1}{2}$ of a day? $\frac{1}{14}$ wk

9 What part of an hour is $\frac{1}{3}$ of $\frac{1}{4}$ of an hour? $\frac{1}{12}$ h

10 Garden: $\frac{7}{10}$ of a square kilometre. $\frac{2}{3}$ of that is planted in corn. What part of a square km is planted in corn? $\frac{14}{30}$ or $\frac{7}{15}$ km^2



11 Loaf of bread: $\frac{9}{10}$ kg. Used $\frac{1}{2}$ loaf for dinner. How many kilograms left? $\frac{9}{20}$ kg

12 $\frac{5}{8}$ of the children were girls. $\frac{1}{2}$ of the girls went to the library. What part of the children went to the library? $\frac{5}{16}$

13 Drank $\frac{2}{3}$ glass of milk. The glass holds $\frac{1}{2}$ litre. Drank what part of a litre? $\frac{2}{6}$ or $\frac{1}{3}$

14 8 puppies in a litter.

A What part of the litter is 1 pup? $\frac{1}{8}$

B What part is 2 puppies? $\frac{2}{8}$ or $\frac{1}{4}$

C If 3 pups are male, what part is female? $\frac{5}{8}$ What part is male? $\frac{3}{8}$

More practice, page A-29, Set 55

Using the Exercises

You might find it necessary to work through several of the problems on page 297 with the children. For example, some children may need guidance in realizing that the first problem is asking them to find $\frac{1}{2}$ of $\frac{3}{8}$ kilometre. Note that this word-problem set contains work both with multiplication and with addition and subtraction of fractional numbers. Therefore, you might want to alert your children to watch carefully as they work each problem to decide which operation they should use for each problem. Be sure the children have an opportunity to discuss not only the arithmetic involved in the various word problems but also the word problems themselves and the situations described.

Assignments (page 297)

Minimum: 1–6. Average: 1–10.
Maximum: 1–14.

Follow-up

Short stories based on data about artificial satellites may prove stimulating and arouse interest in research in this area. For example, the following questions pertain to the table illustrated below.

1. If a satellite weighed 4120 kilograms, how much of that was needed for power supply?
2. If the scientific payload of Vanguard I was $1\frac{3}{4}$ kilograms, how much did the entire satellite weigh?
3. Explorer XVII weighed 188 kilograms. About how much of that satellite was scientific payload? How much did the shell of Explorer XVII weigh?

The weight of some of the earlier scientific satellites was distributed according to the following table.

Scientific payload	$\frac{1}{5}$
Weight of the shell	$\frac{1}{4}$
Communication equipment	$\frac{3}{10}$
Power supply: batteries	$\frac{1}{8}$
solar cells	$\frac{1}{8}$

Workbook, page 103

PAGES 298–299

Objective

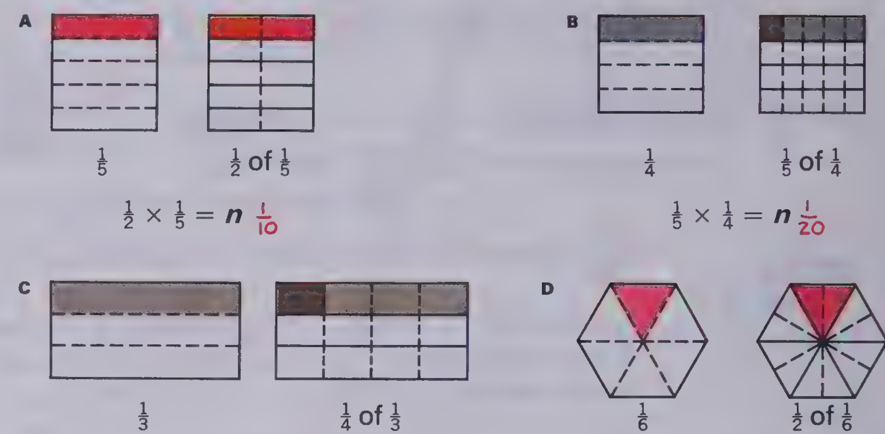
The child will demonstrate his ability to work with the concepts presented in this chapter.

Preparation

To prepare for this lesson, you might review basic operations with fractions, stressing those which caused most difficulty.

Reviewing the Ideas

1. Study the figures. Then solve the equations.



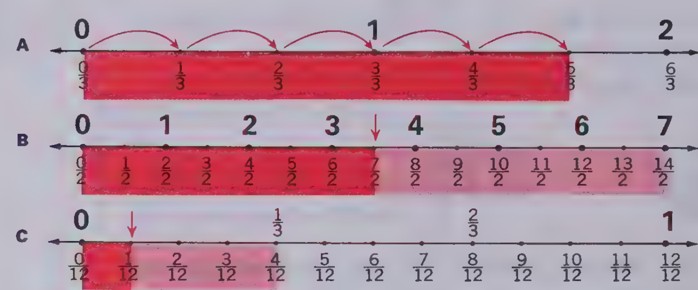
A $\frac{1}{2} \times \frac{1}{5} = n$ $\frac{1}{10}$

B $\frac{1}{5} \times \frac{1}{4} = n$ $\frac{1}{20}$

C $\frac{1}{4} \times \frac{1}{3} = n$ $\frac{1}{12}$

D $\frac{1}{2} \times \frac{1}{6} = n$ $\frac{1}{12}$

2. Study the number lines. Then give the products.



A $5 \times \frac{1}{3} = n$ $\frac{5}{3}$

B $\frac{1}{2} \times 7 = n$ $\frac{7}{2}$

C $\frac{1}{4} \times \frac{1}{3} = n$ $\frac{1}{12}$

3. Give the number for a. Then give the number for b.
   - A Since $7 \times \frac{1}{3} = a$, we know that $\frac{1}{3} \times 7 = b$. $\frac{7}{3}$, $\frac{7}{3}$
   - B Since $\frac{1}{3} \times \frac{1}{6} = a$, we know that $\frac{1}{6} \times \frac{1}{3} = b$. $\frac{1}{18}$, $\frac{1}{18}$
   - C Since $5 \times \frac{1}{3} \times \frac{1}{2} = a$, we know that $\frac{5}{3} \times \frac{1}{2} = b$. $\frac{5}{6}$, $\frac{5}{6}$
   - D Since $3 \times 4 \times \frac{1}{2} \times \frac{1}{5} = a$, we know that $\frac{3}{2} \times \frac{4}{5} = b$. $\frac{12}{10}$, $\frac{12}{10}$

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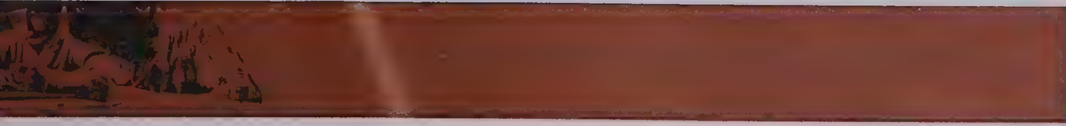
Discussion

The exercises on pages 298 and 299 may be used as evaluation or as review. Either way, you will want to give your children an opportunity to discuss the exercises after they have worked them.

The *Think* problem can be used for those children who finish the exercises quickly. They should see that since $\frac{1}{4}$ of the animals are horses and $\frac{1}{2}$ are cows, the remaining $\frac{1}{4}$ are pigs. If there are 8 pigs, then there must be 8 horses. These 16 make up half the number of animals, so the other half, the cows, must number 16 also. Expect many variations in the way children approach this problem. Urge some of them to share their thinking with the rest of the class.



4. Find the products.

A $\frac{2}{3} \times \frac{3}{5} = (2 \times 3) \times (\frac{1}{3} \times \frac{1}{5}) = n$ $\frac{6}{15}$
C $\frac{1}{8} \times \frac{3}{4} = (1 \times 3) \times (\frac{1}{8} \times \frac{1}{4}) = n$ $\frac{3}{32}$

B $\frac{2}{5} \times \frac{2}{5} = (2 \times 2) \times (\frac{1}{5} \times \frac{1}{5}) = n$ $\frac{4}{25}$
D $\frac{5}{8} \times \frac{2}{3} = (5 \times 2) \times (\frac{1}{8} \times \frac{1}{3}) = n$ $\frac{10}{24}$

5. Solve each short story.

A Recess: $\frac{1}{4}$ hour.
Talked for $\frac{1}{2}$ of it.
Talked what part of an hour? $\frac{1}{8}$ h

B Track: $\frac{1}{4}$ km around.
Ran 2 km.
How many times around? 8

C Cake: $\frac{7}{10}$ kg sugar.
How much for 2 cakes? $\frac{14}{10}$ or $1\frac{2}{5}$ kg

D $\frac{1}{3}$ of a pie left.
Ate $\frac{1}{2}$ of the part left.
Ate what part of the pie? $\frac{1}{6}$

6. Give the products.

A $\frac{3}{4} \times \frac{1}{6}$ $\frac{3}{24}$	D $\frac{1}{2} \times \frac{6}{7}$ $\frac{6}{14}$	G $\frac{4}{7} \times \frac{5}{3}$ $\frac{20}{21}$	J $\frac{1}{10} \times \frac{1}{10}$ $\frac{1}{100}$
B $\frac{5}{8} \times \frac{1}{3}$ $\frac{5}{24}$	E $\frac{3}{4} \times \frac{3}{10}$ $\frac{15}{24}$	H $\frac{3}{10} \times \frac{3}{4}$ $\frac{15}{24}$	K $\frac{2}{3} \times \frac{1}{2}$ $\frac{2}{6}$
C $\frac{1}{3} \times \frac{4}{5}$ $\frac{4}{15}$	F $\frac{2}{3} \times \frac{7}{4}$ $\frac{14}{12}$	I $\frac{1}{2} \times \frac{2}{5}$ $\frac{2}{10}$	L $\frac{7}{10} \times \frac{1}{4}$ $\frac{7}{32}$

7. Give the quotients.

A $1 \div \frac{1}{8} = n$ 8　B $2 \div \frac{1}{2} = p$ 4　C $2 \div \frac{1}{4} = r$ 8　D $3 \div \frac{1}{2} = s$ 6

think

Farmer Brown had some animals.
$\frac{1}{4}$ were horses. $\frac{1}{2}$ were cows.
The rest were pigs. He had 8 pigs.
How many horses and how many cows did he have? 8 horses, 16 cows

Resources for Active Learning

Math Activity Cards, D35, Macmillan. [A recipe]

Applied Mathematics Cards, Group 2/11, Schofield and Sims. [Measurements and fractions] (Available from Mafex Associates, Willowdale)

Workbook, page 104

PAGES 300–301

Objective

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

Review with the children any topics from the text with which they have had particular difficulty. Since the children have just completed a chapter involving multiplication and division of fractional numbers, you may find this a good time for a thorough review of addition and subtraction of fractional numbers. If so, it may be advantageous to present on the chalkboard a few examples involving addition and subtraction of fractional numbers and then give the children an opportunity to work these in a step-by-step fashion, explaining each step to the class.

If you prefer, you might use the preparation period to focus attention on the interesting "people facts" on page 301.

Keeping in Touch with Computing, Measurement, Fractional numbers

1. Compute the sums.

A $693 + 48 + 976 + 9283$ 11 000 C $68.34 + 2.26 + 389.45$ 460.05

B $57.2 + 3.6 + 349.1 + 8.3$ 418.2 D $8\frac{1}{5} + 7\frac{2}{5} + 4\frac{3}{5} + 8\frac{4}{5}$ 29

2. Give the sums, products, differences, and quotients.

A	B	C	D	E	F
$964 - 485$	35×27	$983 + 47$	$807 - 269$	$6954 + 9765$	395×207
479	945	1030	538	16 719	81 765

G $7\overline{)5234}$ 747 R5 H $30\overline{)728}$ 24 R8 I $59\overline{)2130}$ 36 R6 J $43\overline{)3340}$ 77 R29

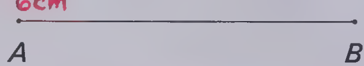
3. Solve the equations.

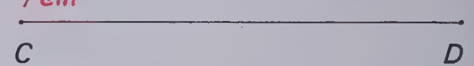
A $\frac{1}{3} + \frac{3}{4} = n$ $\frac{13}{12}$ C $\frac{5}{3} + \frac{0}{5} = n$ $\frac{5}{3}$ E $0.4 + 0.3 = n$ 0.7 G $1.4 - 0.5 = n$ 0.9

B $\frac{5}{6} - \frac{1}{4} = n$ $\frac{7}{12}$ D $\frac{4}{10} + \frac{3}{10} = n$ $\frac{7}{10}$ F $8\frac{4}{5} - 2\frac{1}{4} = n$ $6\frac{11}{20}$ H $5\frac{3}{5} + 8\frac{2}{3} = n$ $14\frac{4}{15}$

4. Give the length of each segment to the nearest centimetre.

A 6cm B 7cm

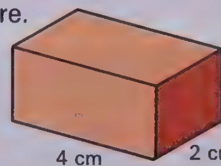




5. Give the volume and surface area of this figure.

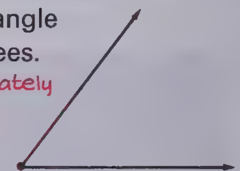
V = 16 cm³

SA = 40 cm²



6. Give the measure of this angle in degrees.

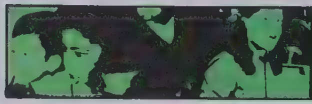
Approximately 53°



7. Find the differences.

A 12 wk 8 days − 7 wk 6 days = 5 wk 2 days

B 17 h 6 min − 9 h 8 min = 7 h 58 min



You are invited to explore

ACTIVITY CARD 15 Page 340

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Discussion

Have the children do the exercises. When they have finished, allow time for discussion and checking papers. If you discussed addition and subtraction of fractional numbers as a part of your preparation period, the children may find it most helpful to have several parts of exercises 1 and 2 presented on the chalkboard and explained. In the presentation of exercise 1, one of the important things you will want to stress for the children is the lining up the decimal points in order to arrive at the correct sums.

You may find it helpful to provide children with additional problems similar to those in exercise 7. Use other units to provide the same type of exercise; for example, you might use quarts and cups, gallons and pints, and so on.

People

Oldest
113 years

Heaviest
485.5 kg

Shortest
58.1 cm

Strongest
Lifted 2724 kg

1. The height of the tallest man is how much greater
   A than the height of the tallest woman? 12.7cm
   B than the height of the shortest man? 212.4cm

Tallest

257.8 cm   270.5 cm

2. The oldest man died in 1814. In what year was he born? 1701

3. Of course, the strongest man did not lift 2724 kg over his head, but he did lift it off the ground. How much greater is the weight he lifted than the weight of the heaviest man? 2238.5 kg

4. Some strong men can put as much as 170 kg over their heads. One of the best lifts by a small man occurred when a 57.7 kg man put slightly more than $2\frac{1}{2}$ times his own weight over his head. About how much was that? $144\frac{1}{4}$ kg

5. Twelve times the weight of the tallest man is about as much as the strongest man can lift. About what does the tallest man weigh? 227 kg

6. When the heaviest man was 10 years old, he weighed 172 kg. About how many times your weight is this? Answers will vary.

*The records given, although subject to dispute, are reasonably accurate.

301

Follow-up
Encourage children to find other interesting data and to make up problems based on the information they find.

Using the Exercises
The problems on page 301 provide an opportunity for interesting discussion as well as review of operations with both fractions and whole numbers. Exercises of this type are provided to stimulate the children's interest in arithmetic and its applications to various interesting situations.

CHAPTER 15 | Graphing and Geometry

Pages 302–321

General Objectives

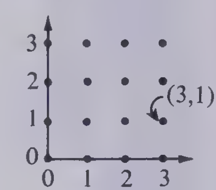
To develop skill in graphing co-ordinates
To build understanding of symmetric figures and reflections
To introduce tessellations of a plane
To introduce negative integers
To extend the idea of graphing co-ordinates which include negative integers
To introduce the idea of graphing number sentences or functions

The first lesson in this chapter reviews locating points in the first quadrant of the co-ordinate plane. Subsequent lessons introduce symmetric figures and reflections, rotations, translations, similarity, tessellations, and integers. Rotations are introduced through consideration of fractional clockwise turns of figures on a geoboard. Translations and a study of similar figures build upon the child's understanding of co-ordinates and graphing. The children are introduced to tessellations in the following lesson. Finally, negative numbers are introduced and points with negative co-ordinates are plotted in the co-ordinate plane. The last lesson introduces the operations of addition and subtraction of numbers by presenting graphs of number pairs using simple functions. The chapter concludes with a page which reviews the topics in the chapter and a page of cumulative review exercises.

Mathematics

In this chapter, geometry is used to picture, or graph, certain arithmetic relationships. To do this, we use the *Cartesian co-ordinate system*, named after the French mathematician, Rene Descartes (1596-1650).

To set up a co-ordinate system in the plane, we choose two perpendicular number lines that intersect at zero and then associate points on the plane with number pairs. In the beginning, we use number rays with the whole-number points labelled. Having chosen two rays, we can locate a point for any pair of whole numbers, provided we agree on which number in the pair refers to which ray. It is conventional to agree that the first number refers to the horizontal ray and the second number refers to the vertical ray.

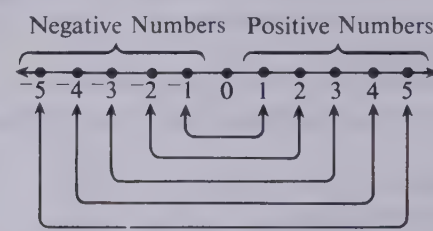


As negative numbers are introduced, the number rays become number lines labelled with positive and negative numbers, and intersecting at the co-ordinates (0, 0).

While studying the last part of this chapter, the children will also translate input and output numbers from function tables into the corresponding pairs of co-ordinates, and then graph these co-ordinates.

Each function rule produces a set of number pairs, and the graph of these pairs often reveals a pattern. This idea of picturing or graphing a special set of number pairs associated with a function is one of the most important ideas in mathematics. The graphs of most of the functions in this chapter are straight lines. Such functions, called linear functions, are the paramount concern of elementary algebra. More complicated functions and relations may produce patterns such as parabolas, hyperbolas, circles, and ellipses.

Two other important mathematical concepts presented in this chapter are negative numbers and their use in extending graphing to the entire plane. Negative numbers are first presented to the children as "reflections" of the positive numbers on the number line.



In the diagram, each whole number is paired with a corresponding negative number. As shown on the diagram, we consider the negative numbers to be those less than zero, which suggests many applications for their use: temperatures above and below zero, in and out of debt, and so on. In mathematics, negative numbers are solutions for equations of the following type:

$$4 + n = 0$$
$$n = {}^{-}4$$

Pairs of numbers are introduced in this chapter as co-ordinates that indicate various locations on the plane.

Teaching the Chapter

Materials

Acetate, 7-by-12-cm pieces (1 per child)
Cardboard
Construction paper or manilla tagboard cards, 7-by-12-cm (optional)
Dot paper, geoboard paper
Graph paper (1-cm grid is suitable for most work)
Overhead projector
Pattern blocks (if available)
Rulers
Scissors
Tracing paper (transparent)
Transparencies of a grid for use on overhead projector

Transparent geoboard for use with overhead projector, or transparencies on which geoboard exercises may be done

Vocabulary

axis
co-ordinate axis
co-ordinate plane
co-ordinates
graph
line of symmetry
mirror image
negative number
positive number
reflection
rotation
similar figures
symmetric figure
tessellation
translation

The most important material for this chapter is graph paper. It would be helpful to have available an abundant supply of graph paper, dot paper, or geoboard paper. You might wish to prepare your own from a duplicating master (see *Duplicator Masters*, pages 61–66). It would be extremely tedious for children to have to make their own grids, although it is important that they learn to draw in the co-ordinate axes properly.

Lesson Schedule

Since this is the next to the last chapter in the book, adjust your time schedule according to the interest of the children and the time remaining in the school year. The chapter is planned to be covered in a week, but some classes may need or want to spend two weeks investigating the many interesting activities.

Evaluation of Progress

Since one of the main purposes of this chapter is to provide children with stimulating and enjoyable geometric activities, you should not be overly concerned with the children's proficiency in graphing or with their ability to use the new vocabulary easily. The children should enjoy the many activities of this chapter (even though some of them require a good deal of concentration) if they are introduced with sincere interest and enthusiasm.

Resources for Active Learning

General Activities

[The first five references suggest a variety of interesting co-ordinates games.]

Developmental Math Cards, 1-12, Addison-Wesley

Inquiry in Mathematics via the Geo-board, "Tic-Tac-Toe," Geo-Cards 24/1-7, Walker (Available from Fitzhenry and Whiteside)

Madison Project: *Discovery in Mathematics:* A Text for Teachers, "The Point-Set Game (Go)," pp. 55–66, Addison-Wesley

Mathex: Graphing and Probability No. 6, "Alpha-Hits" and "Battleship," pp. 3–6, Encyclopaedia Britannica Publications Ltd.

Notes on Mathematics in Primary Schools, "Nine Men's Morris," pp. 231–233, Cambridge University Press (Available from Macmillan of Canada)

Experiments in Mathematics, Stage 1, pp. 30-31, Houghton Mifflin [Curve stitching] (Available from Thomas Nelson & Sons)

Franklin Series: *Making and Using Graphs and Nomographs*, "Maps . . . ," pp. 49-70, Lyons and Carnahan (Available from McGraw-Hill Ryerson)

Freedom to Learn, "Geometry—Congruence and Symmetry," pp. 153–154, Addison-Wesley

Mathex: Geometry No. 9, "Symmetry," pp. 43–47, Encyclopaedia Britannica Publications Ltd.

Mathex: Graphing and Probability No. 6, "Graphing Equations," pp. 7–11, Encyclopaedia Britannica Publications Ltd.

Notes on Mathematics in Primary Schools, "Position Fixing and Co-ordinates," pp. 104–112, Cambridge University Press (Available from Macmillan of Canada)

Nuffield Project: *Computation and Structure* 3, ". . . truth-sets," pp. 14–18; *Graphs Leading to Algebra* 2; *Pictorial Representation* 1; *Shape and Size* 3, "Another look at symmetry," pp. 23–54, Wiley

Manipulative Devices

Geoboards (Addison-Wesley)
Geo Strips (Math Media; Selective Educational Equipment)
Number Line (school supplier)
Pattern Blocks (McGraw-Hill Ryerson; Selective Educational Equipment)
Sigma Chips (Sigma, Scott Scientific)
Spirograph (Cuisenaire Co.)

Commercial Games

Battleship (local supplier)
Go Game (Gomoku) (Math Media; World Wide Games)
Madison Project: Independent Exploration Material (Math Media)
3-D Tic-Tac-Toe (Childcraft; Creative Publications; World Wide Games)
Vectors (Selective Educational Equipment)

PAGES 302–303

Objectives

Given a pair of co-ordinates, the child will be able to graph the corresponding points on a co-ordinate plane.

Given points on a co-ordinate plane, the child will be able to identify the co-ordinates.

Preparation

Materials

graph paper (*Duplicator Masters*, page 65)

It would be helpful to review with the children how a horizontal and vertical number line can be used to form a co-ordinate plane. For example, exhibit on the chalkboard a horizontal number line, reminding children that for every number we think of, there is a corresponding point on the number line. Then draw the vertical axis and help children realize that it is simply a number line in a vertical position. You might want to help the children begin the grid they will need in the investigation.

Investigation

For children who have previously graphed points on a grid, this investigation will simply be a review. Others may need some guidance as they study the example and directions. When they connect the dots they have drawn, instruct the children to connect the last dot (H) to the first dot (A). For those who finish quickly, you might present another set of dots such as the following:

(3,0), (1,3), (3,6), (5,3), (3,0) *(rhombus)*

Or, (3,1), (1,3), (1,5), (3,7), (5,5), (5,3), (3,1) *(hexagon)*

15 Graphing and Geometry

Can you graph points?

Investigating the Ideas

To graph point A with co-ordinates (4, 3)

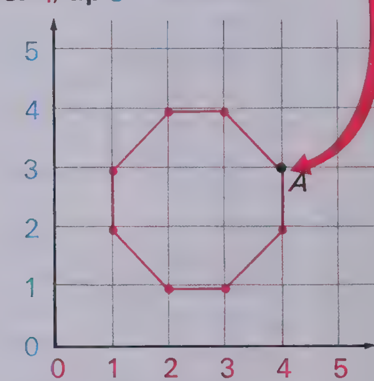
1 LOOK at its **co-ordinates.** (4, 3)

2 THINK about its location. **Over 4, up 3**

3 MARK the dot.

Make a grid like this and graph these points.

A: (4,3)	*E:* (1,2)
B: (4,2)	*F:* (1,3)
C: (3,1)	*G:* (2,4)
D: (2,1)	*H:* (3,4)



? What geometric figure can you form by connecting the dots in order?
Octagon (See above.)

Discussing the Ideas

1. Explain how you would graph these points: See Discussion.

A (5,5)	D (0,0)
B (2,3)	E (4,0)
C (3,2)	F (0,4)

2. How can you show that the points for (5,3) and (3,5) are different points? See Discussion.

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Discussion

Use the first part of the discussion period to answer any questions children might have concerning the investigation activity. Then have volunteers demonstrate how to graph points by graphing those in discussion exercise 1 on a grid drawn on the chalkboard or overhead projector. Exercise 2 emphasizes the conventional agreement that the first number in a co-ordinate pair indicates the distance from zero on the horizontal axis, and the second number indicates the distance from zero on the vertical axis.

It would also be helpful to discuss how to graph fractional points such as $(3\frac{1}{2}, 2\frac{1}{2})$. Make sure the children realize that every fractional number can be matched to some point on the number line. You might develop this by asking children to name some of the points between 2 and 3, such as $\frac{9}{4}$, $2\frac{1}{3}$, $\frac{5}{2}$, $2\frac{3}{4}$. Point out that since every fractional number can be matched to some point, the number line is so full of points with fractional numbers that we would use a solid line to represent all the fractional numbers between two and three.

Using the Ideas

1. Give the missing number. Then give the co-ordinates.

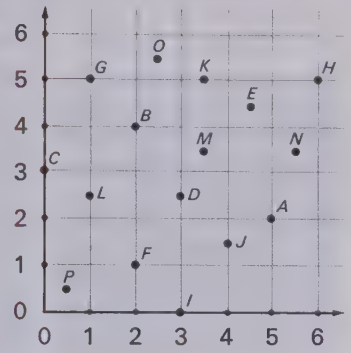
   A Point A is 5 over and ▒ up. 2
   The co-ordinates of A are (5, 2) ?.

   B Point B is ▒ over and 4 up. 2
   The co-ordinates of B are (2, 4) ?.

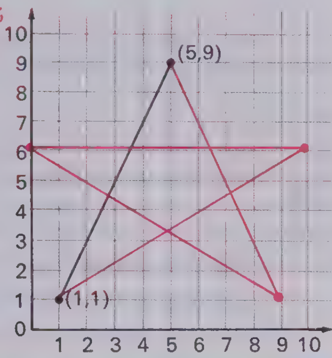
   C Point C is 0 over and ▒ up. 3
   The co-ordinates of C are (0, 3) ?.

   D Point D is ▒ over and $2\frac{1}{2}$ up. 3
   The co-ordinates of D are (3, $2\frac{1}{2}$) ?.

   E Point E is $4\frac{1}{2}$ over and ▒ up. $4\frac{1}{2}$
   The co-ordinates of E are ($4\frac{1}{2}$, $4\frac{1}{2}$) ?.



2. Give the co-ordinates of points F through P. F(2,1); G(1,5); H(6,5); I(3,0); J(4,$1\frac{1}{2}$); K($3\frac{1}{2}$,5); L(1,$2\frac{1}{2}$); M($3\frac{1}{2}$,$3\frac{1}{2}$); N($5\frac{1}{2}$,$3\frac{1}{2}$); O($2\frac{1}{2}$,$5\frac{1}{2}$); P($\frac{1}{2}$,$\frac{1}{2}$)



3. The first 2 points have been graphed and connected. Use your graph paper to copy and complete the picture by connecting these points in the order given: (1,1), (5,9), (9,1), (0,6), (10,6), (1,1). See graph.

4. Invent a picture and list the co-ordinates for the points of the picture. Then see if a classmate can draw the picture. See Using the Exercises.

> **think**
>
> One morning a snail starts climbing up a board that is 309 cm long. Each day he climbs 61 cm. Each night he slips back 30 cm. On what day (8th, 9th, or 10th) does he reach the top? 9th

Using the Exercises
Before assigning the exercises on page 303, point out that some of the points graphed in exercises 1 and 2 are not directly on the lines of the grid because they represent points which match fractional numbers. Provide graph paper for exercises 3 and 4. For exercise 4, encourage children to give points not only for other pictures but also for geometric figures and designs.

By drawing a picture for the *Think* problem, children might note that, although the snail has gained 31 centimetres each day, by the beginning of the ninth day he has only 61 centimetres to go. Therefore, at the end of the ninth day, having advanced 61 centimetres, he is already at the top and we do not count the slipping back that night. Hence, he makes it to the top of the board on the ninth day.

Assignments (page 303)
Minimum: 1–2. Average: 1–3.
Maximum: 1–4.

Follow-up
Children might enjoy graphing and connecting one of the following sets of points to see what picture results. These may then be colored and displayed.

(Witch)	*(Clown's Face)*
1. (9, 0)	1. (0, 9)
2. ($12\frac{1}{2}$, 4)	2. (2, 2)
3. ($13\frac{1}{2}$, 6)	3. (10, 0)
4. (16, 3)	4. (18, $\frac{1}{2}$)
5. (17, 7)	5. (22, 8)
6. (22, 6)	6. (19, 14)
7. (26, 0)	7. (24, 18)
8. (22, 6)	8. (18, 20)
9. (27, 5)	9. (19, 33)
10. (22, 12)	10. (17, 31)
11. (24, 12)	11. ($18\frac{1}{2}$, 28)
12. (20, $17\frac{1}{2}$)	12. (13, 27)
13. ($24\frac{1}{2}$, 34)	13. (18, 23)
14. ($6\frac{1}{2}$, $24\frac{1}{2}$)	14. (9, 23)
15. (1, 24)	15. (19, 33)
16. (4, $21\frac{1}{2}$)	16. (9, 23)
17. (4, $16\frac{1}{2}$)	17. (18, 20)
18. (5, 19)	18. (3, 25)
19. (6, 18)	19. (4, 19)
20. (5, 17)	20. (6, $18\frac{1}{2}$)
21. (6, 18)	21. (0, 9)
22. (5, 19)	22. (0, 12)
23. (4, $16\frac{1}{2}$)	23. ($2\frac{1}{2}$, 13)
24. ($2\frac{1}{2}$, $15\frac{1}{2}$)	24. ($2\frac{1}{2}$, $16\frac{1}{2}$)
25. (2, 9)	25. (6, $18\frac{1}{2}$)
26. ($5\frac{1}{2}$, 13)	26. (19, 14)
27. ($4\frac{1}{2}$, $11\frac{1}{2}$)	27. (13, 16)
28. (5, 10)	28. (12, 10)
29. ($5\frac{1}{2}$, $10\frac{1}{2}$)	29. (11, 10)
30. ($6\frac{1}{2}$, 10)	30. (10, 17)
31. (6, $10\frac{1}{2}$)	31. ($4\frac{1}{2}$, 16)
32. (7, 12)	32. (6, 9)
33. ($8\frac{1}{2}$, 9)	33. (4, 7)
34. ($5\frac{1}{2}$, 9)	34. (5, 8)
35. ($6\frac{1}{2}$, $7\frac{1}{2}$)	35. (7, 5)
36. (8, 8)	36. (11, 3)
37. (6, 3)	37. (15, 14)
38. ($11\frac{1}{2}$, 6)	38. (18, 7)
39. ($12\frac{1}{2}$, 4)	39. (19, 6)
40. ($11\frac{1}{2}$, 6)	40. (17, 8)
41. (13, 7)	41. (19, 14)
42. ($13\frac{1}{2}$, 6)	42. (22, 12)
43. ($11\frac{1}{2}$, 11)	43. (21, 10)
44. (11, 8)	44. (24, $8\frac{1}{2}$)
45. (9, 17)	45. (21, $5\frac{1}{2}$)
46. (4, $21\frac{1}{2}$)	
47. (9, 17)	
48. (22, 12)	

Workbook, page 105

PAGES 304–305

Objective

Given the graph of half of a symmetric figure, the child will be able to graph the other half.

Preparation

Materials

graph paper (*Duplicator Masters*, page 65); compass or carbon paper

Because of the nature of the investigation, you might have the children begin it immediately.

Investigation

Distribute graph paper to the children and help them study the investigation directions to see how they should label the axes from 0 to 8, and cut along the lines marked 8. When they fold the paper, they should fold the right side in back of the left so that they can see the numbers along the vertical grid. In order to mark both sides of the graph at once, they might use the tip of a compass to prick a hole or they might work on top of a piece of carbon paper, carbon side up. If neither compasses nor carbon are available, they might be able to use the carbon technique by replacing the carbon with a piece of paper which they have covered with pencil lead. In any case, be sure to stress the importance of connecting the dots in the given order.

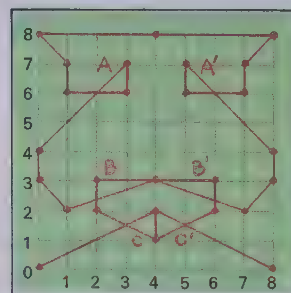
● *Can you graph symmetric figures?*

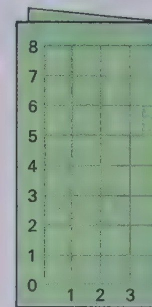
Investigating the Ideas

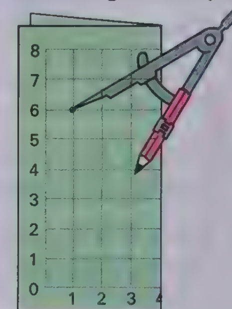
1 Cut out a piece of graph paper.

2 Fold it like this.

3 "Graph" each point with the compass tip. (Push through both parts.)







Use the steps above to graph these points:
(4,8), (0,8), (1,7), (1,6), (3,6), (3,7), (0,4), (0,3), (1,2), (4,3), (2,3), (2,2), (4,1), (4,2), (0,0).
Connect them to form half of a figure. Now unfold the paper.

? Can you use the other holes to help you draw the other half of the figure? See above.

Discussing the Ideas

1. The line along the fold is called the **line of symmetry**.
   - A When you graphed the point (1,6) with your compass tip, what other point did you automatically graph on the other side of the line of symmetry? (7, 6)
   - B Answer the question in A for each of the points in the Investigation. See Answers, T.E. page 304.
2. How many different triangles can you find in the left half of the completed figure above? Can you find each triangle in the right half of the figure that is congruent to a triangle in the left half? See triangles labelled A, A'; B, B'; C, C' in illustration above.

304

Discussion

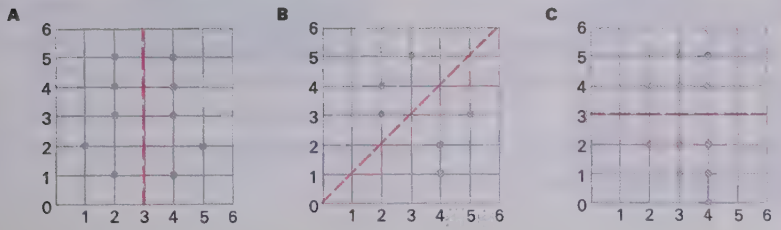
One of the most important points of this lesson is to help children recognize the line of symmetry in a figure and realize that from this dividing mark each symmetric half of the figure can be seen. Name a point on the left side of the fold and ask the children to find its matching point on the right side. Then elicit from them the observation that both points are the same distance from the fold, that is, from the line of symmetry. Then they should try to connect the points on the right side of the fold to make a matching half.

Answers, discussion exercise 1B

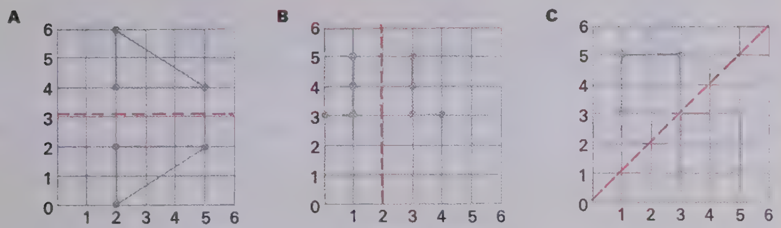
1B (4, 8) ≈ (4, 8)
(0, 8) ≈ (8, 8)
(1, 7) ≈ (7, 7)
(1, 6) ≈ (7, 6)
(3, 6) ≈ (5, 6)
(3, 7) ≈ (5, 7)
(0, 4) ≈ (8, 4)
(0, 3) ≈ (8, 3)
(1, 2) ≈ (7, 2)
(4, 3) ≈ (4, 3)
(2, 3) ≈ (6, 3)
(2, 2) ≈ (6, 2)
(4, 1) ≈ (4, 1)
(4, 2) ≈ (4, 2)
(0, 0) ≈ (8, 0)

Using the Ideas

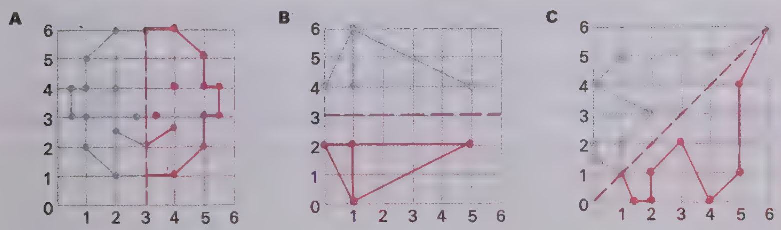
1. Suppose you folded each picture along the dashed line. In which pictures would the dots match each other? A and C



2. Where could you fold each picture so that the two parts will match? See dashed lines below.



3. Copy each figure on graph paper. Then draw the "other half."



4. Draw half of a symmetric figure. Ask a classmate to draw the other half. Figures will vary.

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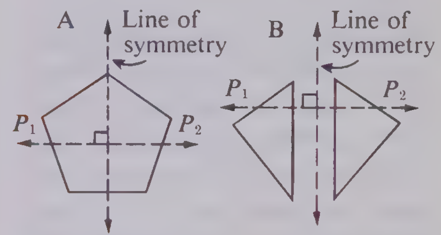
Mathematics

A plane geometric figure is said to have a *line of symmetry* if there is a line in the plane which separates the figure so that, for each point P_1 of the figure on one side of the line, there is a corresponding point P_2 of the figure, such that the line is the perpendicular bisector of $\overline{P_1P_2}$.



The separating line is called the *line of symmetry* or *axis of symmetry*. If a figure has a line of symmetry, then the figure is a line-symmetric figure. The two matching parts of the figure may be connected as in figure A or as in figure B.

This definition of symmetric figures means that, informally, when given a figure drawn on a piece of paper, we may test it for symmetry by folding the paper along what we think is a line of symmetry to see whether the two halves match. Another method of checking symmetry of figures is to place a plane mirror in a vertical position along a presumed line of symmetry to see if the reflected image falls directly on the other half of the figure. For this reason, line symmetry is sometimes called *reflection symmetry*.

Using the Exercises

Assign the exercises on page 305 as independent work. Encourage children to try various ways of proving to themselves that they are choosing the correct answer. Some may copy the figures on graph paper and fold them along the line of symmetry to see if the points match. Others may be able to compare distances from the line of symmetry by using a compass.

Exercise 4 may be extended to include other related activities such as those suggested in the follow-up section.

Assignments (page 305)
Minimum: 1, 3. Average: 1–3.
Maximum: 1–4.

Follow-up

We often consider faces to be symmetric along a line passing vertically down the centre of the face between the eyes, but the actual study of faces shows that most faces are not truly symmetric. Encourage children to think of ways to test for symmetry in pictures of faces by carefully lining a co-ordinate grid on top of the face (making sure the line of symmetry is properly placed in the centre before the other grid lines are drawn). They can then compare the position of eyebrows or ears, using the points on their grid as reference.

PAGES 306–307

Objective

Given a figure or half of a figure, the child will be able to draw its reflection, or mirror image.

Preparation

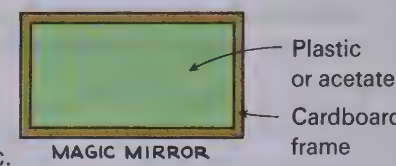
Materials

pieces of acetate or plastic, 7 by 12 centimetres; cardboard for frames (optional)

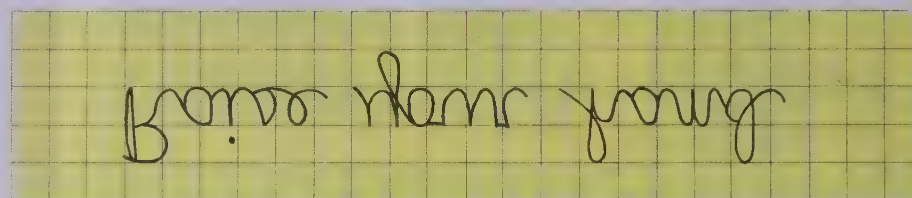
To prepare for this lesson, guide children in the use of their "mirrors." When the plastic is placed so that it is perpendicular (standing straight up) on the line of symmetry, the reflection of the figure will appear on it. It might also be necessary to back the plastic with cardboard or with dark construction paper. You might want to have the children use the writing in exercise 1 to practice with their "mirrors." If actual mirrors are available, you might prefer to use them.

Let's explore line symmetry.

For these exercises you will need a "magic mirror." To make one, cut out a piece of acetate or plastic 8 by 13 centimetres. If necessary, make a cardboard frame to hold the plastic.

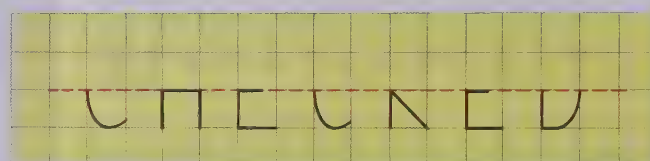


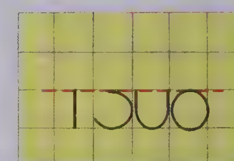
1. Here is a SECRET MESSAGE. Can you use your mirror to figure out what it says? Raise your hand



Can you make some messages for others to figure out?

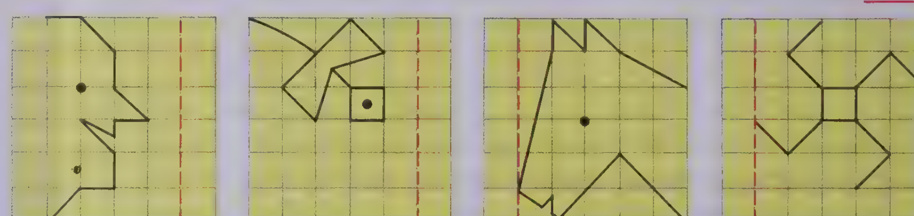
2. Here are a "half word" and a "half numeral." Name them. CHECKED; 1308





How many more "half words" and "half numerals" can you make?

3. Can you draw the reflection ("the other half") of each figure? Check your drawing with your mirror. Make some more like these. See Discussion.



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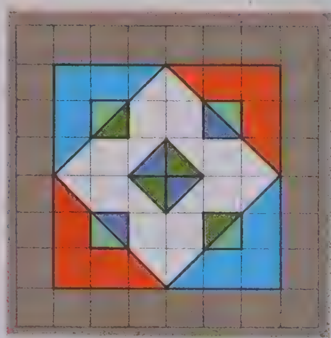
Discussion

As suggested in the preparation, the message in exercise 1 might be used for practicing the procedure with the mirrors. If children are able to use only the plastic without a cardboard backing, they will easily be able to make secret messages of their own. They need only write out the message, turn it upside down, place their mirror to form a reflection, and copy it behind the plastic in mirror-writing form.

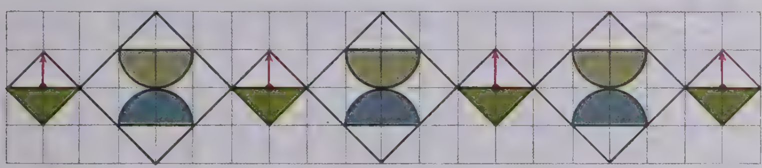
Exercises 2 and 3 are also based on the use of a mirror to find the reflection of a figure. If graph paper the same size as that in the text is not available, suggest to the children that they trace the grid and then try to draw the reflections.

LINE-SYMMETRIC PATTERNS

1. This graph-paper design is line-symmetric. (One half is a reflection of the other half.) Its two halves are exactly alike and will fit upon each other. How? Fold diagonally. Can you use your graph paper to make other line-symmetric designs?



2.

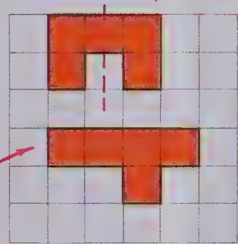


Strip designs like this can be used as borders in decorating. Where can you fold it so that one half is a reflection of the other half? Can you use your graph paper and make other border designs that are line-symmetric?

Fold vertically through centre.

3. A **pentomino** is made up of 5 squares. Each of its squares can touch another one only along one complete common side. Here are two of the twelve possible pentominoes. One has a line of symmetry and the other does not. Which one? Can you find other pentominoes with lines of symmetry and draw them on graph paper?

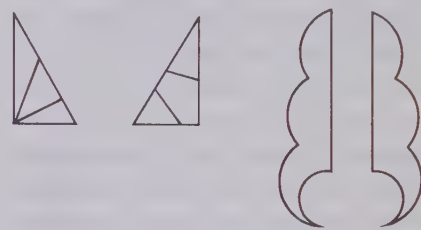
See Answers, T.E. page 307.



307

Follow-up

Commercially prepared mirror cards would be useful in providing further practice with symmetry and reflections. If these are not available, children might make a set for use in class. Various tasks could be designed. For example, some cards might show half of a figure with the question: "Can you name the whole figure? Check your answer with a mirror." Other cards might show drawings of two half-figures side by side with the question: "Do these halves match? Check your answer with a mirror."



Resources for Active Learning

Franklin Series: *Mirror Magic*, "Symmetry . . . :" pp. 58–83, Lyons and Carnahan (Available from McGraw-Hill Ryerson)

Inquiry in Mathematics via the Geo-board, "Polyminoes," Geo-Cards 14/1-6, Walker. (Available from Fitzhenry and Whiteside)

Math Activity Cards, C44, Macmillan.

Maths Mini-lab, Cards 122–124, Selective Educational Equipment.

Developmental Math Cards, 1-36, Addison-Wesley. [Symmetry and 3-D]

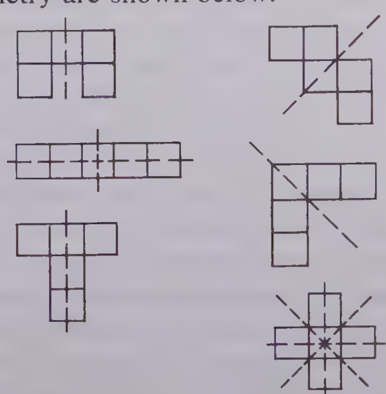
Using the Exercises

The exercises on page 307 are intended to encourage the children to create their own designs. You might want to combine such a page with an art lesson; if so, stress how artistically pleasing symmetric designs can be.

Use the terms *reflection, image, symmetry*, and *line of symmetry* in an informal manner; they need not be taught formally. Your use of these terms in discussions with the children, and questions about them, should enable children to understand their meaning.

Answers, Exercise 3, page 307

The pentominoes with lines of symmetry are shown below.



Assignments (page 307)

Minimum: 1 or 2. Average: 1–2. Maximum: 1–3.

PAGES 308–309

Objective

Given a figure on the geoboard, the child will be able to rotate it 90°, 180°, or 270°.

Preparation

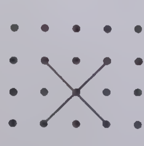
Materials

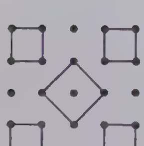
geoboard (1 per child); dot paper which simulates the geoboard (*Duplicator Masters*, page 64); elastic bands for the geoboards; transparent geoboard for use with overhead projector

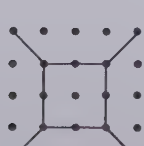
To prepare for this lesson, clarify the meaning of the term *rotation*. For example, remind the children that we speak of the earth rotating about its axis as it revolves around the sun. A top rotates when it is spun; we make a complete rotation if we turn ourselves completely around once, returning to our original position. If children understand what is meant by a complete rotation they should be able to work through the investigation without much difficulty.

Investigation

The arrow illustrated on the geoboard is not intended as an example of the kind of figure children are asked to make on their geoboard. That is, it does not look exactly the same after the $\frac{1}{4}$ rotation as it did in its original position. The children must make a figure which *does* look exactly the same after they have rotated the board in clockwise direction a quarter of a turn. The kind of figures which are acceptable will be figures such as these:

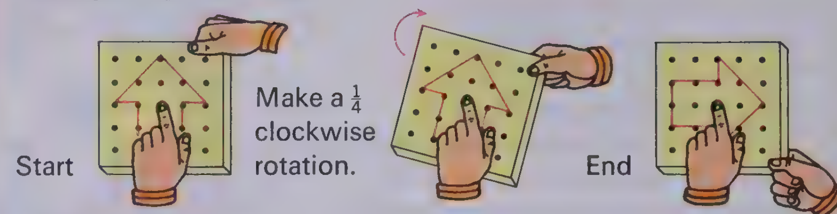






What changes do rotations make?

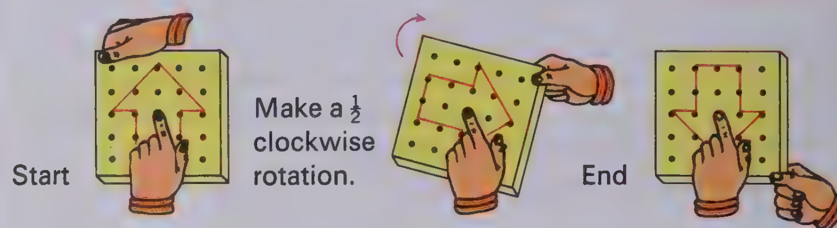
Investigating the Ideas



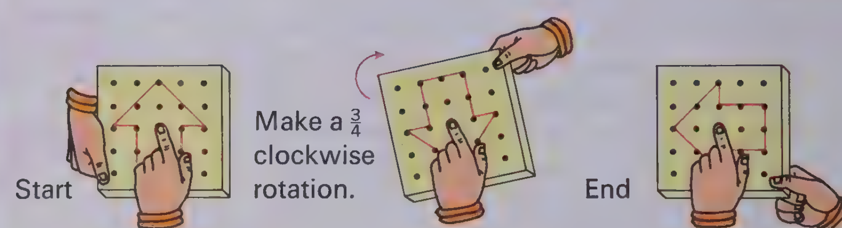
? Can you show on your geoboard and draw on dot paper some figures that will look exactly the same at the end of a $\frac{1}{4}$ rotation as at the beginning? See Investigation.

Discussing the Ideas

1. Here is a $\frac{1}{2}$ rotation. Which of your geoboard figures look the same after a $\frac{1}{2}$ rotation as at the beginning? See Discussion.



2. Here is a $\frac{3}{4}$ rotation. Can you show a geoboard figure that looks the same after a $\frac{3}{4}$ rotation as at the beginning? See Discussion.

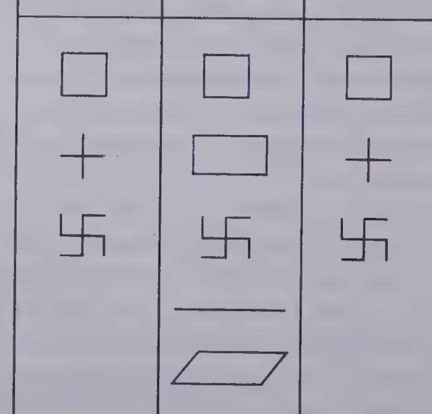


308

Discussion

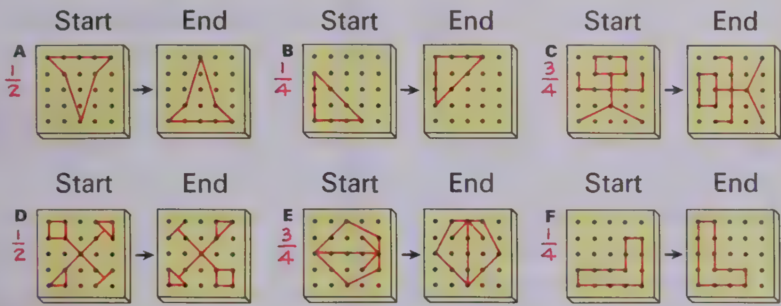
It would be helpful to use a transparent plastic geoboard on the overhead projector so that children could demonstrate the figures they have made. Use their examples and let children judge whether or not the $\frac{1}{4}$ rotation changes the appearance of the figure. Some of these same figures may then be used for discussion of exercises 1 and 2. It would be helpful to make a table to display the results of this discussion. The sample table at the right shows a few possible choices of figures.

$\frac{1}{4}$ rotation	$\frac{1}{2}$ rotation	$\frac{3}{4}$ rotation

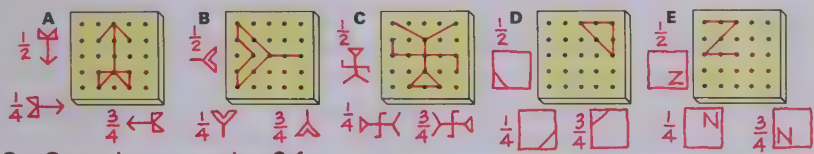


Using the Ideas

1. Each part shows a geoboard at the start and at the end of a rotation. Tell whether it was a $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$ rotation.

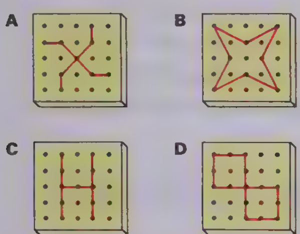


2. Draw a picture on dot or graph paper to show what each geoboard will look like after a $\frac{1}{2}$ rotation.



3. Complete exercise 2 for a $\frac{1}{4}$ rotation and for a $\frac{3}{4}$ rotation. See ex. 2.

4. Which of these will look the same after a $\frac{1}{2}$ rotation? a $\frac{1}{4}$ rotation?



All look same after $\frac{1}{2}$ rotation.
A and B look same after $\frac{1}{4}$ rotation.

think

Suppose you put 2¢ in a bank on February 1, 4¢ on February 2, 8¢ on February 3, and so on, doubling the amount each day. After you made your last deposit on Valentine's Day, would you have enough money to buy a $5 box of candy? How much would you have? Yes ; $327.66

309

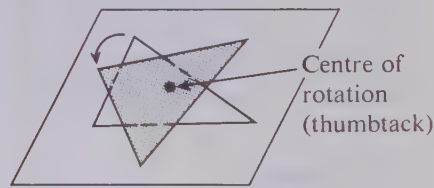
Using the Exercises
Have the children do the exercises on page 309 independently. They may want to check their answers by copying the design of a figure onto their geoboard and trying different rotations. This is particularly recommended for exercises 2, 3, and 4, where children must decide what the geoboard will look like after a particular rotation.

For those who try the *Think* problem, suggest that they guess how much money they would have before they start computing the doubled amounts. They will probably be surprised to find how much money they would have.

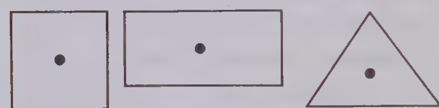
Assignments (page 309)
Minimum: 1–2. Average: 1–3.
Maximum: 1–4.

Mathematics
A physical model of a rotation of a geometric figure may be made as follows:

Cut out a triangular region from tagboard. Use a single thumbtack to pin it to a piece of thick cardboard. Mark around the triangular shape. Now turn the triangle to a new position and mark around it again. Each triangle is a rotational image of the other with respect to the centre of rotation (marked by the thumbtack).



A figure is said to have rotational symmetry with respect to its centre of rotation if it appears not to have been moved by the rotation. In other words, all the points of the figure fall again on the figure. Thus a square has a 90° rotational symmetry and every rectangle has 180° rotational symmetry with respect to their centres. An equilateral



triangle has 120° rotational symmetry when the centre of rotation is the centroid or centre of gravity of the triangle.

Follow-up
Children might enjoy choosing a figure and using various rotations of it to make designs. Distribute stiff paper on which you have duplicated a variety of shapes, such as squares, triangles, and rectangles, each with a dot at its centre. The children should select a figure, cut it out, and pin it or tack it to a piece of paper backed by cardboard. Then they should trace the outline of the figure in its original position and in its position after each of several different rotations. After the resulting designs have been colored, they could serve as an attractive display for the bulletin board or chalkboard tray.

PAGES 310–311

Objective

Given the graph of a figure in the co-ordinate plane, the child will be able to show a translation of it.

Preparation

Materials

graph paper, 1-cm grid (*Duplicator Masters*, page 62)

If appropriate for your class, you might mention the meaning of the word *translations* in reference to changing from one language to another. Since children might be familiar with this meaning, explain that in this lesson the use of the word has a different meaning, a geometric meaning.

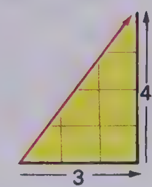
Investigation

Emphasize to the children that they should be very careful in cutting out the "point slider." You might want to guide them in making it, beginning with a 4-by-5 rectangle, drawing a diagonal from corner to corner, and cutting along this line. Then instruct the children to copy the figure *ABCD* made with solid lines onto the paper. They should then be able to use their "right-4, up-5 point slider" to translate the figure to its new position. You can check the children's work by noticing the co-ordinates of the new figure: point *A* moves from (1, 4) to (5, 9); *B* from (3, 4) to (7, 9); *C* from (4, 1) to (8, 6); and *D* from (0, 1) to (4, 6).

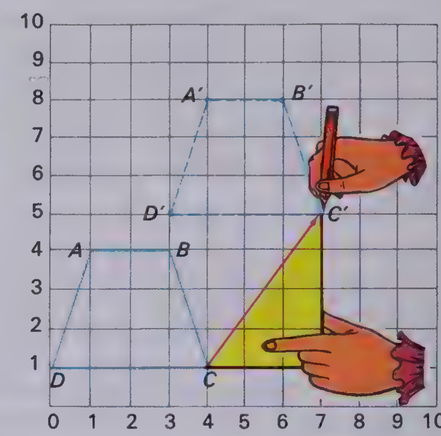
Let's explore translations.

Investigating the Ideas

Anita made a "point slider" like this from graph paper.



She called it a "right-3, up-4 point slider" and used it as shown on the graph to "slide" each point of the blue figure to a new position.



See Investigation.

? Can you make a "right-4, up-5 point slider" and use it to find the new position of the blue figure after sliding each point? Show your results on graph paper.

Discussing the Ideas

1. Explain why Anita called her "point slider" a "right-3, up-4 point slider." Because it moved each point 3 to the right and 4 up.
2. When we "slide" a figure to a new position, we call this motion a **translation** of the figure. The red arrow on the "point slider" shows the direction and distance of the translation. In the picture above, how far was each point of the blue figure moved to obtain the new position? (Mark off the scale units along the edge of a piece of paper to make a ruler.)
3. Explain how you could use a "right-4, up-5 point slider" as a "left-4, down-5 point slider." Rotate slider 180° (a "½ rotation"). See Discussion.

310

Discussion

Exercise 1 refers children to a study of the illustration in the text. As you discuss this and exercise 2, stress that the translation changes the co-ordinates of the figure according to a particular pattern. For example, list the co-ordinates of the figure *ABCD* beside those of the new figure *A′B′C′D′*.

A: (1,4) → A': (4,8)
B: (3,4) → B': (6,8)
C: (4,1) → C': (7,5)
D: (0,1) → D': (3,5)

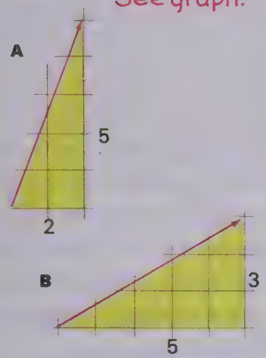
Relate the "right-3, up-4 point slider" to the pattern of adding 3 to the first number of each original co-ordinate pair and 4 to the second. You might also use the translation children showed in the investigation to point out the pattern of adding 4 to the first number and 5 to the second when the "right-4, up-5 point slider" is used.

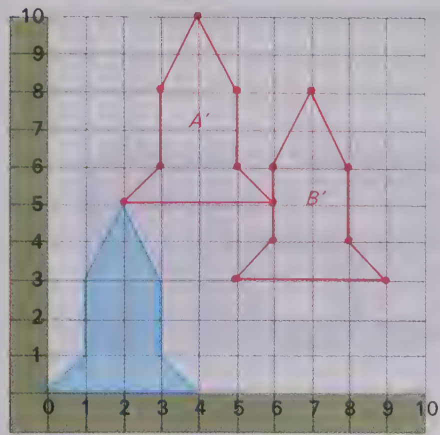
Exercise 3 simply emphasizes the idea that the translation "left 4, down 5" is the opposite, or the inverse, of "right 4, up 5."

Throughout your discussion, stress that the shape of the figure does not change, only its position.

Using the Ideas

1. Copy this figure on your graph paper. Then draw on your graph paper the final positicn of the figure after using the translation indicated. See graph.





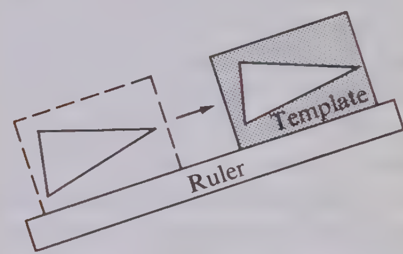
2. The beginning and final positions of the figures are given. Which "point sliders" were used? A "Right-6, up-4" B "Right-3, up-2"

★ 3. Can you draw a picture on your graph paper and show its new position after each of these translations? Drawings will vary.
A left 3, down 2 B right 3, down 4 C left 4, up 5

311

Mathematics
Suppose a figure is given in a plane. Now imagine that all points of the figure are moved in *the same direction and the same distance*. Then we say that the figures in its final position is a translation of the original figure.

A physical model of a translation may be shown as follows. Cut out a triangular shaped region from a rectangular piece of cardboard to form a template for a triangle. Place an edge of the template along a ruler (see below) and draw a triangle using the template. Slide the template along the ruler any chosen distance.



Now mark another triangle using the template. The second triangle is a translation of the first one.

Observe that when a figure is given in the co-ordinate plane, we may easily perform translations by "moving" every point "over and up" by the same amount. This has the effect of sliding every point in the same direction and the same distance, and is the basis for the feasibility of the "point slider" in the lesson.

Using the Exercises
Make sure children have sufficient graph paper to do the exercises on page 311. Encourage them to work independently. However, before they begin, help them to understand the first exercise. It is not necessary that children make and use the point sliders for each translation. Most should be able to interpret the over-and-up action indicated by each illustrated point slider and either count or add to find the points for the new position of the figure. Note that children will need to use separate sheets of graph paper for these translations in order to avoid overlapping of the resulting figures. It would also be helpful to explain that the answers for exercise 2 should be given with the over-and-up terminology as used in exercise 3.

Assignments (page 311)
Minimum: 1–2. Average: 1–2.
Maximum: 1–3.

Resources for Active Learning
Developmental Math Cards, "Rotations, Reflections, Translations," J#16, Addison-Wesley. [A game]

PAGES 312–313

Objective

Given a figure in the co-ordinate plane, the child will be able to construct a similar figure in the co-ordinate plane by graphing a multiple of the co-ordinates of the given figure.

Preparation

Materials

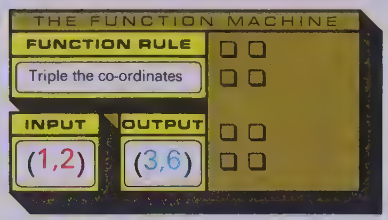
graph paper, 1-cm grid (*Duplicator Masters*, page 62); rulers

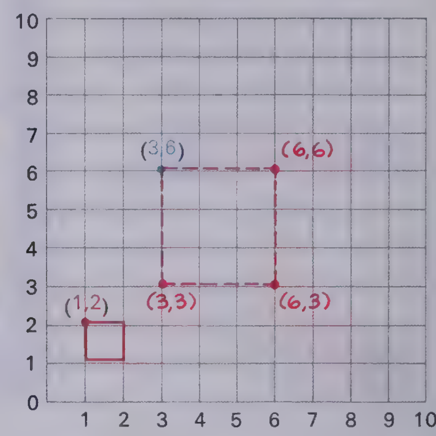
Remind the children that in the rotation and translation of figures, neither the size nor the shape of the figure changes. You might, for example, show them a small square in the centre of the geoboard, and rotate it to show that its shape, size, and position are unchanged. Then to show a translation move the square to another area of the geoboard but keep its original size. Explain that in this lesson they will study still another kind of motion that may be shown for a graphed figure.

Investigation

Before children begin to work on this investigation, make sure they relate the function machine and its rule to the graph. For some children, you might want to restate the function rule as, "Multiply each co-ordinate by three." Note that, in order to complete this investigation successfully, the child must correctly identify and copy the co-ordinates of the original small square.

Investigating the Ideas





Can you copy the red square on your paper and show the figure that would result if **all** the corner co-ordinates are tripled, graphed, and connected? See above.

Discussing the Ideas

1. How would you describe the new figure formed by tripling the co-ordinates? Sample answer: It is the same shape but is larger than original figure.
2. A How many times as long is the side of the new figure as the side of the red square? 3

   B How do the areas of the two figures compare? New figure has 9 times the area of the original figure.
3. What figure do you think would result if the co-ordinates were doubled? quadrupled? Both will be same shape as original. Doubled co-ordinates will give 2 x original length of sides and 4 x original area. Quadrupled co-ordinates will give 4 x original length of sides and 16 x original area.
4. How does the distance from (0,0) to (3,6) compare with the distance from (0,0) to (1,2)? It's 3 times as far.

312

Discussion

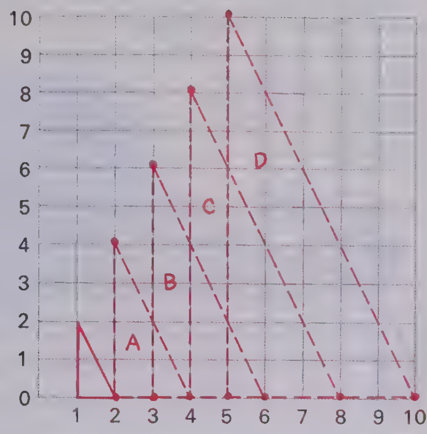
One of the main points of this lesson is for children to realize that similar figures differ in size but not in shape. It should be obvious to children that the shape of the square has been maintained and that the new figure is larger. A study of the unit square of the grid should help children realize that the length of each side of the new larger square is three times that of each side of the original square. However, the area of the new square is three squared, or 9 times that of the original. You might ask children whether they think this would be true of any original shape and a similar figure formed by tripling its co-ordinates. Suggest that they try this with squares of other sizes or with other figures, such as rectangles or parallelograms.

In discussing exercise 3, observe that, no matter what multiple is used, the resulting figure will be similar to the original (in this case, a square). As you discuss exercise 4, instead of using rulers, you might have the child mark the shorter distance, from (0,0) to (1,2), on an edge of paper and see how many times he can "lay it off" on the longer distance.

Using the Ideas

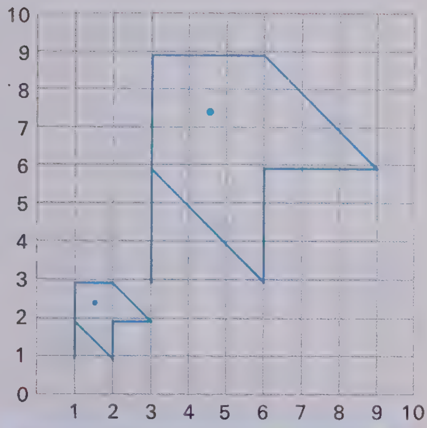
1. On your graph paper show the figure that would result if the co-ordinates of the triangle were
   - A doubled.
   - B tripled.
   - C quadrupled.
   - D multiplied by 5.

   See graph.



2. When the co-ordinates of one figure are multiplied to arrive at the co-ordinates of a second figure, the figures are **similar** to each other (the same shape). Here are two similar figures.
   - A The co-ordinates of the large figure are what multiple of the co-ordinates of the small figure? 3
   - B Show on your graph another figure similar to these figures.

     Figure sizes may vary.

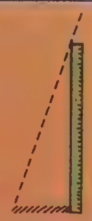


3. On your graph paper make
   - A two rectangles that are similar.
   - B two right triangles that are similar.
   - C two other interesting figures that are similar.

   Figures may vary.

think

A metre stick makes a shadow 20 cm long. How tall is a flagpole that makes a 6-metre shadow?



313

Mathematics
Reflections, translations, and rotations are kinds of motions in geometry which are called *rigid motions* because the figures resulting from any of these motions have exactly the same size and shape as the original figure.

In this lesson a new motion, which could be called an *enlargement*, is introduced. This special motion, achieved by multiplying the co-ordinates of each point of a figure by a fixed number, produces a similar figure which is larger than the original figure, provided the multiplying factor is greater than 1. If the factor is between 0 and 1 a similar but smaller figure would result. Thus an enlargement is not, in general, a rigid motion, except in the case in which the multiplying factor is 1.

Resources for Active Learning
Measure and Find Out, Book 1, Activity 19, Scott Foresman. [Another way to enlarge figures] (Available from Gage Educational Publishing)

Duplicator Masters, page 59
Workbook, page 106

Using the Exercises
Have the children do the exercises on page 313 independently. Besides graphing the figures, they might be asked to record the co-ordinates of each similar figure in number pairs. For example, when the triangle in exercise 1 is quadrupled, the following list would be appropriate:

(1,2) → (4,8); (2,0) → (8,0); (1,0) → (4,0).

The Think problem is a special challenge. Allow any child who wishes to try it, but do not let them be discouraged if they fail to find the answer. After the children have had some time to consider the problem, discuss it with the class. A large graph would help in the discussion. Suggest the children think of 20 cm or (20, 0) corresponding to 6 metres or 600 cm or (600, 0). Then 1 metre corresponds to 30 times (600 ÷ 20) that, or 30 metres of the pole.

Assignments (page 313)
Minimum: 1–2. Average: 1–2.
Maximum: 1–3.

PAGES 314–315

Objective

Given appropriate geometric shapes, the child will be able to use the shapes to make a tessellation of a region.

Preparation

Materials

construction paper; tracing paper; scissors; graph paper

Since children will need time to trace and cut out the shapes they choose, have them begin the investigation immediately. If pattern blocks are available, their use would save children the task of making their own tiles.

Investigation

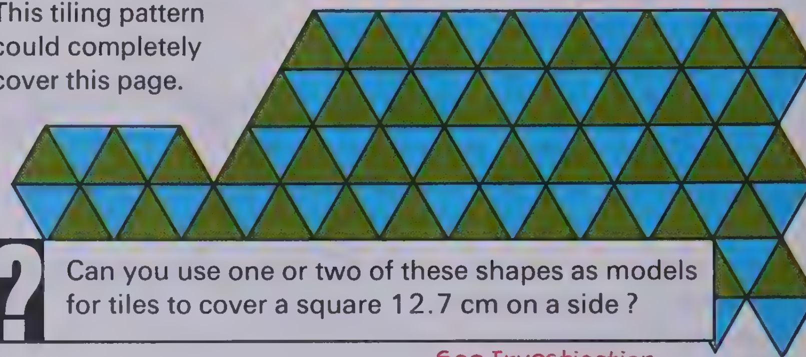
Before children begin working on the investigation, help them to observe how the illustrated triangles fit together.

Children may use any of the given figures. Suggest that they make at least 20 tiles to start with, particularly if they choose one of the smaller shapes. It would be helpful to have them use graph paper for tiles of the T-shaped model. You might guide them in making a template for both the diamond and the hexagon, or even duplicate copies of these shapes. They can then use the template to outline as many shapes as they need.

Distribute 1-cm or 2-cm graph paper to the children and suggest that they outline a 3-by-3-centimetre square on it. Then challenge them to form some tiling patterns with their shapes. They should not try to cover the square region exactly; some overlapping of the border should be expected. The purpose of the boundary is simply to give the children a workable surface to tessellate. Children may use only one of the figures in their tiling pattern, or they may use a combination of two of them. Encourage a variety of patterns, but allow the children to work with the shapes of their choice. It is important that no spaces or open areas are left within the tiling patterns; every part of the square region should be covered with some part of the tiling.

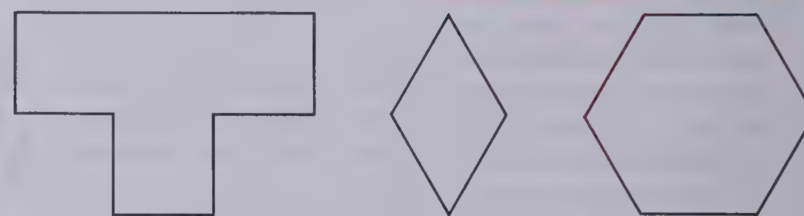
Investigating the Ideas

This tiling pattern could completely cover this page.

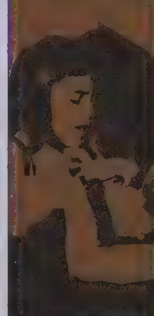


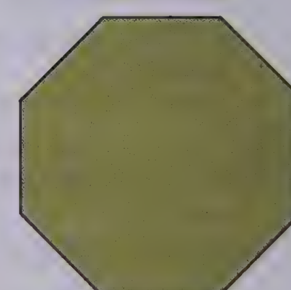
? Can you use one or two of these shapes as models for tiles to cover a square 12.7 cm on a side?

See Investigation.



Discussing the Ideas



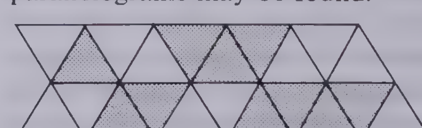


1. A pattern that covers a plane with tiling is called a **tessellation** of the plane.
   - A Can you tessellate a plane with a regular octagon? No
     Explain your answer. See Discussion.
   - B Can you use another shape with the octagon to tessellate a plane? Yes; see Discussion.
2. Which regular polygons could be used to tessellate a plane? Equilateral triangle, square, hexagon

Discussion

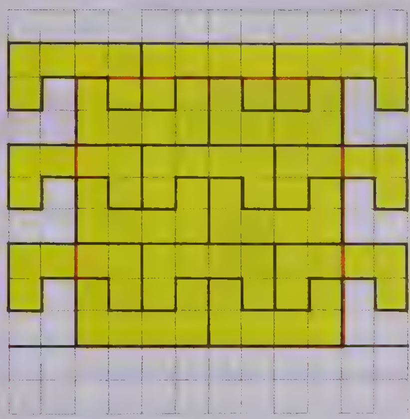
The main point of this lesson is for children to realize that a tessellation covers a plane, leaving no holes or open spaces. If you have a small regular octagon, you might use it on the overhead projector. (A *regular* octagon has 8 sides of *equal* length.) The regular octagon will not tessellate the plane. Let children discover by experimentation that a square with sides of length equal to the sides of the octagon is needed to fill in the "holes."

If time permits, you might have children study the illustration at the top of the page and look for geometric shapes within the design. For example, in this design triangles, rhombus, trapezoids, and parallelograms may be found.



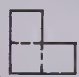
Using the Ideas

1. This picture shows a covering of the red square with yellow pieces made up of 6 small squares. Can you find some different 6-square pieces that will completely cover the square? Show each tessellation.
See Answers, T.E. page 315.

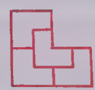


2. Can you invent a figure of your own that could be used to tessellate a plane? Figures will vary.
Show the tessellation on graph paper and color it so that an interesting pattern stands out.

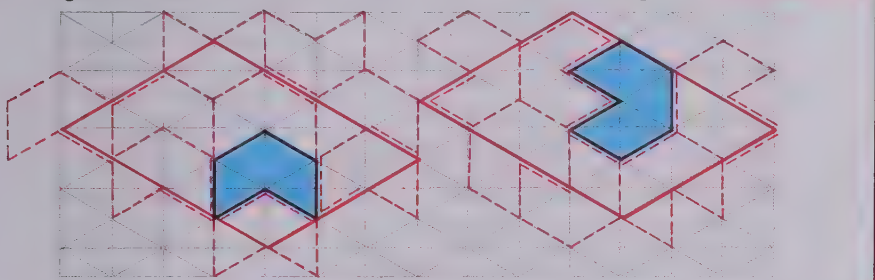
3. A Can this shape → be used to make a tessellation? Yes



B Use four of the shapes to make a figure similar in shape but larger.



4. Can you trace this special graph paper grid and use the blue figure shown to draw a tessellation of each red figure? Dotted lines show sample solutions.



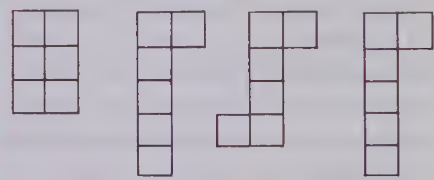
Using the Exercises

The exercises on page 315 are intended to be worked independently. Children will need graph paper for exercises 1 and 2 and tracing paper for exercise 4. You might have children select one of these exercises if there is not sufficient time to do them all. Isometric graph paper, if available, would be interesting for the children to use. You might also use these tessellation activities for an art or free period activity. However, your choice of how to use them should depend not only on the amount of time available but also on the enthusiasm exhibited by the children.

Assignments (page 315)
Minimum: 1–2. Average: 1–3.
Maximum: 1–4.

Follow-up
The study of tessellations should stimulate ideas for many designs and tiling patterns. Refer to the resource materials listed below for several ideas. As always, remember how much children enjoy seeing their work of this kind displayed around the classroom.

Answers, exercise 1, page 315
Possible 6-square pieces which will tessellate the square:



Resources for Active Learning

Experiments in Mathematics, Stage 1, pp. 14–15; Stage 2, "Reptiles," pp. 8-9, Houghton Mifflin. (Available from Thomas Nelson & Sons)

Geometry in the Classroom, New Concepts and Methods, pp. 46-49, 75, Holt, Rinehart and Winston of Canada, Ltd.

Mathex: Geometry No. 9, "Tessellations," pp. 53-56, Encyclopaedia Britannica Publications Ltd.

Notes on Mathematics in Primary Schools, "Tessellations," p. 131-142, Cambridge University Press. (Available from Macmillan of Canada)

Objectives

Given positive and negative numbers as co-ordinates, the child will be able to graph the corresponding points on a co-ordinate plane.

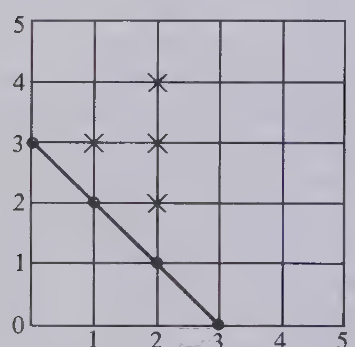
Given points on a co-ordinate plane, the child will be able to give the co-ordinates of each point.

Preparation

Materials

graph paper or 4-quadrant co-ordinate grid (*Duplicator Masters*, page 66)

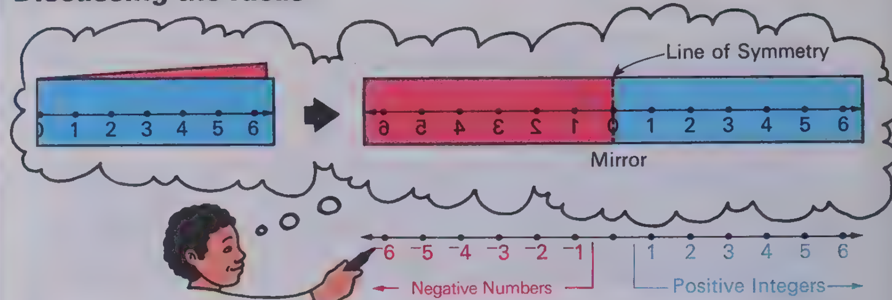
To stimulate children for further work with co-ordinates and graphing, you might play a quick game of co-ordinate Tic-Tac-Toe. Split the class into two teams, X and O. Draw a grid on the chalkboard, clearly labelling both axes and drawing in the vertical and horizontal lines of the grid.



A member of each team gives the position of his team's mark (X or O) by naming in proper order the co-ordinates of the location he chooses. The first team to get four X's or four O's in a vertical, horizontal, or diagonal row wins.

Let's explore whole numbers and graphing

Discussing the Ideas



This whole number line shows numbers for the points **opposite** the **positive whole numbers.** Numerals for these numbers are in red. These "new" numbers we call **negative whole numbers.**

You can think of whole numbers in two ways. They tell you "how many." But they also tell you "what direction" (above or below zero). Positive and negative numbers are useful in describing many things in the world around you. They are also useful in graphing.

Positive numbers indicate temperatures above 0°, negative numbers indicate those below 0°.

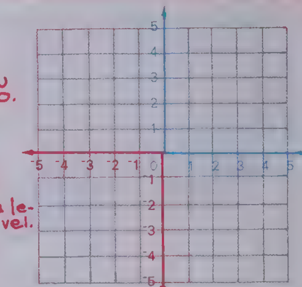
1. Explain how positive and negative numbers are used on a thermometer.

2. Explain how positive and negative numbers can be used for telling the amount of time before and after blast-off.

In countdowns negative numbers are used to tell the amount of time before blast-off, positive to tell the amount of time elapsed after blast-off.

3. How can you use positive and negative numbers in a game when your score goes below zero?

Use negative numbers to record scores below zero.



4. How can you use positive and negative numbers to talk about places above and below sea level? Use positive numbers to indicate altitudes above sea level, negative to indicate depths below sea level.

5. How can you use two crossed number lines to give co-ordinates for any point in a plane? See Discussion.

316

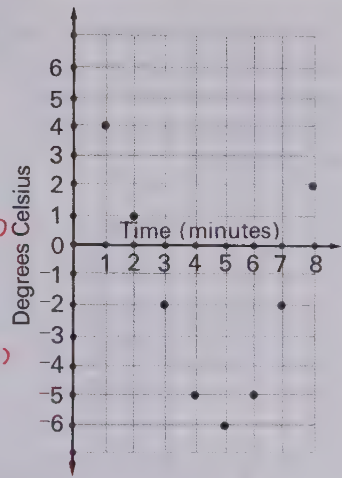
Discussion

Negative numbers are introduced by illustrating them on the number line as reflections of positive whole numbers. By means of this introduction, each positive number can be thought of as having an opposite. We read the symbol $^{-}1$ as "negative one." Negative 1 is the opposite of positive 1, $^{-}2$ is the opposite of 2, $^{-}10$ is the opposite of 10, and so on. Point out that any whole number and its opposite are equidistant from zero on the number line, but they lie on opposite sides of zero.

Use the text material to introduce the set of the positive numbers, zero, and the negative numbers. Exercises 1 through 4 provide situations to be discussed as examples of how positive and negative numbers might be used. You might develop the ideas in exercise 5 by using the two intersecting number lines for some games of co-ordinate Tic-Tac-Toe or Battleship (see the follow-up section).

Using the Ideas

1. A scientist was conducting an experiment with a certain type of motor oil. He was testing the oil at very low temperatures. The graph shows the temperature at various times (in minutes).



   A What was the temperature of the oil at 1 min? What are the co-ordinates of that point? 4°C (1,4)

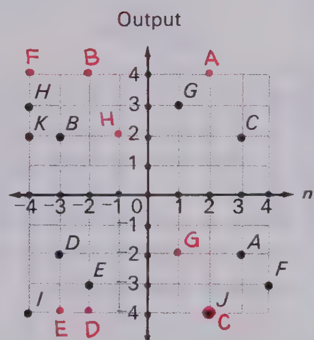
   B What was the temperature of the oil at 3 min? What are the co-ordinates of that point? $^-2$°C (3,$^-2$)

   C What are the co-ordinates for the lowest recorded temperature? (5,$^-6$)

   D At what two periods of time was the temperature 5 degrees below zero? At 4 min and 6 min

   E What would you guess about the temperature at $7\frac{1}{2}$ minutes? It's about 0°C.

2. In the graph on the right, the co-ordinates of point *A* are (3, $^-2$) and the co-ordinates of *B* are ($^-3$, 2). Give the co-ordinates of points *C* through *K*.
See below.



3. Draw a pair of axes like those for exercise 2. Graph the following co-ordinates. Label them *A*, *B*, *C*, and so on. (Answers indicated in red on graph above)

A (2, 4)	C (2, $^-4$)	E ($^-3$, $^-4$)	G (1, $^-2$)
B ($^-2$, 4)	D ($^-2$, $^-4$)	F ($^-4$, 4)	H ($^-1$, 2)

Ex. 2 C (3,2); D($^-3$,$^-2$); E($^-2$,$^-3$); F(4,$^-3$); G(1,3); H($^-4$,3); I($^-4$,$^-4$); J(2,$^-4$); K($^-4$, 2)

317

Using the Exercises

Have the children do the exercises on page 317 independently. You might reemphasize that the co-ordinate which shows the horizontal (left or right) direction is always given first in a pair. The children need draw only one pair of axes for exercise 3. However, if you wish to provide them with further practice, you might have them graph other sets of co-ordinates. Capable children might do some research to find appropriate data for graphing. They might list some locations and graph their heights above and below sea level.

Assignments (page 317)

Minimum: 1–2. Average: 1–2. Maximum: 1–3.

Mathematics

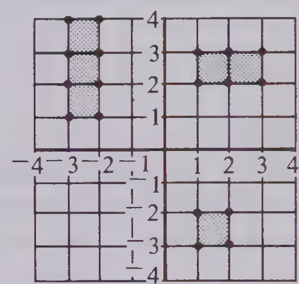
The two important mathematical concepts presented in this lesson are the negative numbers and the use of these numbers to extend graphing to the entire co-ordinate plane.

Follow-up

Children may enjoy the game of Treasure Hunt (an adaptation of Battleship). Each player or team, makes two co-ordinate grids. On one, he marks the locations of his treasures; on the other, he records the points he used to find his opponent's treasure. Treasures of various sizes may be used; for example:

Chest	2-by-4 array of dots
Pot of gold	2-by-3 array of dots
A diamond	2-by-2 array of dots

Each player takes turns calling out the co-ordinates of a point where he thinks one of his opponents' treasures may be located.



Each player must inform the other when he has struck one of his treasures. For example, player A may have "hidden" his diamond at (1,$^-2$), (1,$^-3$), (2,$^-2$) and (2,$^-3$). Even though player B may find the point (1,$^-2$), he may need several more turns before he can locate the other three points of the treasure. The player who finds the most treasures is the winner.

Resources for Active Learning

Math Activity Cards, "Addresses," C1, Macmillan.

Teaching Aids for Elementary Mathematics, "Graphing Pictures," pp. 132–137, Holt, Rinehart and Winston.

PAGES 318–319

Objective

Given a simple function rule involving positive and negative numbers, the child will be able to find and graph the input-output numbers of the function.

Preparation

Materials

graph paper or 4-quadrant co-ordinate grid (*Duplicator Masters*, page 66) 7-by-12-cm cards (optional)

To prepare for this lesson, you might quickly review the use of function rules with whole numbers. For example, tell the children that, when you think 7, you will write 12; when you think 9, you will write 14; and so on. Continue to give input numbers until the children are able to name the number you will write. Finally, ask them to explain your rule. A brief oral activity of this kind should simply remind children of various function rules they have previously studied. You might also want to review the graphing studied in the previous lesson.

Investigation

For children to benefit most from this investigation, have them work in groups of two or three so that they can study and discuss the material with one another. The graph should aid them in deciding the outputs for $^-1$, $^-2$, $^-3$, and $^-4$. Give only that guidance which is absolutely necessary; encourage children to think through the investigation with each other or by themselves.

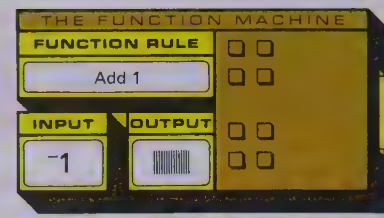
You might challenge those who finish quickly with the following problem:

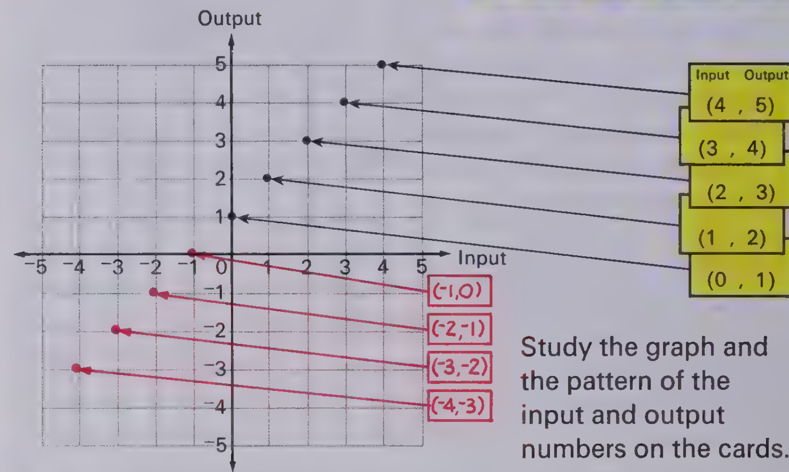
Find co-ordinate pairs for this rule: Add 4. Some sample cards for this rule are shown below. Can you find others?

Input	Output
2	6
0	4
$^-2$	2

Investigating the Ideas

Each time this function machine operates, it produces an input-output card. We can mark a point on the graph for each card.





Study the graph and the pattern of the input and output numbers on the cards.

? Can you draw the input-output cards that the machine would produce for inputs $^-1$, $^-2$, $^-3$, and $^-4$? See above.

Show their points on a graph. See above.

Discussing the Ideas

1. What do you notice about the set of points on the graph? They lie in a straight line.
   Describe other cards that will come from the machine.
   ($^-5$,$^-4$), ($^-6$,$^-5$), (5,6), (6,7), etc.
2. How can the Investigation help you find these sums? See Discussion.
   A $^-1 + 1$ 0 B $^-2 + 1$ $^-1$ C $^-3 + 1$ $^-2$ D $^-4 + 1$ $^-3$

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Discussion

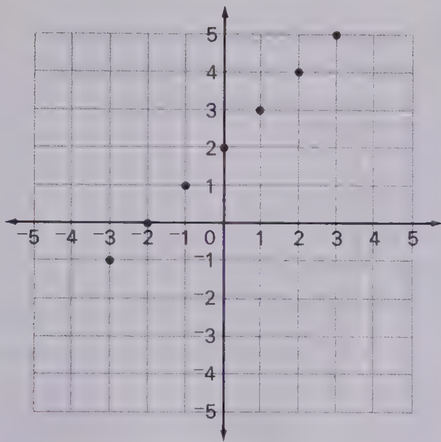
If children have done their graphing carefully, they should notice that, for exercise 1, all of their points are in a straight line. It would be helpful to have them use a ruler and connect the dots with a line. Then you might show them how to locate another co-ordinate pair farther out on the line, such as (6,7) or ($^-5$, $^-4$). Use of a grid transparency with the overhead projector would be helpful in your discussion of exercises 1 and 2. As you discuss exercise 2, point out the relation of each addition phrase to the input-output cards the children made in the investigation and to the points they graphed. It would also be helpful to describe situations in which such phrases might occur. For example, to explain $^-4 + 1 = {}^-3$, present the following situation: "If the thermometer reads $^-4$ degrees and the temperature rises 1 degree, what does the thermometer read?" ($^-3$) Similarly, for $^-3 + 1 = {}^-2$, you might describe this situation: "If you owed each of three people one dollar, and someone gave you one dollar which you immediately gave to one of the persons, how much in debt would you be now?" (two dollars in debt, or $^-2$)

Using the Ideas

1. Complete the function table. The graph may help you.

Function Rule

Add 2	
Input	Output
3	5
2	4
1	3
0	2
⁻1	1
⁻2	0
⁻3	⁻1



Use the table or graph to help you solve these equations.

A $2 + {}^{-}2 = n$ 0 B $2 + {}^{-}1 = n$ 1 C $2 + {}^{-}3 = n$ ⁻1 D $2 + {}^{-}4 = n$ ⁻2

2. Give the missing numbers in the table. Then list and graph the input-output pairs.

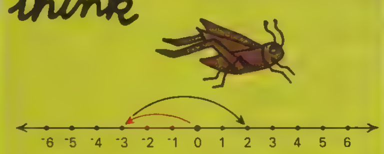
A Function Rule

Add 3	
Input	Output
3	6
2	5
1	4
0	3
⁻1	2
⁻2	1

★B Function Rule

Subtract 1	
Input	Output
4	3
3	2
2	1
1	0
0	⁻1
⁻1	⁻2

think



If a cricket jumps 3 left then 5 right he ends up at

3 left	then	5 right		he ends up at
⁻3	+	5	=	2
⁻4	+	4	=	? 0
⁻2	+	4	=	? 2
⁻6	+	3	=	? ⁻3

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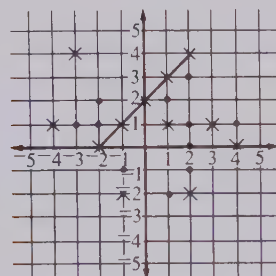
Follow-up

The co-ordinate Tic-Tac-Toe game may be adapted to include negative numbers. Exhibit a co-ordinate system using the numbers −5 through 5 either on the chalkboard, on a large chart that can be covered with acetate, or on a transparency that can be used with an overhead projector. For such a grid children should try to get five ×'s or five O's in a row, column, or diagonal. Strategies to block the other team while continuing to build a string of ×'s or O's will arise as the children play the game. Eventually, they should become skillful enough to make every contest a draw.

You can make the game more difficult by extending the game board to a larger grid and by requiring six ×'s or O's in a row. You might also reverse the rules and say that the first team which has three ×'s or O's in the row loses. In this case, the grid should not extend beyond ⁻4 and 4.

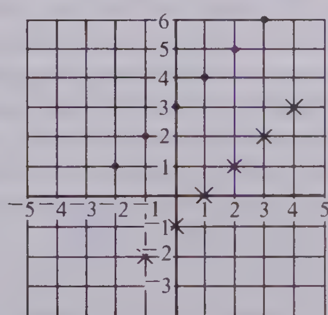


A Sample Winning Game of Tic-Tac-Toe

Using the Exercises

Have the children do the exercises on page 319 independently. They will have to make their own graph for both parts of exercise 2. Regarding the second table, you might remind the children that subtraction "undoes" addition and may be interpreted as a movement to the left on the number line.

Answers, exercise 2, page 319



2A Graphed by •.
B Graphed by ×.

Assignments (page 319)
Minimum: 1. Average: 1–2.
Maximum: 1–2.

Resources for Active Learning

Notes on Mathematics in Primary Schools, "Crossroads," pp. 239–240, Cambridge University Press. [A game] (Available from Macmillan of Canada)

Madison Project: *Discovery in Mathematics:* A Text for Teachers, pp. 71–74, Addison-Wesley.

Duplicator Masters, page 60
Workbook, page 107

PAGES 320–321

Objectives

The child will demonstrate his ability to work with the concepts presented in this chapter.

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

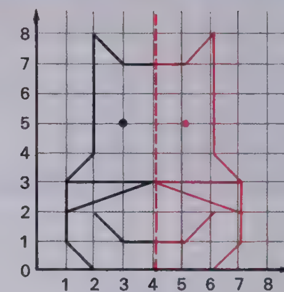
Materials

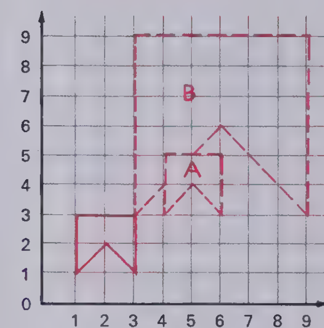
graph paper (*Duplicator Masters,* page 61 or 63)

Depending on the needs of the class, review topics which caused difficulty during the study of this chapter. For example, you might review the meaning of the terms *symmetric figure, rotation,* and *translation.*

Reviewing the Ideas

1. Copy this figure on your graph paper. Then draw the other half to make a symmetric figure.
See graph.





2. Draw the picture that results when you

A move each point of this figure over 3 and up 2. See graph.

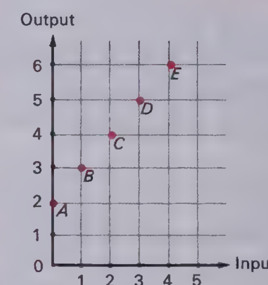
B triple the co-ordinates of the points. of this figure to get a new figure.
See graph.

3. Copy the table on your paper. Complete your table by giving the number pairs for points *C, D,* and *E.* Give a function rule for this set of number pairs.

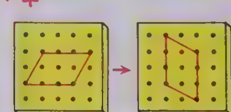
Function Rule

n+2

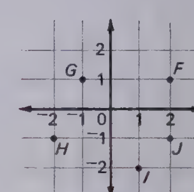
	Input	Output
A	0	2
B	1	3
C	2	4
D	3	5
E	4	6



4. What rotation does the pair of geoboards show?
$\frac{1}{4}$ or $\frac{3}{4}$



5. Give the co-ordinates of points **F** through **J**.
F(2,1) I(1,⁻2)
G(⁻1,1) J(2,⁻1)
H(⁻2,⁻1)



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Discussion

Page 320 may serve as an evaluation instrument, or you might use it as a basis for discussion to review this chapter. In either case, give children ample time to do the required graphing. When they have finished, check their work carefully and allow time for questions.

Keeping in Touch with Addition, Subtraction, Multiplication, Division, Fractional numbers

1. Solve the equations.

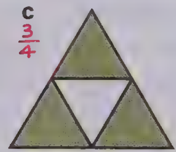
A $n \times 7 = 21$ 3	**I** $7 \times n = 63$ 9	**Q** $(6 \times 4) + 6 = n$ 30
B $5 \times n = 30$ 6	**J** $40 = n \times 8$ 5	**R** $(8 \times 6) + 3 = n$ 51
C $28 = 4 \times n$ 7	**K** $n = 5 \times 9$ 45	**S** $56 = (9 \times 6) + n$ 2
D $n = 5 \times 8$ 40	**L** $6 \times n = 54$ 9	**T** $67 = (9 \times 7) + n$ 4
E $6 \times n = 42$ 7	**M** $56 = n \times 8$ 7	**U** $48 = (n \times 5) + 3$ 9
F $n \times 3 = 24$ 8	**N** $7 \times 9 = n$ 63	**V** $46 = (n \times 7) + 4$ 6
G $n = 9 \times 6$ 54	**O** $72 = 8 \times n$ 9	**W** $81 - (n \times 9) = 0$ 9
H $49 = 7 \times n$ 7	**P** $n = 8 \times 8$ 64	**X** $56 - (n \times 7) = 0$ 8

2. Give the fractional part of the region that is shaded.

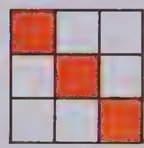
A $\frac{5}{6}$

B $\frac{5}{8}$

C $\frac{3}{4}$



D $\frac{3}{9}$



3. Find the sums.

A $\frac{1}{2} + \frac{3}{4}$ $\frac{5}{4}$ or $1\frac{1}{4}$ **B** $\frac{3}{8} + \frac{1}{4}$ $\frac{5}{8}$ **C** $\frac{2}{3} + \frac{1}{6}$ $\frac{5}{6}$ **D** $\frac{3}{10} + \frac{1}{2}$ $\frac{8}{10}$ or $\frac{4}{5}$

4. Find the differences.

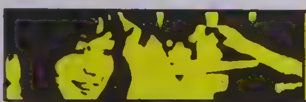
A $\frac{7}{8} - \frac{3}{4}$ $\frac{1}{8}$ **B** $\frac{9}{10} - \frac{2}{5}$ $\frac{5}{10}$ or $\frac{1}{2}$ **C** $\frac{11}{20} - \frac{1}{5}$ $\frac{7}{20}$ **D** $\frac{7}{4} - \frac{3}{2}$ $\frac{1}{4}$

5. Find the sums and differences.

A $\begin{array}{r} 3\frac{1}{8} \\ +5\frac{3}{4} \\ \hline 8\frac{7}{8} \end{array}$ **B** $\begin{array}{r} 8\frac{1}{2} \\ -1\frac{1}{4} \\ \hline 7\frac{1}{4} \end{array}$ **C** $\begin{array}{r} 7\frac{1}{5} \\ +3\frac{9}{10} \\ \hline 11\frac{1}{10} \end{array}$ **D** $\begin{array}{r} 6\frac{1}{4} \\ -3\frac{2}{3} \\ \hline 2\frac{7}{12} \end{array}$

6. Give the products.

A $\frac{1}{2} \times 3$ $\frac{3}{2}$ or $1\frac{1}{2}$ **B** $\frac{1}{3} \times \frac{1}{4}$ $\frac{1}{12}$ **C** $\frac{2}{3} \times \frac{4}{5}$ $\frac{8}{15}$ **D** $\frac{3}{2} \times \frac{2}{3}$ $\frac{6}{6}$ or 1



You are invited to explore

ACTIVITY CARD 16
Page 341

Using the Exercises

Page 321 may be assigned as independent work or you might use it to review basic operations. Again, have children check their work carefully and allow time to correct any incorrect ideas. If children do exercises incorrectly, it is important that they realize why their work is considered incorrect.

Workbook, page 108 (Use with text page 320.)

CHAPTER 16 | A New Mathematical System

Pages 322–332

General Objectives

To create interest by exploring new mathematical systems
To introduce children to a mathematical system having a limited number of elements
To provide additional opportunities for children to work with the basic principles
To introduce exercises to stimulate children's imagination

The new mathematical system introduced in this chapter is clock (modulo 12) arithmetic. Several examples of addition on the clock provide intuitive background for addition and subtraction on a "twelve clock." Then multiplication is presented as repeated addition, and the inverse is developed for use in finding missing factors.

Children are led to discover the operations for a "four clock," and then are introduced to a special clock using a weekday calendar. Pages 328 and 329 review the concepts of the modular system introduced in the chapter.

Finally, the last three pages of the chapter provide a review of the skills and concepts that form the nucleus of Book 5. These pages may be used for either evaluation or review.

Mathematics

Clock arithmetic is also called *remainder,* or *modular, arithmetic.*

Consider the set of whole numbers 0, 1, 2, 3, ... Suppose we imagine dividing each whole number by 12 and consider the remainders for each of these divisions. The set of whole numbers which give remainder 0 when divided by 12 (denoted by R_0) is

$$R_0 = 0, 12, 24, 36, 48, \ldots$$

The set of whole numbers which give remainder 1 when divided by 12 is

$$R_1 = 1, 13, 25, 37, 49, \ldots$$

Since there are exactly 12 different remainders possible when dividing by 12, we get 12 disjoint remainder classes:

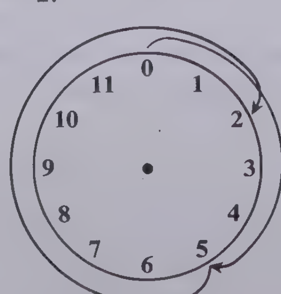
$$R_0, R_1, R_2, R_3, \ldots R_{11}$$

Now, using only the 12 remainders 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, we can define the operation of addition on these numbers as follows.

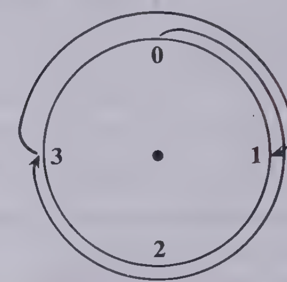
> The sum of any two of the remainders is the remainder when the ordinary sum of the numbers is divided by 12.

Thus, in clock arithmetic, $6 + 8 = 2$ because the remainder when the ordinary sum of 6 and 8 is divided by 12 is 2. It can be shown that when addition is defined in this manner, the system is closed, addition is associative and commutative, and 0 is the identity element. Subtraction can be defined as the inverse of addition. It is also possible to define multiplication and show that it is quite similar to ordinary multiplication. By using divisors other than 12, other clock-arithmetic systems can be constructed.

Remainder arithmetic can be easily demonstrated by means of clock faces. In "twelve-clock" arithmetic, the face is the usual clock face with 12 replaced by 0. If it is 5 o'clock now, 9 hours later it will be 2 o'clock, and we write: $5 + 9 = 2$.



If we use a "four clock" we can reason that if it is 3 o'clock now, then 2 hours from now it will be 1 o'clock, and we write: $3 + 2 = 1$.

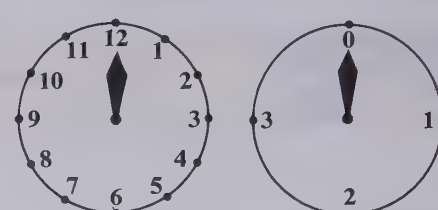


Subtraction can be thought of as hours *before* a given time.

Teaching the Chapter

Materials

Cardboard
Colored chalk
Overhead projector (if available) and transparencies with two clock faces (as shown below)
Paper fasteners
Paper plates
Two large cardboard clocks, with a single hand, labelled as shown



Vocabulary

clock arithmetic
modular system

Large cardboard clocks, made by capable children, will be helpful models for demonstrating the operations in the modular systems. Individual clocks which you or the children can make from paper plates, with cardboard hands fastened on by brass paper fasteners, may create additional interest and will help those who are having trouble understanding the operations in modular systems.

The term *modular arithmetic* need not be used with all the class.

More capable children who wish to pursue the subject in greater depth should be aware that modular arithmetic may also be called *remainder arithmetic* as well as clock arithmetic. A few helpful references are listed in the Books to Explore section in the appendix of the children's text (pages A30–A32).

Lesson Schedule

Though these lessons are designed to take a little more than a week, modify your plans according to the interest of your class and the time left in your school year. You may wish to rearrange the material so that the final tests are given before the modular systems are introduced.

Evaluation of Progress

Since the purpose of these pages is to create interest and stimulate exploration into finite mathematical systems, no formal evaluation is necessary. The chapter review on pages 328 and 329 is included to give you insight into the children's understanding of basic operations and principles for whole numbers and their ability to check these ideas out for application to the clock systems. However, children are not expected to master modular systems simply on the basis of this brief introduction.

The Keeping in Touch lesson on pages 330–332 can be used as a year-end test. The lesson is designed to cover, at least to some extent, all the topics from the book. You may choose to have the children work on this test over two classroom periods, for it requires a considerable amount of work. The problems on page 332 give children an opportunity to review word-problem interpretation skills.

Resources for Active Learning

General Activities

A Cloudburst, Vol. 2, Nos. 5543–5563, Midwest Publications

Nuffield Project: *Shape and Size* 3, "Patterns on Circles," pp. 11–18, Wiley [Clock patterns]

Nuffield Project: *Computation and Structure* 2, "Time," pp. 82–90; *Computation and Structure* 4, "Modular Arithmetic," pp. 13–21; *Problems*—Green Set, No. 15, Wiley

Teaching Aids for Elementary Mathematics, "Clock Arithmetic," pp. 62–63; pp. 66–67, Holt, Rinehart and Winston

Commercial Games

Operational Systems Games—clock arithmetic (McGraw-Hill Ryerson)

TUF (Creative Publications; Cuisenaire Co.; TUF)

PAGES 322–323

Objective

Given simple addition equations in "clock arithmetic," the child will be able to find the answer in "clock arithmetic."

Preparation

Since this investigation introduces an entirely new kind of arithmetic, it would be best to have the children begin immediately to study the investigation. The only previous understanding necessary is the use of the number line. If you think it necessary, you might review briefly the way the number line is used to show addition.

Investigation

The children may need to study the examples in the investigation very carefully before they will be able to write other equations using the number circle. If some children seem to be having difficulty, ask them to show $5+4$ on the number circle. Then ask them to show $5+8$ or $5+9$ on the number circle. Repeat your questioning, first using sums less than 12 and then using a sum greater than twelve. In this manner, the children should begin to realize that they can simply count around the circle and read off the answer from their landing position.

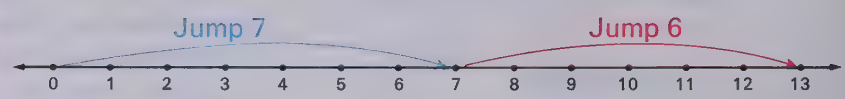
If some children grasp the use of the number circle quickly, you might challenge them to solve a more difficult equation, such as $7+8+9$ or $12+4+11$.

16 A New Mathematical System

Are there other kinds of arithmetic?

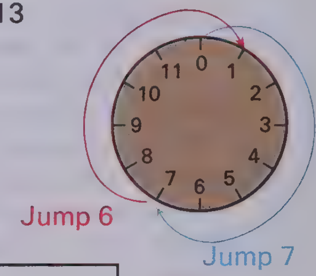
Investigating the Ideas

Here is an "ordinary" number line showing the sum of 7 and 6.



$7 + 6 = 13$

Here is a "number circle" showing the "sum" of 7 and 6.



$7 + 6 = 1$

? Can you use this number circle to write some other strange-looking equations? Equations will vary. See Investigation.

Discussing the Ideas

1. If the 0 is replaced by a 12, arithmetic on the number circle above is called "clock arithmetic." Can you explain why? See Discussion.
2. Explain why $7 + 6 = 1$ in clock arithmetic. See Discussion.
3. Bill wrote "$9 + 6 = 3$" on his paper. Jane said, "You made a mistake in your clock arithmetic. You must have meant to write $9 - 6 = 3$." Was Bill's equation correct in clock arithmetic? See Discussion.
4. Give an equation that is correct in both clock arithmetic and the arithmetic we usually study. Sample answer: $3+4=7$ (Any sum less than or equal to 12.)
5. Explain why $9 + 12 = 9$ in clock arithmetic. See Discussion.

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Discussion

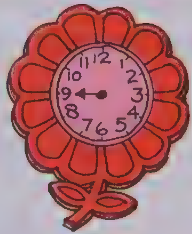
Discussion exercise 1 is intended to help children relate the number circle they used in the investigation to their ordinary use of a clock. As you work through exercises 1 and 2, point out how we think about time in hours. For example, six hours after 7 o'clock is 1 o'clock. You might find it helpful to use a large cardboard model of a clock on which there is one hand that can rotate around the clock. If you use a cardboard hand attached firmly to the face of the clock with a pronged paper-fastener, it should remain stationary at the various positions in which it is set. Have the children use the clock to show several of the equations they wrote in the investigation.

For discussion exercise 3, children should observe that 6 hours after 9 o'clock is 3 o'clock and 6 hours before 9 o'clock is 3 o'clock. You might have the children add and subtract 6 from other numbers to prompt the generalization that in clock arithmetic the same result is obtained by adding six or by subtracting six from a given number.

For exercise 4, the children should discover that, so long as the addends in a given equation in regular arithmetic are clock-arithmetic numbers with a sum less than or

Using the Ideas

Jane does her clock problems by thinking about a clock with only one hand. Bill does his by counting to 12 and then starting over again:



1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 1, 2, 3, . . .

See if you can do these clock problems.

1. Tell what time it will be

A 4 hours after 9 o'clock. 1 o'clock	F 6 hours after 9 o'clock. 3 o'clock
B 2 hours after 11 o'clock. 1 o'clock	G 5 hours after 7 o'clock. 12 o'clock
C 8 hours after 8 o'clock. 4 o'clock	H 7 hours after 5 o'clock. 12 o'clock
D 3 hours after 4 o'clock. 7 o'clock	I 12 hours after 3 o'clock. 3 o'clock
E 9 hours after 6 o'clock. 3 o'clock	J 2 hours after 12 o'clock. 2 o'clock

2. Write clock equations for each part of exercise 1. For example, for exercise 1A the equation is $9 + 4 = 1$.
B 11+2=1; C 8+8=4; D 4+3=7; E 6+9=3; F 9+6=3; G 7+5=12; H 5+7=12; I 3+12=3; J 12+2=2

3. In "regular" arithmetic, the sum of a number and zero is that number. Which clock number acts like zero? 12
Write two addition equations using this clock number.
Sample equations: 8+12=8; 12+4=4

4. Bill gave this problem to his father:

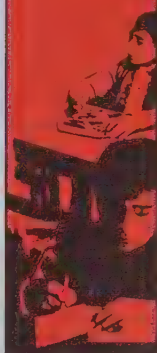
$$\blacksquare + \blacksquare = \blacksquare$$

He said, "You have to put the same clock number in each box." Can you find this clock number? 12

5. Solve these clock equations. Remember that you are working with clock numbers.

A $6 + 7 = v$ 1	F $6 + 10 = a$ 4	K $8 = 6 + c$ 2
B $5 + 7 = s$ 12	G $10 + 6 = f$ 4	L $2 = 9 + q$ 5
C $9 + 9 = t$ 6	H $9 + (4 + 6) = y$ 7	M $8 + 12 = k$ 8
D $6 + 9 = m$ 3	I $(9 + 4) + 6 = n$ 7	N $12 + 12 = g$ 12
E $8 + n = 2$ 6	J $1 = 11 + b$ 2	O $6 + 6 = d$ 12

323



equal to 12, then the clock-arithmetic equation will be the same. For example:

Regular arithmetic: $5 + 6 = 11$
Clock arithmetic: $5 + 6 = 11$

Exercise 5 points out that, in clock arithmetic, adding 12 is the same as adding 0 in regular arithmetic.

Using the Exercises
Have the children do the exercises on page 323 independently or in groups of two or three. When they have finished, check and discuss their solutions. In exercise 2, be sure that the children see the relationship between the equations that are written and the part of exercise 1 to which they apply. Allow the children considerable freedom in discussing the solutions to exercise 5.

Assignments (page 323)
Minimum: 1, oral; 2–3.
Average: 1, oral; 2–4, 5A–E.
Maximum: 1–5.

Follow-up
Suggest to the children that they make an addition table to show sums in clock arithmetic. If they cannot make a large table, encourage them to vary the numbers they put on their table so they will not all be the same.

+	4	5	6	7	8
4	8	9	10	11	12
5	9	10	11	12	1
6	10	11	12	1	2
7	11	12	1	2	3
8	12	1	2	3	4

+	8	9	10	11	12
8	4	5	6	7	8
9	5	6	7	8	9
10	6	7	8	9	10
11	7	8	9	10	11
12	8	9	10	11	12

Resources for Active Learning
Applied Mathematics Cards, "Time . . .," Group 1/15, 16, 18, 19, Schofield and Sims. (Available from Mafex Associates, Willowdale)

Workbook, page 109

PAGES 324–325

Objective

Given subtraction and multiplication problems in clock arithmetic, the child will be able to write and solve an equation for each.

Preparation

To prepare for this lesson, you might show the number-line equation from the previous lesson and use it to remind children of the relation between addition and subtraction. Stress how subtraction "undoes" addition and how on the number line this is shown by moving to the left.

Investigation

Suggest to the children that they draw a clock to use in this investigation. Allow them to work together in trying to figure out what subtraction means in clock arithmetic.

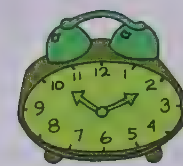
If some children have difficulty in finding the missing number in the table, you might write the facts from the function table in equation form: $2 - ? = 10$; $3 - ? = 11$; $1 - ? = 9$; $8 - ? = 4$; $4 - ? = 12$. You might also point out how subtraction "undoes" addition. If some children still cannot complete the rule (subtract 4), write some of the related addition equations: $10 + 4 = 2$; $11 + 4 = 3$; $9 + 4 = 1$.

The function tables which the children make may be based on either addition or subtraction. Challenge children who complete a function table to make up not just another function table but one related to their first table, to show how addition and subtraction are related.

Can other operations be performed in clock arithmetic?

Investigating the Ideas

Study this clock-number function table and give the missing number.



Function Rule

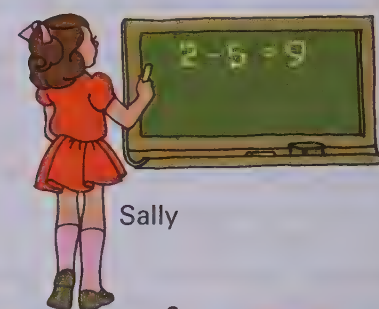
Subtract 4

Input	Output
2	10
3	11
1	9
8	4
4	12

? Make up a function table for clock arithmetic and see if a classmate can guess your rule.

See Investigation.

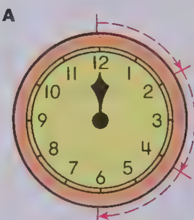
Discussing the Ideas



Sally

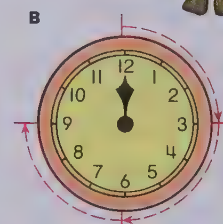
1. Use the clock to show that Sally's equation is correct.
   See Discussion.
2. A multiplication equation is given for picture A. Give multiplication equations for B and C.

A



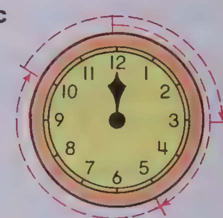
$3 \times 2 = 6$

B



$3 \times 3 = 9$

C



$3 \times 5 = 3$

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Discussion

The children's work in the investigation should provide material for discussion. Have volunteers use the display clock to demonstrate their rule and examples. Help them realize that they can think of subtraction in clock arithmetic in terms of counterclockwise moves around the face of the clock. Stress this point as you discuss exercise 1. It would also be helpful to show the equation $9 + 5 = 2$, and use both equations with the clock to show how subtraction "undoes" addition.

As you discuss multiplication in exercise 2, stress that multiplication may be thought of as repeated addition. With this definition of multiplication, we need special definitions for zero and one factors. We could define zero times a clock number as 12 times the clock number; hence, 12 times any clock number is 12, and one times a clock number is simply that clock number, whatever it is. It would be helpful to give the children multiplication equations to solve by showing the operation on the clock.

Using the Ideas

1. Tell what time it will be (Answers for ex. 2 are in parentheses below.)

   A 5 hours before 2 o'clock. 9 o'clock

   B 6 hours before 2 o'clock. 8 o'clock (2 − 6 = 8)

   C 3 hours before 12 o'clock. 9 o'clock (12 − 3 = 9)

   D 12 hours before 3 o'clock. 3 o'clock (3 − 12 = 3)

   E 7 hours before 4 o'clock. 9 o'clock (4 − 7 = 9)

   F 4 hours before 7 o'clock. (7 − 4 = 3) 3 o'clock

   G 6 hours before 3 o'clock. (3 − 6 = 9) 9 o'clock

   H 3 hours before 6 o'clock. (6 − 3 = 3) 3 o'clock

   I 12 hours before 5 o'clock. (5 − 12 = 5) 5 o'clock

   J 3 hours before 3 o'clock. (3 − 3 = 12) 12 o'clock

2. Write clock equations for each part of exercise 1. (Example: A $2 - 5 = 9$) See exercise 1.

3. Remember that you are working with clock numbers as you solve these clock equations. Check each equation by addition.

A $2 - 6 = s$ 8	E $9 + 9 = b$ 6	I $3 - 12 = c$ 3
B $8 + 5 = t$ 1	F $9 - 9 = y$ 12	J $3 - 5 = k$ 10
C $1 - 5 = m$ 8	G $12 + 12 = n$ 12	K $1 - g = 11$ 2
D $9 + 4 = n$ 1	H $12 - 12 = f$ 12	L $5 - d = 6$ 11

4. Solve the clock equations. Use the clock face or repeated clock-arithmetic addition.

A $4 \times 2 = t$ 8	E $5 \times 12 = m$ 12	I $3 \times 7 = b$ 9
B $2 \times 5 = r$ 10	F $8 \times 12 = a$ 12	J $7 \times 3 = c$ 9
C $6 \times 2 = s$ 12	G $12 \times 2 = f$ 12	K $2 \times 10 = q$ 8
D $4 \times 5 = n$ 8	H $12 \times 4 = y$ 12	L $10 \times 2 = d$ 8

★ 5. Find the missing factors. More than one answer is possible in parts E, F, G, and H.

A $n \times 5 = 10$ 2	D $r \times 5 = 3$ 3	G $f \times 2 = 8$ 4 or 10
B $a \times 7 = 9$ 3	E $b \times 3 = 12$ 4, 8, or 12	H $y \times 4 = 12$ 3, 6, 9, or 12
C $c \times 1 = 12$ 12	F $t \times 6 = 12$ 2, 4, 6, 8, 10, or 12	I $d \times 5 = 15$

325

Using the Exercises
Have the children do the exercises on page 325 independently or with a partner. When they have finished, check their work and discuss some parts of each exercise. As you discuss exercise 3, emphasize the fact that the easiest way to find differences is to think about missing addends. Exercise 5 is starred because nothing has been said so far about missing factors for clock numbers. However, the children should be able to do these by thinking about multiplication.

Assignments (page 325)
Minimum: 1, oral; 2–3.
Average: 1, oral; 2–4.
Maximum: 1–5.

Follow-up
For enrichment, some children may want to make a multiplication table and observe why some parts of exercise 5 (E, F, G, and H) on page 325 have more than one answer and others do not. Remember that, in clock arithmetic, children should consider only the numbers on the clock, that is, 1 to 12.

×	1	2	3	4	5
1	1	2	3	4	5
2	2	4	6	8	10
3	3	6	9	12	3
4	4	8	12	4	8
5	5	10	3	8	1

Workbook, page 110

PAGES 326–327

Objective

Given a "four clock," the child will be able to perform addition, subtraction, and multiplication operations.

Preparation

To prepare for this lesson, you might review clock arithmetic by playing "What's My Rule."

Tell children that you will be using the ordinary "twelve clock" and you will give them the input and the output; they must find the rule. For example, under appropriately labelled columns, write the input and output numbers according to your rule and have children supply the output.

Input	Output
7	3
5	1
2	10
9	5

Note that here you may be thinking either "subtract 4" or "add 8," so both rules are acceptable.

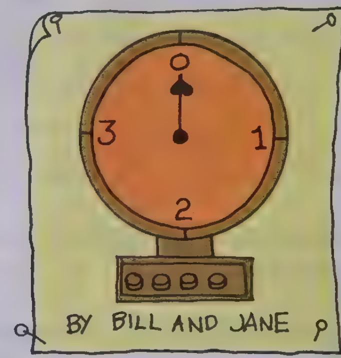
Investigation

In this investigation, children must again study the text material and use it to figure out what equations they should write. Make sure they understand that they are to use only the digits 0, 1, 2, and 3. If a child does not understand what to do, give him sample equations to complete, such as $1 + 2 = ?\,(3)$; $2 + 2 = ?\,(0)$; $3 + 2 = ?\,(1)$. If necessary, relate adding to jumps on the clock, as was done with the "twelve clock" on page 322.

● *Are there other clock arithmetics?*

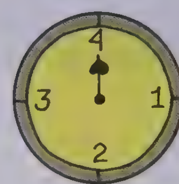
Investigating the Ideas

Bill and Jane made a clock like the one in the picture. Since every answer would be 0, 1, 2, or 3, they decided to use only those numbers in their equations.



? Can you use this clock to write some clock equations? Sample equations: $3+3=2$; $1-2=3$ See Investigation.

Discussing the Ideas



1. Suppose Jane and Bill's clock had a "4" on it. Solve and explain each of these equations. See Discussion.

   A $3 + 4 = n$ 3
   B $2 + 4 = r$ 2
   C $1 - 4 = s$ 1
   D $4 - 4 = a$ 4
   E $2 \times 4 = c$ 4
   F $3 \times 4 = q$ 4

2. Can you explain why Jane and Bill decided to put a "0" in place of a "4" on their clock? See Discussion.

3. Make a special clock of your own and write some equations for your clock. Check your work with a classmate. Answers will vary.

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Discussion

Have volunteers write some of their equations from the investigation for all to see and have other children check to see whether they are correct. Leave these equations on display so that some of them can be compared to the equations in exercise 1. For example, $3 + 4 = n$ and $3 + 0 = n$ have the same sum ($n = 3$); similarly, $1 - 4 = 1$ and $1 - 0 = 1$; $3 \times 4 = 0$ and $3 \times 0 = 0$. Use exercises 1 and 2 to help children realize that just as 12 or 0 is the identity element for addition in "twelve-clock" arithmetic, 0 or 4 is the identity element for addition in "four-clock" arithmetic. They should see that the circular pattern of the numbers is exactly the same as it was when they used the regular clock, except that there are only four numbers.

Encourage children to use different clock numbers. This exercise might be extended as suggested in the follow-up section.

Using the Ideas

Remember, this clock has only four numbers: 0, 1, 2, and 3.

1. Tell what time it will be (Answers for ex. 2 are in parentheses below.)
   - A 2 hours after 0 o'clock. 2 o'clock (0+2=2)
   - B 3 hours after 0 o'clock. 3 o'clock (0+3=3)
   - C 1 hour after 2 o'clock. 3 o'clock (2+1=3)
   - D 1 hour after 3 o'clock. 0 o'clock (3+1=0)
   - E 2 hours after 2 o'clock. 0 o'clock (2+2=0)
   - F 2 hours after 3 o'clock. 1 o'clock (3+2=1)
   - G 3 hours after 2 o'clock. 1 o'clock (2+3=1)
   - H 3 hours after 3 o'clock. 2 o'clock (3+3=2)
   - I 2 hours after 1 o'clock. 3 o'clock (1+2=3)
   - J 3 hours after 1 o'clock. 0 o'clock (1+3=0)
2. Write clock equations for each part of exercise 1.
   See exercise 1.
3. Tell what time it will be (Answers for ex. 4 are in parentheses below.)
   - A 2 hours before 0 o'clock. 2 o'clock (0−2=2)
   - B 1 hour before 0 o'clock. 3 o'clock (0−1=3)
   - C 2 hours before 3 o'clock. 1 o'clock (3−2=1)
   - D 2 hours before 2 o'clock. 0 o'clock (2−2=0)
   - E 1 hour before 3 o'clock. 2 o'clock (3−1=2)
   - F 2 hours before 1 o'clock. 3 o'clock (1−2=3)
   - G 3 hours before 1 o'clock. 2 o'clock (1−3=2)
   - H 3 hours before 2 o'clock. 3 o'clock (2−3=3)
4. Write clock equations for each part of exercise 3.
   See exercises 3.
5. Solve these clock equations.

A $2+1=c$ 3	D $3+2=m$ 1	G $1-3=n$ 2	J $1+n=0$ 3
B $2+2=v$ 0	E $3\times 3=f$ 1	H $3+1=t$ 0	K $3+e=0$ 1
C $0-2=d$ 2	F $2-3=s$ 3	I $0-1=b$ 3	L $2\times 2=w$ 0

★6. Copy and complete the multiplication and addition tables.

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

×	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

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Follow-up
Suggest to the children that they make both an addition and a multiplication table for the clock they choose. These might be made large enough to display on the bulletin board or elsewhere in the room.

Workbook, page 111

Using the Exercises
Assign the exercises on page 327 for children to do independently or with a partner. Remind them that they are to think of a clock with only four numbers. When they have finished, allow time for checking papers and discussion. Although exercise 6 is starred, you may find that it would make an intriguing class activity. However, the faster children should be given a chance to do this exercise on their own prior to any class discussion. Exhibit the two tables on the chalkboard, and have various children come to the front of the room and fill in the entries.

Assignments (page 327)
Minimum: 1, oral; 2–4.
Average: 1, oral; 2–5.
Maximum: 1–6.

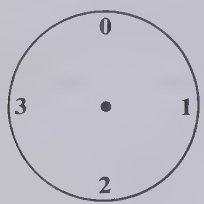
PAGES 328–329

Objective

The child will demonstrate his ability to work with the concepts presented in this chapter.

Preparation

To prepare for this lesson, you might review clock arithmetic by drawing a "four clock" on the chalkboard. Ask the children to think of an equation and then show it on the clock.



Reviewing the Ideas

1. Copy and complete each addition table for "twelve-clock" numbers.

A

+	6	8
6	12	2
8	2	4

B

+	7	9
5	12	2
10	5	7

2. Solve these equations for "twelve-clock" numbers.

A $7 + 6 = a$ 1   C $1 + 12 = c$ 1   E $8 + 6 = e$ 2

B $9 + 9 = b$ 6   D $10 + 4 = d$ 2   F $12 + 11 = f$ 11

3. Find the differences of these "twelve-clock" numbers.

A $8 - 7 = a$ 1   C $8 - 9 = c$ 11   E $8 - 11 = r$ 9

B $8 - 8 = b$ 12   D $8 - 10 = d$ 10   F $8 - 12 = f$ 8

4. Find the products of these "twelve-clock" numbers.

A $4 \times 4 = a$ 4   C $5 \times 5 = c$ 1   E $3 \times 4 = e$ 12

B $2 \times 8 = b$ 4   D $11 \times 2 = d$ 10   F $6 \times 3 = f$ 6

5. Find two different "twelve-clock" numbers for n in this equation.

$n \times 6 = 12$ 2, 4, 6, 8, 10, or 12

6. Find the sums and products of these "four-clock" numbers.

A $3 + 2 = m$ 1

B $2 + 3 = n$ 1

C $2 \times 1 = r$ 2

D $2 \times 2 = s$ 0

E $2 + 2 = t$ 0

F $1 + 3 = u$ 0

G $2 \times 3 = a$ 2

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

"Four-clock" numbers

328

Discussion

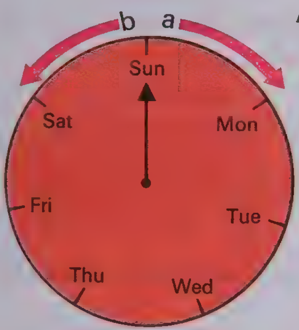
Page 328 reviews the basic ideas covered in this chapter, while page 329 presents an extension of these ideas. Most children would benefit from working through the addition tables in exercise 1 together. The multiplication equations in exercise 4 should also be worked together with most children. You might even develop a multiplication table for "twelve-clock" numbers.

The remaining exercises might be assigned as independent work. However, discuss any which the children find troublesome.

The special clock in exercise 7 is based on the same concepts as the number clocks in the previous lessons. Guide the children to decide possible meanings of the "a" and "b" directions. For example, "$\underrightarrow{\text{3a}}$" could mean *3 days after*, and "$\underrightarrow{\text{5b}}$" could mean *5 days before*. Note that there are two rules possible in part C: $\underrightarrow{\text{2a}}$ and $\underrightarrow{\text{5b}}$. Also, note that the arrow in symbols such as $\underrightarrow{\text{3a}}$ and $\underrightarrow{\text{5b}}$ does not indicate a specific direction; it is the letter "a" or "b" which indicates the direction to be clockwise or counterclockwise.

The *Think* problem should be extremely challenging, even for the very able children. Expect those who solve it to say that the rule is

7. Study this special "clock" and the function tables. Try to find how the function rules work. Then give the missing symbols.



A Function Rule $\xrightarrow{3a}$

Input	Output
Mon	Thu
Wed	Sat
Sun	Wed
Fri	Mon
Sat	Tue

B Function Rule $\xrightarrow{5b}$

Input	Output
Mon	Wed
Wed	Fri
Sun	Tue
Fri	Sun
Sat	Mon

C Function Rule $\xrightarrow{2a}$ or $\xrightarrow{5b}$

Input	Output
Sun	Tue
Fri	Sun
Wed	Fri
Sat	Mon
Thu	Sat

8. Study the example. Then give the missing day.

Example: Mon $\xrightarrow{3a}$ Thu

A Thu $\xrightarrow{2a}$ Sat **B** Fri $\xrightarrow{2b}$ Wed **C** Sat $\xrightarrow{5a}$ Thu

think

Here is a very special function table. The rule is not easy to guess but your work in clock arithmetic may help. Copy the table and give the missing rule and numbers.
See Discussion.

Function Rule

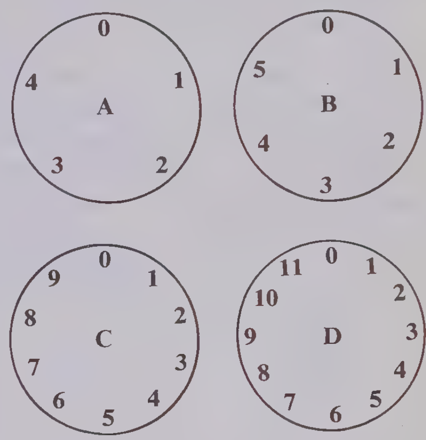
n	Output
7	1
2	2
5	2
6	0
1	1
0	0
9	0
14	2

"The remainder when you divide by 3" or "Change the number to a 'three-clock' number." Some bright youngsters may wish to check out the operations for this system, too.

Follow-up

If the children previously made and displayed the clocks using different numbers, suggest that they make puzzle questions to match them. For example, they might use statements similar to the following to match one of the clocks illustrated below.

On which clock does $6 - 2 = 4$?
On which clock does $1 + 4 = 0$?
On which clock does $9 + 3 = 2$?



Workbook, page 112

PAGES 330–332

Objective

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

Review with the children any topics from the text with which they have had difficulty. Since this is the last cumulative review lesson in the text, you will want to give special attention to the preparation period.

Keeping in Touch with

Computing | Functions
Measurement | Principles
Geometry | Fractional numbers
Place value | Number theory

1. Give the correct sign (<, >, =).

A $375 \;\square\; 300+60+5$ >

B $4793 \;\square\; 4000+800+90+3$ <

C $762 \;\square\; (7\times100)+(6\times10)+5$ <

D $9685 \;\square\; (97\times100)+(8\times10)+5$ <

2. Solve the equations.

A $8\times0=n$ 0

B $54\times1=a$ 54

C $96\div1=t$ 96

D $(9\times6)\times5=9\times(n\times5)$ 6

E $57+n=63+57$ 63

F $0\div42=r$ 0

G $8\times9=(n\times9)+(4\times9)$ 4

H $(t\times8)\div8=60$ 60

I $56-(n\times7)=0$ 8

3. Give the number for a.
Then give the number for b.

A $(54+39)+78=a$ 171 $\rightarrow$ $54+(39+78)=b$ 171

B $50\times4\times10=a$ 2000 $\rightarrow$ $50\times40=b$ 2000

C $(3\times30)+(3\times2)=a$ 96 $\rightarrow$ $3\times32=b$ 96

D $24\times9=a$ 216 $\rightarrow$ $b\div24=9$ 216

E $50\times283=a$ 14,150 $\rightarrow$ $49\times283=b-283$ 14,150

4. Give the missing numbers.

Function Rule: $(2\times n)+9$

	n	Output
A	6	$\square$ 21
B	$\square$ 10	29
C	37	$\square$ 83

5. A jug holds 57.75 cubic centimetres. **Estimate** the number of cubic centimetres in 8 jugs. 480 cm³

6. Compute the following.

A $65+37$ = 102

B 96×8 = 768

C $83-48$ = 35

D 47×24 = 1128

E $307-38$ = 269

F 72×72 = 5184

G $875+367$ = 1242

H $4987+9634$ = 14621

I $3004-1865$ = 1139

J 637×456 = 290472

K $5083-2987$ = 2096

L $7\overline{)168}$ 24

M $60\overline{)540}$ 9

N $9\overline{)4653}$ 517

O $61\overline{)3275}$ 53 R42

P $45\overline{)3569}$ 79 R14

330

Discussion

Pages 330–332 may be used as a year-end evaluation or simply as review. If you use these pages as a year-end test, you will want to check the papers before having the exercises discussed in class. Be sure to give the children ample time to do the exercises you assign. If you use these pages as a review of the year's work, you will probably wish to spend considerable time having the various exercises presented on the chalkboard and explained to the class.

7. A What is the greatest common factor of 21 and 28? 7
   B Give in lowest terms: $\frac{21}{28}$ $\frac{3}{4}$
   C Construct a set of 10 equivalent fractions that includes $\frac{21}{28}$ and the lowest-terms fraction for $\frac{21}{28}$. $\left\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}, \frac{18}{24}, \frac{21}{28}, \frac{24}{32}, \frac{27}{36}, \frac{30}{40}\right\}$
   D Draw a number line and locate the point for $\frac{21}{28}$ on the number line. See below.
   E Which fraction in the set of fractions equivalent to $\frac{21}{28}$ would you use if you wanted to add $\frac{21}{28}$ to $\frac{5}{12}$? $\frac{9}{12}$

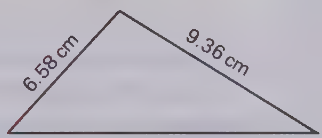
0 1

$\frac{0}{28}$ $\frac{7}{28}$ $\frac{14}{28}$ $\frac{21}{28}$ $\frac{28}{28}$

8. Find the sums and differences.
   A $\frac{3}{8} + \frac{2}{8}$ $\frac{5}{8}$
   B $4\frac{1}{5} + 6\frac{3}{5}$ $10\frac{4}{5}$
   C $\frac{17}{50} - \frac{1}{5}$ $\frac{4}{12}$ or $\frac{1}{3}$
   D $2\frac{6}{10} + 3\frac{1}{5}$ $5\frac{8}{10}$ or $5\frac{4}{5}$
   E $8\frac{1}{2} - 4\frac{2}{3}$ $3\frac{5}{6}$
   F $37\frac{3}{5} + 48\frac{9}{15}$ $86\frac{3}{15}$ or $86\frac{1}{5}$
   G $\frac{3}{10} + \frac{41}{100}$ $\frac{17}{21}$
   H $\frac{2}{3} + \frac{3}{4}$ $\frac{17}{12}$ or $1\frac{5}{12}$
   I $5\frac{3}{5} + 6\frac{1}{4}$ $11\frac{13}{24}$
   J $8\frac{3}{5} - 6\frac{1}{10}$ $2\frac{8}{12}$ or $2\frac{2}{3}$

9. Find the sums and differences.
   A $0.8 + 0.4$ 1.2
   B $0.28 + 0.56$ 0.84
   C $5.46 + 2.97$ 8.43
   D $8.43 - 2.76$ 5.67
   E $93.675 + 4.869$ 98.544
   F $67.07 - 0.34$ 66.73
   G $9.687 + 8.34$ 18.027
   H $0.56 + 5.3$ 5.86
   I $4.684 - 0.89$ 3.794

10. Solve the problems.
   A Spelling scores: 84, 95, 88, 97. Average score? 91
   B 75 kilometres per hour. Drive 13 hours. How many kilometres? 975 km
   C Height on birthday: 88.5 cm. Height next birthday: 91.25 cm. How much taller? 2.75 cm
   D Travel 513 kilometres in 9 hours. How many kilometres per hour? 57 km/h
   E Find the perimeter. 28.41 cm



Follow-up

Children might enjoy decoding the following message:

Code

A	C	E	I	J	N
$\frac{1}{100}$	$\frac{10}{27}$	583	$4\frac{7}{12}$	1403	76 R5

O	R	T	U	V	Y
437	10.68	$9\frac{7}{8}$	1041	351.3	43898

Explain that, if they match the letters of the code to the answers for these problems in the given order, they will decode the message: *Enjoy your vacation.* (Answers for the problems are shown for your convenience.)

1. 53×11 *(583)*
2. $7\overline{)537}$ *(76 R5)*
3. $469 + 934$ *(1403)*
4. $906 - 469$ *(437)*
5. 467×94 *(43,898)*
6. 934×47 *(43,898)*
7. $323 + 114$ *(437)*
8. $1608 - 567$ *(1041)*
9. $18.24 - 7.56$ *(10.68)*
10. $8.6 + 342.7 = (351.3)$
11. $\frac{1}{20} \times \frac{1}{5} = (\frac{1}{100})$
12. $\frac{5}{6} \times \frac{4}{9} = \frac{20}{54} = (\frac{10}{27})$
13. $\frac{1}{4} \times \frac{1}{25} = (\frac{1}{100})$
14. $5\frac{3}{4} + 4\frac{1}{8}$ *($9\frac{7}{8}$)*
15. $9\frac{1}{4} - 4\frac{2}{3}$ *($4\frac{7}{12}$)*
16. $2309 - 1872$ *(437)*
17. $12\overline{)917}$ *(76 R5)*

(See pages 330–331 for teaching suggestions for this page.)

Add, Subtract, Multiply, or Divide?

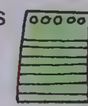
No numbers are given in these short story problems. Use **A**, **S**, **M**, or **D** to tell which operation (**A**ddition, **S**ubtraction, **M**ultiplication, or **D**ivision) you would use to solve each problem.

1 Bicycling: Rode |||| km. Pushed |||| km. How far in all? A

2 Hiking: Walked and ran |||| km. Ran only |||| km. Walked how far? S

3 Bought |||| candy bars for |||| cents. How much for one candy bar? D

4 |||| packages of notebook paper. |||| sheets in each package. How many sheets of paper in all? M



5 John had |||| marbles, Bill had |||| marbles, and Mike had |||| marbles. How many marbles all together? A

6 |||| rocks in each box. |||| rocks in all. How many boxes of rocks? D

7 Dorothy bought |||| stamps. Paid |||| cents for each stamp. How much money did Dorothy spend? M

8 Bought a bicycle for ||||. Sold it for ||||. Sold it for how much less? S

9 Drove |||| km. Used |||| litres of gas. Drove how many km on 1 litre of gas? D

★ **10** Given four different test grades. What is the average test score? A, D (Hint: May use more than one operation.)

You are invited to explore

ACTIVITY CARD 17 Page 341

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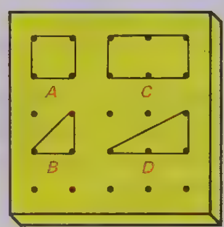
Mathematical Activities

How to Use the Activity Cards

Do you like to explore things for yourself? These Activity Cards will give you some exciting experiences with mathematics. Each card presents a different idea for you to explore. Often you will find that a card will give you ideas for additional activities on your own.

ACTIVITY CARD 1



Every right triangle has an area that is half of some square or rectangle.

The area of square A is 1, so the area of triangle B is __?__. $\frac{1}{2}$

The area of rectangle C is 2, so the area of triangle D is __?__. 1

How many **right triangles** of different sizes can you find on the geoboard and draw on dot paper? What is the area of each?

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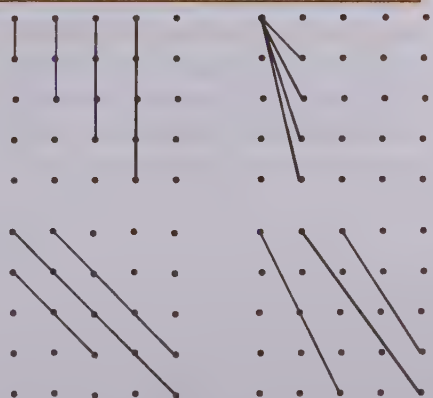
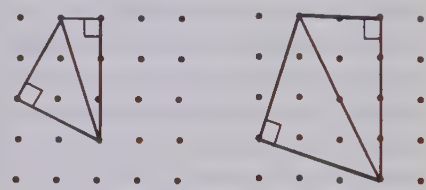
Activity Card 1

The object of this card is to encourage children to find the area of right triangles by thinking about half the area of the corresponding square or rectangle. There are 14 possible right triangles that can be formed on the 5×5 geoboard. The figure at the right illustrates the 14 different segments that can be formed on a 5×5 geoboard. Except for the segment that is exactly one unit long and the segment that is three units long, each of these segments can be the hypotenuse of a right triangle. This gives them 12 possible right triangles using these



segments. Since 2 of the segments will yield two different right triangles, the total of possible right triangles is 14. The segments which can be the hypotenuse of two different right triangles are the ones constructed as follows: connect the second dot from the left on the top row with the dot that is found by going over one to the right and down three. This segment is the hypotenuse of two different right triangles. The other segment that is the hypotenuse of two different right triangles can be found as follows: connect the second dot from the left on the top row with the dot that is two over to the right and down four. (See illustrations below.)



You should not expect the children to find all 14 different right triangles; simply encourage them to find as many as they can.

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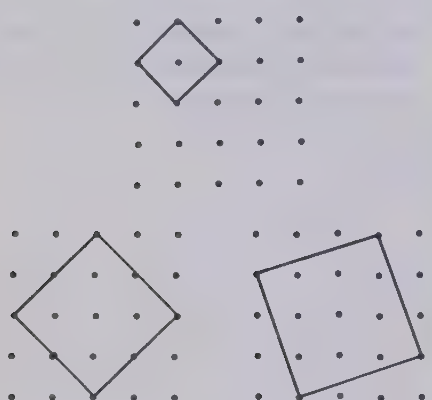
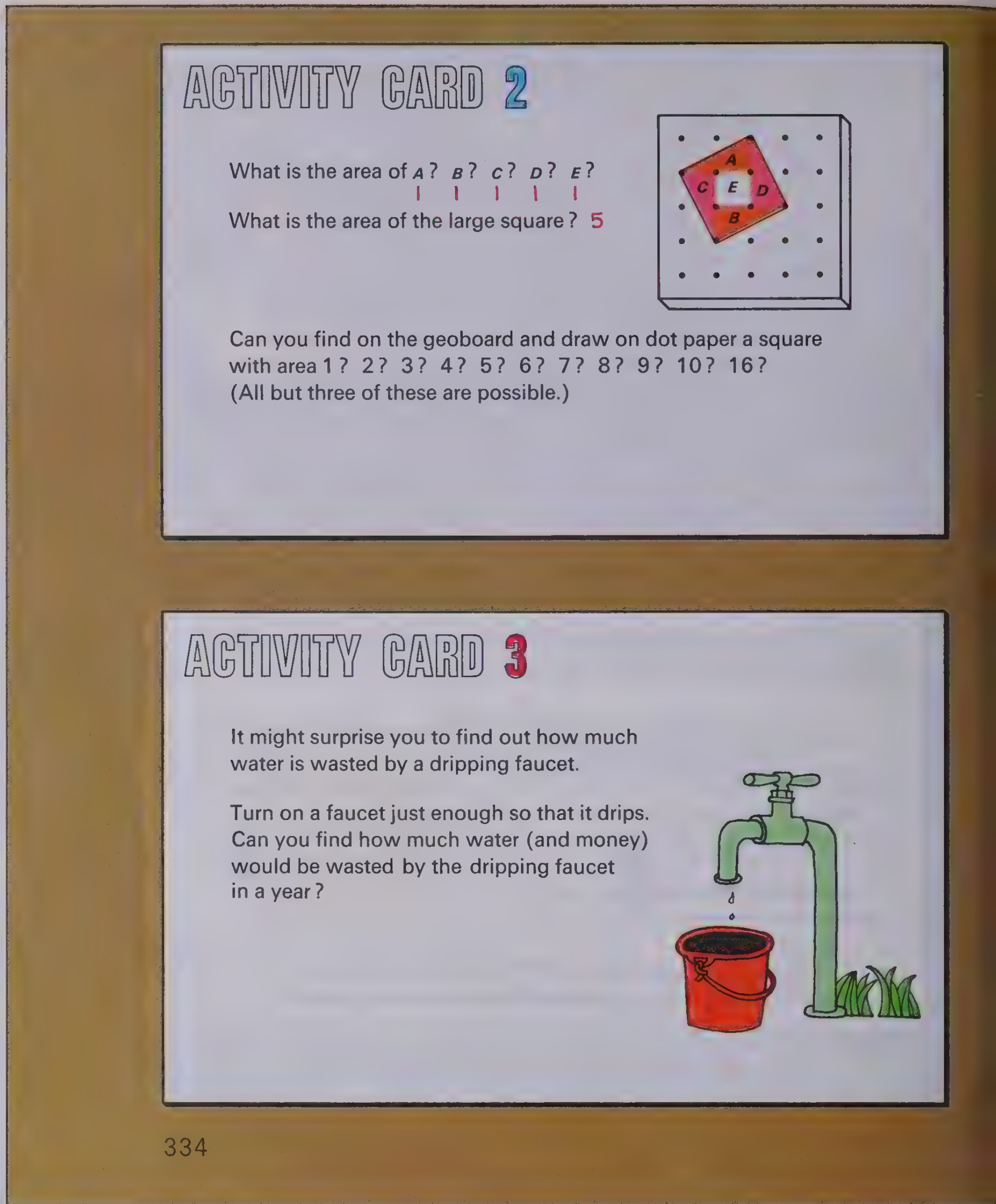
Activity Card 2

This card provides the child with an opportunity to find all possible squares that can be formed on a 5×5 geoboard and to compute their areas. The questions are intended to lead the child to see how he can find the area of some of the squares by piecing together smaller figures of known area. For instance, each triangle has area 1 because each is $\frac{1}{2}$ of a rectangle of area 2; and, since the area of E is 1, the area of the large square is 5. Of the areas suggested in the last question, all are possible except squares of areas 3, 6, and 7. Squares of the less-obvious areas (2, 8, 10) can be formed as shown below.



Activity Card 3

This card involves some painstaking investigation by the child. The child has first to establish a dripping faucet and then find the amount of time it takes for this faucet to fill a given-size container. Of course, this can be done in a wide variety of ways, depending upon the methods used for timing and the size of the container used to catch the drops of water. Following this, then, the child must expand his data to include the time element of a year and find out the cost of water in his local community. Certainly, a considerable amount of thought and creativity are involved if the child is to perform this investigation card on his own. You may find it necessary to give some guidance and hints concerning methods of procedure to some of the children. You should, however, encourage as much original and creative work as possible on this type of card.



ACTIVITY CARD 2

What is the area of A? B? C? D? E?
1 1 1 1 1

What is the area of the large square? 5

Can you find on the geoboard and draw on dot paper a square with area 1? 2? 3? 4? 5? 6? 7? 8? 9? 10? 16?
(All but three of these are possible.)

ACTIVITY CARD 3

It might surprise you to find out how much water is wasted by a dripping faucet.

Turn on a faucet just enough so that it drips. Can you find how much water (and money) would be wasted by the dripping faucet in a year?

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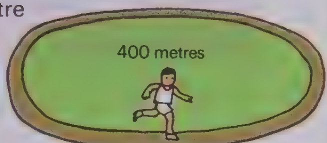
ACTIVITY CARD 4

How fast do you

- ▶ walk?
- ▶ run?
- ▶ ride a bicycle?

Try figuring out at least one of these. (It will help if you measure off a 100-metre track or perhaps use a school track, which may be about 400 metres.)



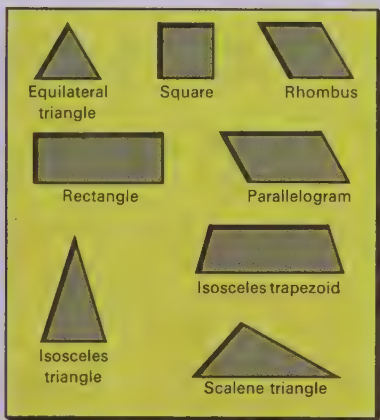
ACTIVITY CARD 5

Cut out figures from a sheet of cardboard so the holes look like this.

Mark an **F** on the front and a **B** on the back of each cutout figure.

In how many different ways can you put each figure back into its hole?

(Draw a picture to show the different ways for each.)



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Activity Card 4

Like card 3, this card calls for a considerable amount of creativity and inventiveness on the part of the child. First, the child has to lay out some type of track of known measure. Of course, the easiest thing to do is to use a school track, or a hundred-metre track, as suggested on the card. Then, a fairly accurate timing device must be found, so that the child can time whatever event interests him. Depending upon the children's ability to work with fractions and decimals, you may want to suggest the possibility of rounding the numbers and using estimation for a considerable amount of the arithmetic.

Activity Card 5

This card involves working with two kinds of symmetry. One is the familiar line of symmetry, and the other is symmetry involving rotation. For example, in the equilateral triangle there are three different lines of symmetry, that is, three different ways the triangle could be folded so that the halves would exactly match. There are three possible rotations of the triangle that make it possible to put the triangle back into its hole. These three rotations can be made with either the front or the back facing upward; hence, there are six possible ways the equilateral triangle can be put back into the hole. You should caution the children to mark each of the figures with an F on the front and a B on the back so that they can identify the different ways they find to put the figure back into its hole. This is a fairly long card for the children to work with, so you may want to encourage them to take their time and spread the work out over several days. Also, it would be helpful if you would encourage the children to make a table including the names of the figures and pictures to show the different ways the figures can be placed back into the holes. Of course, you should be aware that in recording the results of this investigation the children may have some difficulty in drawing the pictures accurately and may not find all of the possibilities. The answers for the various figures are as follows: equilateral triangle, 6; square, 8; rhombus, 4; rectangle, 4; parallelogram, 2; isosceles triangle, 2; isosceles trapezoid, 2; scalene triangle, 1.

Activity Card 6

This card involves logical reasoning with emphasis on the words *and, or,* and *not*. The chart below indicates the answers for this investigation. (Here the red ring is represented by dashed lines.) Notice

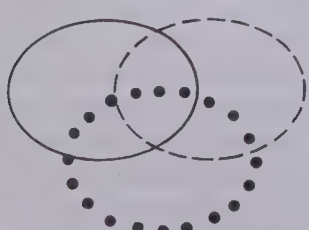
in ◌ but not ○	in neither ◌ nor ○
G, I, J, K	*A, H*

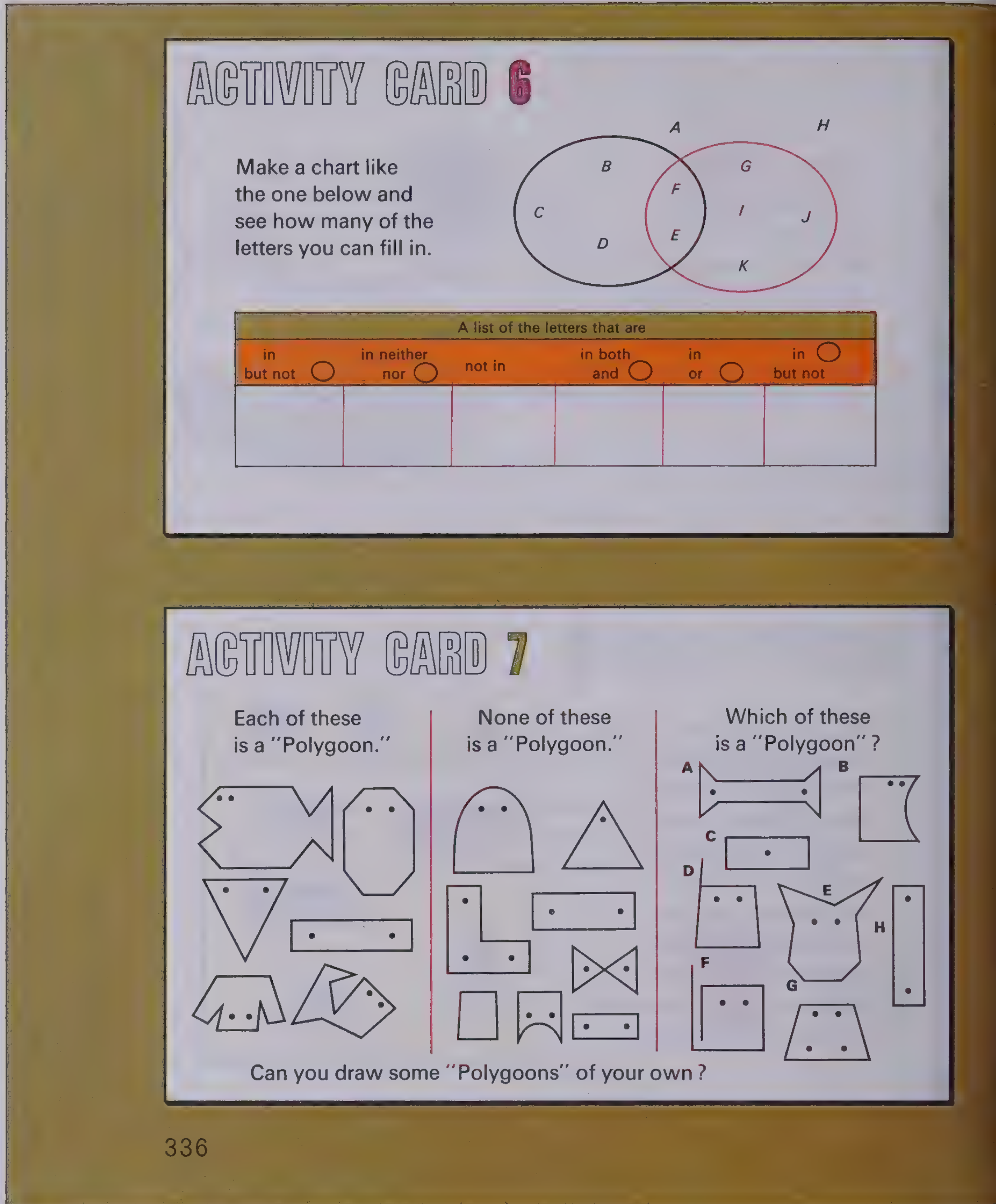
not in ◌	in both ◌ and ○
A, B, C, D, H	*F, E*

in ◌ or ○	in ○ but not ◌
B, C, D, E, F, G, I, J, K	*B, C, D*

that only *F* and *E* are in both the red and the black ring; that is, use of the word *and* in this instance indicates that the letters must satisfy both conditions at the same time. Note also that when the word *or* is used, this indicates that the letters must satisfy only one condition. Therefore, all of the letters except *A* and *H* are inside the red ring or the black ring.

As an extension of this activity, you might suggest that the children draw three rings intersecting as in the accompanying figure. Then they should place letters in various positions and describe the location of various letters in relation to the three different rings.





ACTIVITY CARD 6

Make a chart like the one below and see how many of the letters you can fill in.

A list of the letters that are					
in ○ but not ○	in neither ○ nor ○	not in ○	in both ○ and ○	in ○ or ○	in ○ but not ○

ACTIVITY CARD 7

Each of these is a "Polygoon."

None of these is a "Polygoon."

Which of these is a "Polygoon"?

Can you draw some "Polygoons" of your own?

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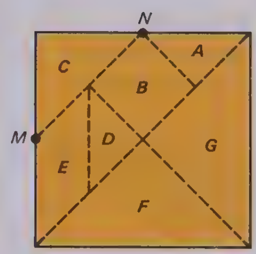
Activity Card 7

This investigation involves logic and reasoning and the child's ability to discern various properties that a given figure has or does not have. For example, the child must look at the various pictures of a "Polygoon" and the various pictures of figures that are not "Polygoons" in order to distinguish what unique combination of properties distinguishes a "Polygoon." Once the child has been given an opportunity to arrive at the conclusions about what a "Polygoon" is, he is asked to identify various "Polygoons" and draw some of his own. The properties of a "Polygoon" are as follows: it is a closed figure; it is made up of line segments; it has exactly two dots in its interior. Therefore, figures A, E, and H are polygoons. B is not a polygoon because it has one curved side; C has just one dot; D has an exterior line segment; F is not closed; and G contains four dots.

ACTIVITY CARD 8

Draw a 8-cm square and cut it into 7 tangram pieces as shown. (M and N are the middle points of the sides.)



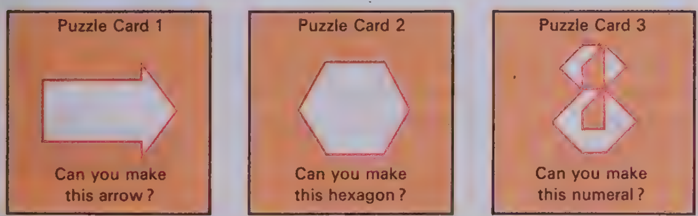
Can you place **all 7** pieces together to form a triangle? a rectangle? a parallelogram? a trapezoid? another interesting figure?

(Draw a picture to show how you made each figure.)

ACTIVITY CARD 9

Can you make a set of at least ten puzzle cards to use with the tangram pieces?

Here are three samples. Each card contains an exact outline of a figure that can be made by using **all seven** tangram pieces.



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Activity Card 8

Several books have been written about tangrams, and many interesting figures (including all the letters of the alphabet) have been made with the tangram pieces. (For further work with tangrams, see the set of tangram cards produced by the Elementary Science Study published by the Webster Division of the McGraw-Hill Book Company.) To make their tangram pieces, the children should use a square of heavy paper or cardboard no smaller than 8 by 8 centimetres. In this way, the pieces will be much simpler to work with. Accuracy of the tangram pieces must be closely controlled and this could best be accomplished by dittoing the tangram pattern and passing it out to the children to use for constructing their pieces. Notice that construction of the tangram pieces involves finding midpoints and right angles. The figures mentioned on the card can easily be made by simply starting with the original square and moving only a few pieces. For example, the large triangle can be made by moving the triangle formed by pieces F and G and placing it beside the triangle formed with pieces A through E. The rectangle can be made by using the triangle formed by pieces A through E as the basic core and placing triangle F on one side and triangle G on the other side. The parallelogram and the trapezoid can be formed by using the triangle formed by pieces A through E as the basic core and by appropriately placing the triangles F and G. Many other interesting figures can be formed by various placements of tangram pieces. You should encourage children to form and draw pictures of some of the various figures they construct.

Activity Card 9

Children should be encouraged to solve the three puzzles given on the card, and they should be given considerable freedom in developing additional cards for use with the tangram pieces. You might emphasize for the children that they are to use all seven of the tangram pieces for each of the patterns given. It would also be most helpful for the children if you were to actually ditto the three patterns given on the card so that the pattern will be the actual size for the tangram pieces that they have. Making the other seven puzzle cards could well become a group project with the children deciding together the shape of puzzle to make out of the tangram pieces.

Some references for additional work with tangram pieces follow:

1. *The Tangram Work Cards* (Educational Aids Division of Invicta Plastic, Leicester, England)
2. *Tangram Puzzle,* An Associated Book on Tangrams (Selective Educational Equipment Co.)
3. Set of Tangram Cards, Elementary Science Study (Webster Division of McGraw-Hill Ryerson)

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Activity Card 10

This activity provides experiences with area, conservation of area, and congruence of geometric figures.

The area of each tangram piece can be found by covering each region of unknown area with regions of known area. For example, the area of square *B* is 2 square units because two of the triangle *A* regions will exactly cover the square. In other words, a region congruent to the square can be constructed from 2 of the small triangular regions. Thus, the areas the children should find are as follows:

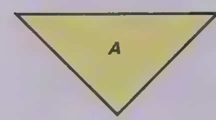
Square *B:* 2 square units
Right triangle *C:* 2 square units
Parallelogram *E:* 2 square units
Right triangle *G:* 4 square units

Several extensions of this activity might be suggested:

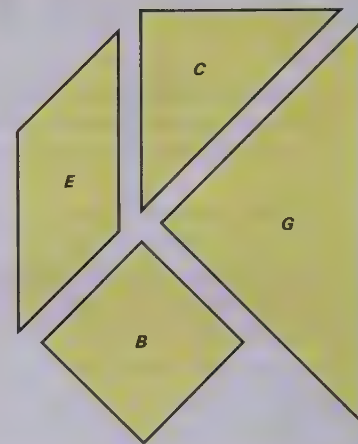
1. Find the area of each region when square *B* is the unit.
2. Find the area of each region when right triangle *G* is the unit.
3. Find the area of each region when right triangle *C* is the unit.
4. Find the area of each region when the total area of the 7 tangram pieces is 1 square unit.

ACTIVITY CARD 10

The area of tangram piece *A* is 1 square unit.



Can you find the area of these other tangram pieces? (Use the tangram pieces you made in **Activity 8.**)

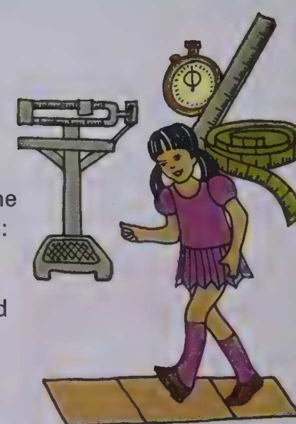


ACTIVITY CARD 11

In how many different ways can you measure yourself?

Make as many different measurements of **you** as you can and make a chart to show the information. Here are just a few suggestions:

Pulse	Length of step
Height	Number of calories used
Weight	Area of bottom of foot
Arm span	Distance you can jump
Grip strength	



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Activity Card 11

This card is designed as an open-ended experience for the children. They should be encouraged to search for unique things to measure about themselves. Some things not mentioned on the card would be breathing rate, comparisons of such things as their waist measurement and their neck measurement, or their height as compared to their arm span. Their lung capacity can be measureed by blowing into a plastic bag or blowing up a large balloon. Other measurements could be shoe size, arm length, span length (span means length from the tip of the little finger to the tip of the thumb when the hand is outstretched), or cubit length (length from the tip of the elbow to the tip of the fingers). This investigation could grow into a year-long study of the changes that take place in the children. If they would keep an accurate chart recording a number of these measurements, they could fill in the chart at various times of the year, observing the changes in themselves.

ACTIVITY CARD 12

Write the letters R, E, and A
on cards and put them in a hat.
Draw them out of the hat.

Can you list all the ways you could
draw them out? There are 6 ways.

	1st	2nd	3rd	
One way is	R	E	A	.

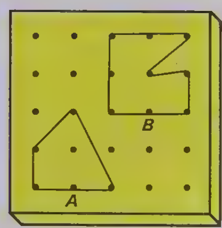
How many of these will form words?

Is it very likely that you will get a word on any one drawing?

ACTIVITY CARD 13

Figure *A* is a 4-sided polygon
(quadrilateral) without "dents."

Figure *B* is a 6-sided polygon
(hexagon) with one "dent."



Can you find on the geoboard and draw
on dot paper a polygon (without "dents")
that has 5 sides? 6 sides? 7 sides?
8 sides? more than 8 sides?

(If you can, give the name of each polygon you find.)

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Activity Card 12

This card concerns the idea of the total number of permutations for 3 objects. As is noted on the card, the number for 3 objects is 6, since if one begins by drawing an R, there are two possible ways to draw the other two letters; if one begins by drawing an E, there are two possible ways for the other two letters to be drawn; and, finally, if one begins by drawing an A, there are two possible ways for drawing the other two letters. Hence, there are a total of six possible arrangements of the letters R, E, and A. There are either three (*are, era, ear*) or four words formed, depending upon whether you wish to consider the name *Rae* as a word.

Activity Card 13

This card provides an opportunity for the children to explore polygons of 5, 6, 7, and 8 sides using the geoboard. You might choose to tell your children about concave and convex polygons, and even possibly give a carefully stated definition. However, for the purpose of this activity, it will suffice simply to say that a concave polygon is one with a dent, and a convex polygon has no dents. In the picture given on the activity card, polygon A is convex and polygon B is concave. You should encourage your children to use a reference book to find names of each of the polygons of 5, 6, 7, and 8 sides. These names are, respectively: pentagon, hexagon, heptagon, and octagon.

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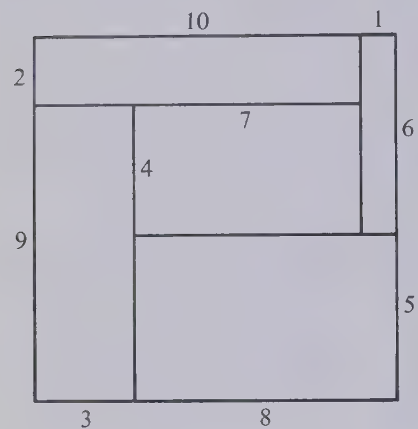
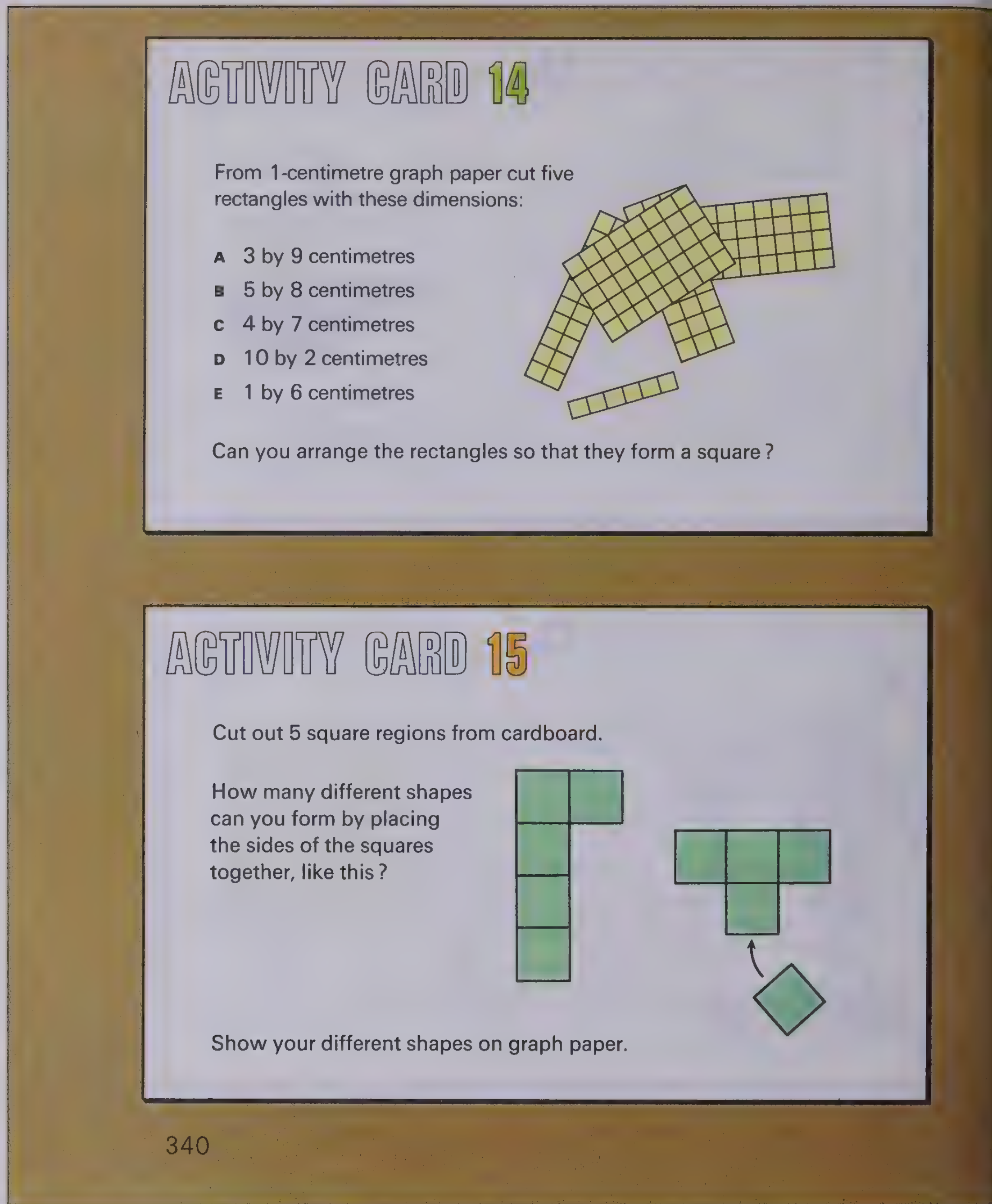
Activity Card 14

This card presents a jigsaw-puzzle approach to the concept of area. Given sufficient time, most children will succeed in arranging the rectangles so that they form a square. The solution is shown below.



Since the total area of the rectangles is 121 square units, the square that they form must have sides of 11 units length. Most children, however, will not analyze the puzzle in this manner; their solution will much more likely be the result of trial and error.



ACTIVITY CARD 14

From 1-centimetre graph paper cut five rectangles with these dimensions:

- A 3 by 9 centimetres
- B 5 by 8 centimetres
- C 4 by 7 centimetres
- D 10 by 2 centimetres
- E 1 by 6 centimetres

Can you arrange the rectangles so that they form a square?

ACTIVITY CARD 15

Cut out 5 square regions from cardboard.

How many different shapes can you form by placing the sides of the squares together, like this?

Show your different shapes on graph paper.

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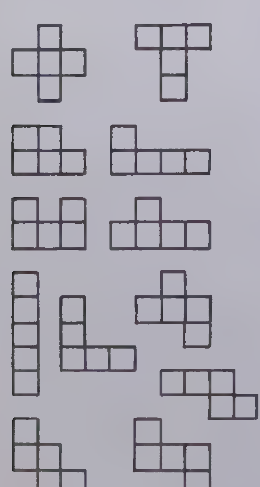
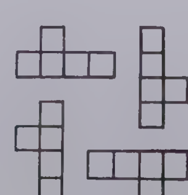
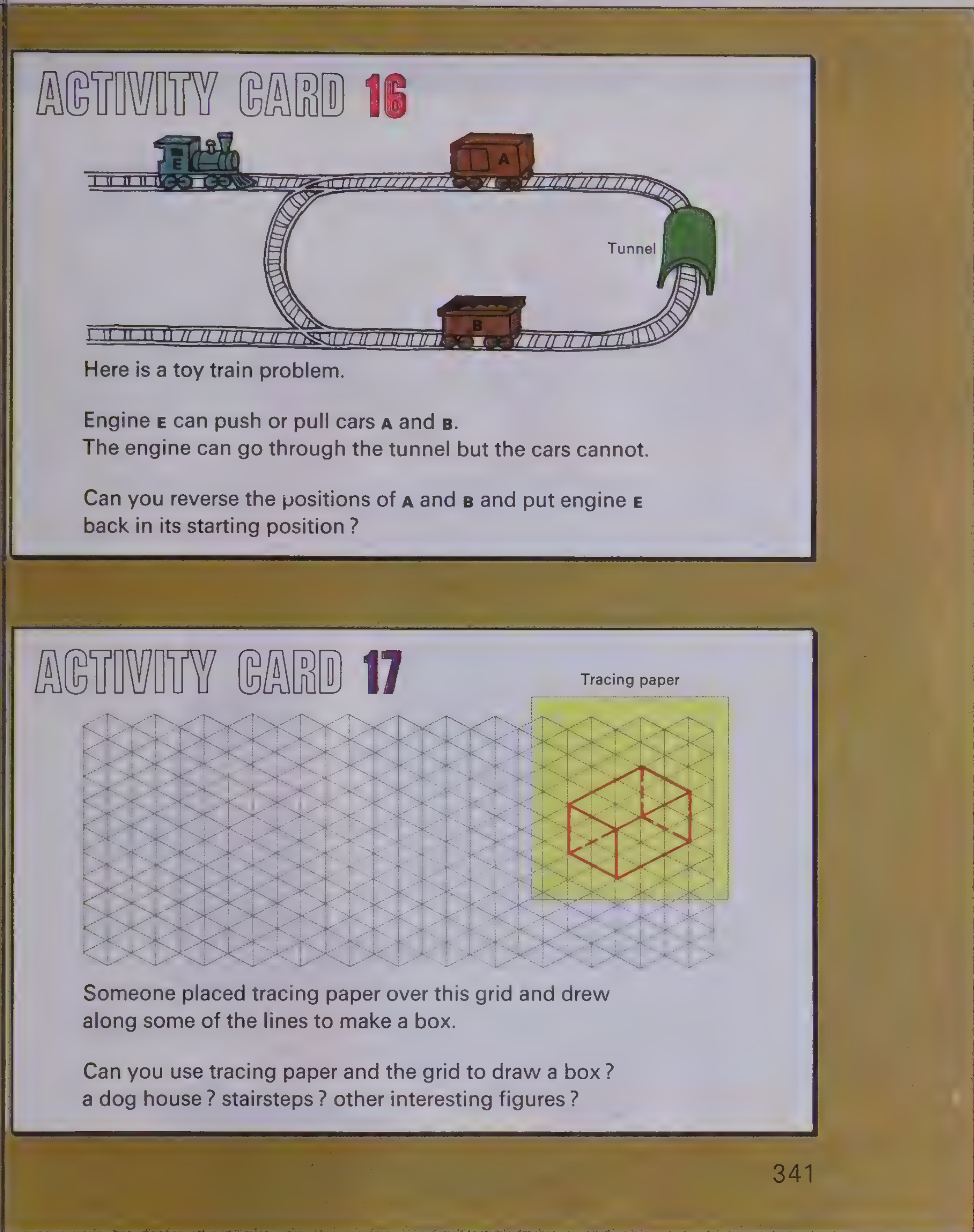
Activity Card 15

Each shape formed from 5 squares is called a *pentominoe*. With the restriction that each square in a pentominoe must share at least one full side with another square, there are exactly 12 different pentominoes, as shown below.



Many children may not be able to find all 12 of the pentominoes, but they will enjoy the challenge of trying to find as many as they can. Care should be taken that each pentominoe is really distinct rather than simply a flip or a rotation of the same pentominoe. For example, the figures below are not really different; each is simply a rotation or flip of the same pentominoe. After having found the 12 different pentominoes, some children might like to try to fit all 12 together to form a 6×10 unit or a 4×15 unit rectangle. This is difficult, but many children will enjoy trying it.





ACTIVITY CARD 16

Here is a toy train problem.

Engine E can push or pull cars A and B.
The engine can go through the tunnel but the cars cannot.

Can you reverse the positions of A and B and put engine E back in its starting position?

ACTIVITY CARD 17

Someone placed tracing paper over this grid and drew along some of the lines to make a box.

Can you use tracing paper and the grid to draw a box? a dog house? stairsteps? other interesting figures?

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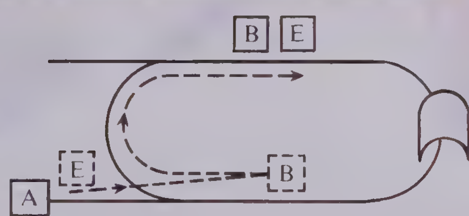
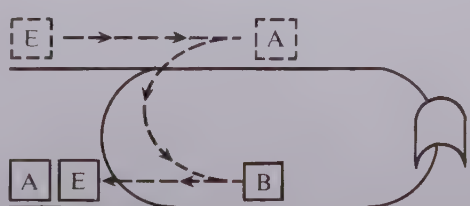
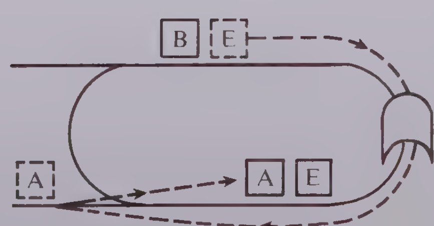
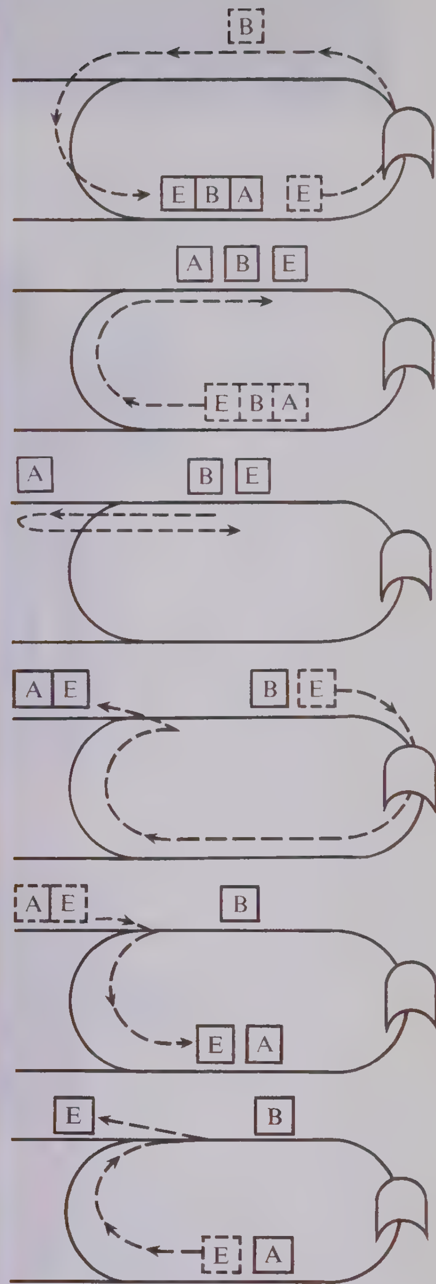
Activity Card 16

In order to solve the problem presented by this card, the children should make a drawing of the track and tunnel, and use blocks or other movable objects to represent engine **E** and cars **A** and **B**. The steps required in order to reverse **A** and **B** and return **E** to its original position are as follows.





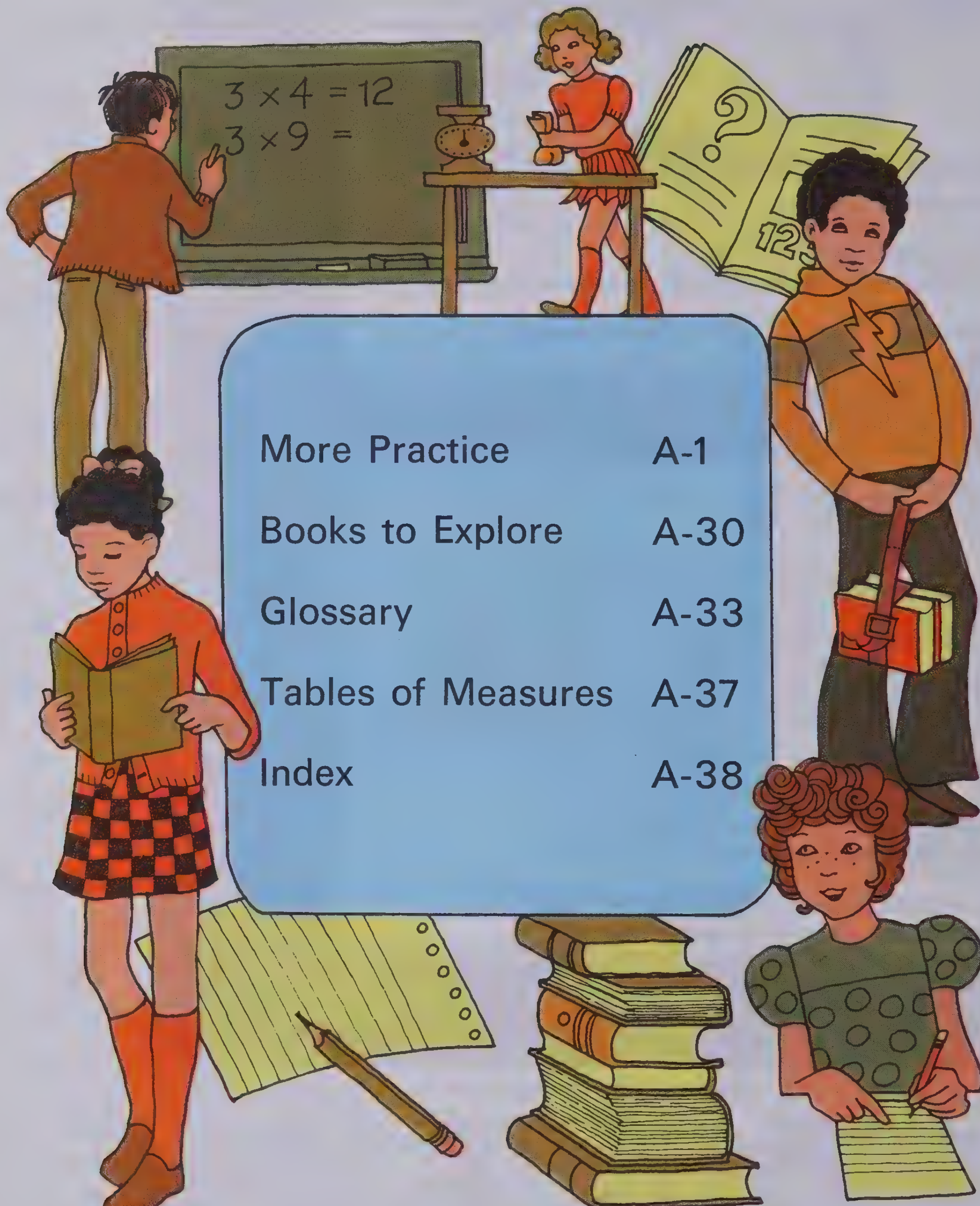




Activity Card 17

This card uses isometric paper to provide a vehicle by which the child can make three-dimensional drawings. The object of the card is to provide an opportunity for experimentation and to help the children visualize three-dimensional figures drawn on two-dimensional paper. Some of the children should be encouraged to draw three-dimensional figures without the use of the grid. Also, some of the children may choose to make a larger grid in order to construct larger three-dimensional figures. You should observe with your children that the grid is made by piecing together equilateral triangles.

Appendix



More Practice

Set 1 | *For use with page 23*

Solve the equations.

1. $6385 = 6000 + n + 80 + 5$ 300
2. $7082 = m + 80 + 2$ 7000
3. $3490 = 3000 + 400 + y$ 90
4. $7233 = 7000 + v + 30 + 3$ 200
5. $4106 = 4000 + 100 + b$ 6
6. $1087 = t + 80 + 7$ 1000
7. $2194 = 2000 + p + 90 + 4$ 100
8. $8183 = 8000 + 100 + s + 3$ 80
9. $9007 = 9000 + n$ 7
10. $6911 = j + 900 + 10 + 1$ 6000
11. $2613 = 2000 + 600 + m + 3$ 10
12. $7508 = 7000 + 500 + b$ 8
13. $9089 = 9000 + n + 9$ 80
14. $8650 = 8000 + 600 + s$ 50
15. $2930 = f + 900 + 30$ 2000
16. $3057 = 3000 + r + 7$ 50
17. $2873 = 2000 + n + 70 + 3$ 800
18. $6704 = 6000 + y + 4$ 700

Reflected answers, Set 1: 1. 300, 2. 7000, 3. 90, 10. 6000, 11. 10, 12. 8

Set 2 | *For use with page 25*

For the numeral 948 306 125, tell what digit is in each of these places.

1. hundreds' 1
2. millions' 8
3. ones' 5
4. hundred millions' 9
5. ten thousands' 0
6. ten millions' 4
7. hundred thousands' 3
8. tens' 2
9. thousands' 6

Use the numeral 607 854 932 to answer each exercise below.

10. 8 is in the __?__ place. hundred thousands'
11. 3 is in the __?__ place. tens'
12. 0 is in the __?__ place. ten millions'
13. 7 is in the __?__ place. millions'
14. 9 is in the __?__ place. hundreds'
15. 6 is in the __?__ place. hundred millions'
16. 4 is in the __?__ place. thousands'
17. 5 is in the __?__ place. ten thousands'

Reflected answers, Set 2: 1. 1, 4. 9, 7. 3, 10. hundred thousands', 14. hundreds'

Set 3 — *For use with page 27*

For each number, write an equation as in the example.
(Example: 7896 = 7000 + 800 + 90 + 6)

1. 2358 2000 + 300 + 50 + 8
2. 5501 5000 + 500 + 1
3. 7483 7000 + 400 + 80 + 3
4. 22 793 20 000 + 2000 + 700 + 90 + 3
5. 52 438 50 000 + 2000 + 400 + 30 + 8
6. 72 230 70 000 + 2000 + 200 + 30
7. 772 853 700 000 + 70 000 + 2000 + 800 + 50 + 3
8. 396 923 300 000 + 90 000 + 6000 + 900 + 20 + 3
9. 790 374 700 000 + 90 000 + 300 + 70 + 4
10. 8 700 108 8 000 000 + 700 000 + 8
11. 4 502 824 4 000 000 + 500 000 + 2000 + 800 + 20 + 4
12. 43 227 038 40 000 000 + 3 000 000 + 200 000 + 20 000 + 7000 + 30 + 8

For each number, write an equation as in the example.
[Example: $3241 = (3 \times 1000) + (2 \times 100) + (4 \times 10) + 1$]

13. 6984 $(6\times1000)+(9\times100)+(8\times10)+4$
14. 7362 $(7\times1000)+(3\times100)+(6\times10)+2$
15. 5083 $(5\times1000)+(8\times10)+3$
16. 17 626 $(1\times10\,000)+(7\times1000)+(6\times100)+(2\times10)+6$
17. 40 261 $(4\times10\,000)+(2\times100)+(6\times10)+1$
18. 341 163 $(3\times100\,000)+(4\times10\,000)+(1\times1000)+(1\times100)+(6\times10)+3$
19. 175 011 $(1\times100\,000)+(7\times10\,000)$
20. 2 419 421 $(2\times1\,000\,000)+(4\times100\,000)+(1\times10\,000)+(9\times1000)+(4\times100$ $+(5\times1000)+(1\times10)+1$

Solve.

21. $900+70+4=x$ 974
22. $1000+200+90+4=n$ 1294
23. $(8\times100)+(3\times10)+7=q$ 837
24. $(2\times1000)+(6\times100)+(1\times10)+9=r$ $+(2\times10)$ $+1$ 2619
25. $(3\times1000)+(2\times10)+6=t$ 3016
26. $(8\times1000)+(3\times100)+(5\times10)+2=z$ 8352

Reflected answers, Set 3: 1. 2000 + 300 + 50 + 8, 4. 20 000 + 2000 + 700 + 90 + 3, 7. 700 000 + 70 000 + 2000 + 800 + 50 + 3, 10. 8 000 000 + 700 000 + 100 + 8, 21. 974, 24. 2619

Set 4 — *For use with page 29*

Give the correct sign (< or >) for each ▒.

1. 73 ▒ 67 >	7. 8384 ▒ 7384 >	13. 795 668 ▒ 759 668 >
2. 206 ▒ 260 <	8. 6287 ▒ 6587 <	14. 437 202 ▒ 437 220 <
3. 437 ▒ 427 >	9. 4391 ▒ 4319 >	15. 661 369 ▒ 661 269 >
4. 734 ▒ 743 <	10. 26 844 ▒ 26 518 >	16. 9 264 577 ▒ 9 354 577 <
5. 896 ▒ 968 <	11. 39 263 ▒ 39 463 <	17. 8 242 357 ▒ 8 224 357 >
6. 519 ▒ 549 <	12. 59 393 ▒ 59 339 >	18. 8 320 811 ▒ 8 328 011 <

Reflected answers, Set 4: 1. >, 7. >, 13. >

Set 5 *For use with page 43*

Tell which operation or operations (+, −, ×, ÷) you would use to find the answer if numbers were given.

1. ||||| insects.
||||| legs each.
How many legs in all? ×

2. Book ||||| pages long.
||||| pages in each chapter.
How many chapters? ÷

3. Had ||||| dollars.
Spent |||||.
How much left? −

4. ||||| boys. ||||| girls.
||||| people on each team.
How many teams? +, ÷

5. Had ||||| records.
Bought ||||| more.
Gave away |||||.
How many records now? +, −

6. Drove ||||| kilometres on Monday.
||||| kilometres on Tuesday.
Used ||||| litres of gas.
How many kilometres to one litre of gas? +, ÷

Reflected answers, Set 5: 1. × , 4. + , ÷

Set 6 *For use with page 45*

Rewrite each equation. Use an operation different from the one given.

1. $8 - 5 = 3$ 3+5=8
2. $6 \times 4 = 24$ 24÷4=6 24÷6=4
3. $35 \div 5 = 7$ 7×5=35
4. $16 + 5 = 21$ 21−5=16 21−16=5
5. $7 \times 7 = 49$ 49÷7=7
6. $36 \div 9 = 4$ 4×9=36
7. $7 + 6 = 13$ 13−6=7 13−7=6
8. $2 \times 9 = 18$ 18÷2=9 18÷9=2
9. $15 - 9 = 6$ 6+9=15
10. $4 \times 7 = 28$ 28÷4=7 28÷7=4
11. $72 \div 9 = 8$ 8×9=72
12. $9 + 8 = 17$ 17−8=9 17−9=8
13. $11 - 6 = 5$ 5+6=11
14. $63 \div 7 = 9$ 9×7=63
15. $9 \times 3 = 27$ 27÷3=9 27÷9=3
16. $8 + 7 = 15$ 15−7=8 15−8=7
17. $15 \div 3 = 5$ 5×3=15
18. $3 \times 4 = 12$ 12÷3=4 12÷4=3
19. $12 - 8 = 4$ 4+8=12
20. $81 \div 9 = 9$ 9×9=81
21. $4 \times 5 = 20$ 20÷4=5 20÷5=4
22. $32 \div 4 = 8$ 8×4=32
23. $18 + 7 = 25$ 25−7=18 25−18=7
24. $25 - 18 = 7$ 7+18=25
25. $6 \times 6 = 36$ 36÷6=6
26. $27 \div 3 = 9$ 9×3=27
27. $35 + 6 = 41$ 41−6=35 41−35=6
28. $8 \times 7 = 56$ 56÷7=8 56÷8=7
29. $63 - 54 = 9$ 9+54=63
30. $5 \times 9 = 45$ 45÷5=9 45÷9=5

Reflected answers, Set 6: 1. $3 + 5 = 8$, 2. $24 \div 6 = 4$ or $24 \div 4 = 6$
11. $9 \times 8 = 72$ 12. $17 - 9 = 8$ or $17 - 8 = 9$,
21. $20 \div 5 = 4$ or $20 \div 4 = 5$, 22. $4 \times 8 = 32$

Set 7 *For use with page 51*

Find the missing addend in the addition equation.
Then write the subtraction equation with the correct difference.

1. $n + 9 = 16$ 7 $16 - 9 = n$ 7	**3.** $r + 4 = 13$ 9 $13 - 4 = r$ 9	**5.** $9 + q = 14$ 5 $14 - 9 = q$ 5	**7.** $s + 28 = 35$ 7 $35 - 28 = s$ 7
2. $8 + w = 13$ 5 $13 - 8 = w$ 5	**4.** $t + 6 = 11$ 5 $11 - 6 = t$ 5	**6.** $y + 74 = 82$ 8 $82 - 74 = y$ 8	**8.** $14 + v = 23$ 9 $23 - 14 = v$ 9

Find the missing factor in the multiplication equation.
Then write the division equation with the correct quotient.

9. $x \times 3 = 12$ 4 $12 \div 3 = x$ 4	**11.** $b \times 8 = 56$ 7 $56 \div 8 = b$ 7	**13.** $4 \times m = 36$ 9 $36 \div 4 = m$ 9	**15.** $c \times 8 = 64$ 8 $64 \div 8 = c$ 8
10. $7 \times w = 42$ 6 $42 \div 7 = w$ 6	**12.** $d \times 9 = 54$ 6 $54 \div 9 = d$ 6	**14.** $8 \times h = 40$ 5 $40 \div 8 = h$ 5	**16.** $k \times 6 = 48$ 8 $48 \div 6 = k$ 8

Reflected answers, Set 7: 1. $n = 7$, 3. $r = 9$, 5. $q = 5$, 7. $s = 7$

Set 8 *For use with page 53*

Find as many of these as you can in 1 minute.

1. 2×4 8	**4.** 3×7 21	**7.** 7×8 56	**10.** 0×0 0	**13.** 9×2 18
2. 7×5 35	**5.** 5×9 45	**8.** 4×5 20	**11.** 8×4 32	**14.** 6×4 24
3. 0×8 0	**6.** 4×7 28	**9.** 3×9 27	**12.** 6×8 48	**15.** 5×5 25

Find as many of these as you can in $1\frac{1}{2}$ minutes.

16. $16 \div 4$ 4	**19.** $25 \div 5$ 5	**22.** $36 \div 9$ 4	**25.** $81 \div 9$ 9	**28.** $63 \div 7$ 9
17. $16 \div 8$ 2	**20.** $32 \div 4$ 8	**23.** $24 \div 6$ 4	**26.** $35 \div 7$ 5	**29.** $0 \div 9$ 0
18. $27 \div 3$ 9	**21.** $40 \div 8$ 5	**24.** $45 \div 9$ 5	**27.** $16 \div 2$ 8	**30.** $6 \div 3$ 2

Reflected answers, Set 8: 1. 8, 4. 21, 7. 56, 10. 0, 13. 18, 16. 4, 19. 5, 22. 4, 25. 9, 28. 9

Set 9 — *For use with page 57*

Solve the equations.

1. $(2 \times 9) + 6 = m$ 24
2. $(4 \times 7) + 5 = k$ 33
3. $(6 \times 3) + 4 = j$ 22
4. $(8 \times 3) + 2 = c$ 26
5. $(4 \times 9) + 7 = s$ 43
6. $(7 \times 5) + 2 = d$ 37
7. $(8 \times 0) + 3 = q$ 3
8. $(2 \times 6) + 6 = t$ 18
9. $(8 \times 5) + 1 = w$ 41
10. $(9 \times 7) + 8 = f$ 71
11. $(4 \times 5) + 9 = b$ 29
12. $(7 \times 7) + 7 = h$ 56
13. $(6 \times 9) + 3 = z$ 57
14. $(5 \times 5) + 4 = a$ 29
15. $(7 \times 8) + 2 = v$ 58
16. $(6 \times 7) + 3 = r$ 45
17. $(8 \times 2) + 9 = u$ 25
18. $(9 \times 9) + 2 = x$ 83
19. $(6 \times 5) + 0 = c$ 30
20. $(4 \times 3) + 6 = t$ 18
21. $(7 \times n) + 4 = 25$ 3
22. $(w \times 4) + 3 = 19$ 4
23. $(4 \times p) + 5 = 37$ 8
24. $(3 \times s) + 6 = 15$ 3
25. $(c \times 1) + 7 = 14$ 7
26. $(6 \times m) + 3 = 39$ 6
27. $(j \times 5) + 6 = 16$ 2
28. $(7 \times x) + 2 = 37$ 5
29. $(i \times 6) + 4 = 28$ 4
30. $(t \times 8) + 1 = 65$ 8

Reflected answers, Set 9: 1. 24, 2. 33, 11. 29, 12. 56, 21. 3, 22. 4

Set 10 — *For use with page 65*

Solve the equations.

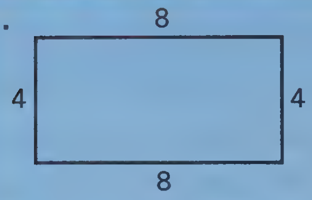
1. $9 \times 9 = (6 \times 9) + (s \times 9)$ 3
2. $7 \times 9 = (5 \times 9) + (r \times 9)$ 2
3. $8 \times 8 = (6 \times 8) + (b \times 8)$ 2
4. $5 \times 12 = (5 \times 10) + (5 \times q)$ 2
5. $4 \times 11 = (m \times 10) + (4 \times 1)$ 4
6. $6 \times 16 = (6 \times 10) + (t \times 6)$ 6
7. $5 \times 13 = (5 \times k) + (5 \times 3)$ 10
8. $7 \times 14 = (7 \times 10) + (7 \times n)$ 4
9. $9 \times 15 = (9 \times 10) + (z \times 5)$ 9
10. $4 \times 18 = (d \times 10) + (4 \times 8)$ 4
11. $3 \times 26 = (3 \times j) + (3 \times 6)$ 20
12. $7 \times 31 = (7 \times 30) + (7 \times a)$ 1
13. $5 \times 43 = (5 \times m) + (5 \times 3)$ 40
14. $2 \times 24 = (2 \times 20) + (n \times 4)$ 2
15. $6 \times 14 = (c \times 10) + (6 \times 4)$ 6
16. $5 \times 21 = (5 \times 20) + (5 \times t)$ 1
17. $8 \times 32 = (8 \times j) + (8 \times 2)$ 30
18. $9 \times 19 = (9 \times 10) + (d \times 9)$ 9
19. $7 \times 23 = (k \times 20) + (7 \times 3)$ 7
20. $4 \times 27 = (4 \times p) + (4 \times 7)$ 20
21. $6 \times 48 = (6 \times 40) + (6 \times c)$ 8
22. $3 \times 62 = (3 \times 60) + (r \times 2)$ 3
23. $5 \times 36 = (h \times 30) + (5 \times 6)$ 5
24. $8 \times 29 = (8 \times x) + (8 \times 9)$ 20

Reflected answers, Set 10: 1. 3, 2. 2, 3. 2, 13. 40, 14. 2, 15. 6

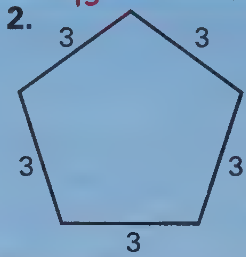
Set 11 | *For use with page 89*

Find the perimeter of each figure.

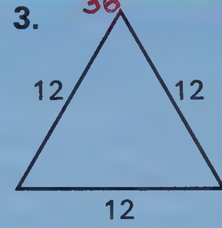
1. 24



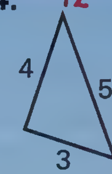
2. 15



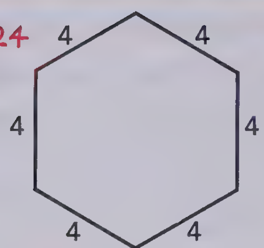
3. 36



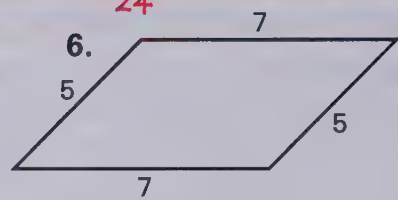
4. 12



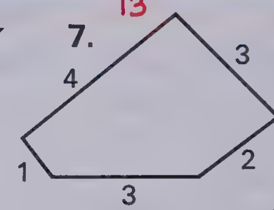
5. 24



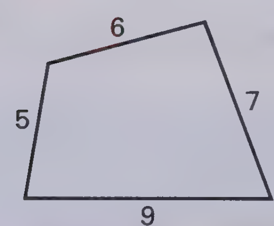
6. 24



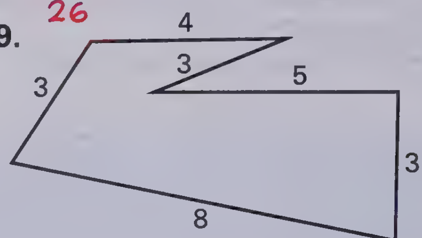
7. 13



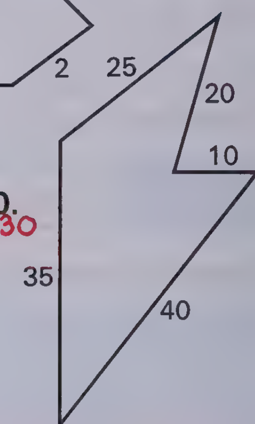
8. 27



9. 26



10. 130



Reflected answers, Set 11: 1. 24, 2. 15, 3. 36, 4. 12

Set 12 | *For use with page 93*

Find the area of the following triangles. The base and height are given.

1. $b = 6$, $h = 4$ 12
2. $b = 7$, $h = 8$ 28
3. $b = 5$, $h = 12$ 30
4. $b = 13$, $h = 18$ 117
5. $b = 9$, $h = 16$ 72
6. $b = 30$, $h = 10$ 150
7. $b = 20$, $h = 17$ 170
8. $b = 22$, $h = 7$ 77
9. $b = 14$, $h = 11$ 77
10. $b = 24$, $h = 10$ 120

Reflected answers, Set 12: 1. 12, 2. 28, 3 30, 4. 117, 5. 72

Set 13 *For use with page 99*

Give the multiple of 10 that you think should go in each |||||.

1. To estimate 33 + 58, we can find the sum 30 + |||||. 60
2. To estimate 87 − 49, we can find the difference ||||| − 50. 90
3. To estimate 62 × 76, we can find the product 60 × |||||. 80
4. To estimate 348 + 213, we can find the sum 350 + |||||. 210
5. To estimate 477 ÷ 63, we can find the quotient ||||| ÷ 60. 480

Give the multiple of 100 that you think should go in each |||||.

6. To estimate 701 + 882, we can find the sum 700 + |||||. 900
7. To estimate 261 − 172, we can find the difference ||||| − 200. 300
8. To estimate 861 × 244, we can find the product 900 × |||||. 200
9. To estimate 778 × 419, we can find the product ||||| × 400. 800
10. To estimate 623 ÷ 196, we can find the quotient 600 ÷ |||||. 200

Reflected answers, Set 13: 1. 60, 6. 900

Set 14 *For use with page 101*

Find the products.

1. 9 × 30 270	11. 70 × 30 2100	21. 300 × 40 12 000	31. 8 × 7000 56 000
2. 8 × 40 320	12. 40 × 70 2800	22. 600 × 70 42 000	32. 90 × 9000 810 000
3. 40 × 7 280	13. 80 × 90 7200	23. 40 × 200 8000	33. 6000 × 40 240 000
4. 50 × 3 150	14. 70 × 60 4200	24. 90 × 300 27 000	34. 20 × 3000 60 000
5. 6 × 50 300	15. 30 × 90 2700	25. 800 × 40 32 000	35. 7000 × 50 350 000
6. 7 × 90 630	16. 50 × 40 2000	26. 500 × 60 30 000	36. 500 × 500 250 000
7. 10 × 30 300	17. 20 × 90 1800	27. 70 × 900 63 000	37. 700 × 800 560 000
8. 40 × 10 400	18. 70 × 50 3500	28. 400 × 40 16 000	38. 200 × 400 80 000
9. 60 × 80 4800	19. 60 × 30 1800	29. 200 × 10 2000	39. 600 × 100 60 000
10. 30 × 50 1500	20. 40 × 90 3600	30. 30 × 800 24 000	40. 900 × 300 270 000

Reflected answers, Set 14: 1. 270, 2. 320, 3. 280, 11. 2100, 12. 2800, 13. 7200, 21. 12 000, 22. 42 000, 23. 8000, 31. 56 000, 32. 810 000, 33. 240 000

Set 15 — *For use with page 103*

Solve.

1. 240 ÷ 6 = ***t*** 40	**10.** 180 ÷ 60 = ***c*** 3	**19.** 320 ÷ 40 = ***b*** 8	**28.** 420 ÷ 60 = ***x*** 7
2. 350 ÷ 7 = ***h*** 50	**11.** 120 ÷ 30 = ***q*** 4	**20.** 360 ÷ 40 = ***s*** 9	**29.** 560 ÷ 70 = ***p*** 8
3. 320 ÷ 4 = ***w*** 80	**12.** 250 ÷ 50 = ***x*** 5	**21.** 350 ÷ 50 = ***f*** 7	**30.** 490 ÷ 70 = ***n*** 7
4. 150 ÷ 3 = ***p*** 50	**13.** 160 ÷ 40 = ***a*** 4	**22.** 420 ÷ 70 = ***r*** 6	**31.** 810 ÷ 90 = ***j*** 9
5. 560 ÷ 7 = ***b*** 80	**14.** 240 ÷ 30 = ***k*** 8	**23.** 560 ÷ 80 = ***m*** 7	**32.** 480 ÷ 60 = ***b*** 8
6. 240 ÷ 8 = ***n*** 30	**15.** 280 ÷ 70 = ***z*** 4	**24.** 640 ÷ 80 = ***j*** 8	**33.** 540 ÷ 90 = ***s*** 6
7. 280 ÷ 7 = ***d*** 40	**16.** 140 ÷ 20 = ***d*** 7	**25.** 540 ÷ 60 = ***y*** 9	**34.** 150 ÷ 30 = ***h*** 5
8. 210 ÷ 3 = ***s*** 70	**17.** 270 ÷ 30 = ***u*** 9	**26.** 630 ÷ 70 = ***i*** 9	**35.** 360 ÷ 60 = ***t*** 6
9. 480 ÷ 6 = ***m*** 80	**18.** 270 ÷ 90 = ***g*** 3	**27.** 720 ÷ 90 = ***d*** 8	**36.** 120 ÷ 60 = ***r*** 2

Reflected answers, Set 15: 1. 40, 2. 50, 10. 3, 11. 4, 19. 8, 20. 9, 28. 7, 29. 8

Set 16 — *For use with page 107*

Choose the best estimate for each problem.

1. There are 52 weeks in one year. Choose the best estimate for the number of weeks in 7 years.

A 420 weeks B 350 weeks C 280 weeks

2. A theater has 47 seats in each row. Choose the best estimate for the number of seats in the theater if there are 32 rows.

A 1500 B 1200 C 150

3. 1 hour is 60 minutes. Choose the best estimate for the number of hours in 5390 minutes.

A 100 B 90 C 80

Reflected answers, Set 16: 1. B

Set 17 *For use with page 115*

Find the sums.

1. 34 +45 79	**2.** 56 +87 143	**3.** 42 +57 99	**4.** 136 +157 293	**5.** 427 +382 809
6. 487 45 +262 794	**7.** 28 856 +28 912	**8.** 382 994 +481 1857	**9.** 1481 464 +168 2113	**10.** 562 2363 +3905 6830
11. 283 636 34 +440 1393	**12.** 1934 759 50 +621 3364	**13.** 3281 5182 56 +354 8873	**14.** 9533 65 536 +2292 12426	**15.** 4116 8476 5022 +4273 21887

Reflected answers, Set 17: 1. 79, 2. 143, 3. 99, 4. 293, 5. 809

Set 18 *For use with page 117*

Find the differences.

1. 39 −24 15	**2.** 51 −35 16	**3.** 416 −39 377	**4.** 165 −97 68	**5.** 367 −39 328
6. 356 −163 193	**7.** 873 −436 437	**8.** 721 −563 158	**9.** 500 −187 313	**10.** 802 −496 306
11. 1500 −59 1441	**12.** 1608 −567 1041	**13.** 5308 −795 4513	**14.** 2834 −242 2592	**15.** 7438 −842 6596
16. 6900 −3486 3414	**17.** 8043 −5964 2079	**18.** 7004 −5437 1567	**19.** 5702 −3986 1716	**20.** 6700 −3985 2715

Reflected answers, Set 18: 1. 15, 2. 16, 3. 377, 4. 68, 5. 328

Set 19 *For use with page 123*

Find the products.

1. 12 × 3 = 36	2. 34 × 5 = 170	3. 17 × 5 = 85	4. 56 × 4 = 224	5. 48 × 7 = 336	6. 76 × 4 = 304
7. 324 × 7 = 2268	8. 572 × 9 = 5148	9. 856 × 4 = 3424	10. 457 × 3 = 1371	11. 863 × 2 = 1726	12. 294 × 7 = 2058
13. 566 × 8 = 4528	14. 354 × 9 = 3186	15. 768 × 6 = 4608	16. 9534 × 6 = 57 204	17. 7604 × 8 = 60 832	18. 6875 × 2 = 13 750
19. 2442 × 8 = 19 536	20. 3905 × 6 = 23 430	21. 5027 × 9 = 45 243	22. 9701 × 4 = 38 804	23. 6762 × 3 = 20 286	24. 1836 × 7 = 12 852

Reflected answers, Set 19: 1. 36, 2. 170, 3. 85, 4. 224, 5. 336, 6. 304

Set 20 *For use with page 125*

Find the products.

1. 24 × 15 = 360	2. 57 × 26 = 1482	3. 33 × 54 = 1782	4. 64 × 25 = 1600	5. 65 × 34 = 2210	6. 67 × 27 = 1809
7. 79 × 18 = 1422	8. 80 × 89 = 7120	9. 253 × 23 = 5819	10. 147 × 45 = 6615	11. 568 × 63 = 35 784	12. 476 × 72 = 34 272
13. 935 × 87 = 81 345	14. 467 × 94 = 43 898	15. 851 × 56 = 47 656	16. 304 × 69 = 20 976	17. 436 × 275 = 119 900	18. 514 × 308 = 158 312
19. 175 × 302 = 52 850	20. 643 × 449 = 288 707	21. 863 × 705 = 608 415	22. 972 × 667 = 648 324	23. 781 × 708 = 552 948	24. 825 × 191 = 157 575

Reflected answers, Set 20: 1. 360, 2. 1482, 3. 1782, 4. 1600, 5. 2210, 6. 1809

Set 21 | *For use with page 139*

Find the average of the numbers in each set.

1. {5, 8, 2} 5	**6.** {21, 34, 20} 25	**11.** {189, 373, 284} 282
2. {6, 7, 5} 6	**7.** {74, 65, 29} 56	**12.** {9, 12, 15, 16} 13
3. {9, 6, 2, 3} 5	**8.** {41, 24, 31} 32	**13.** {107, 56, 88, 97} 87
4. {8, 9, 9, 2} 7	**9.** {64, 80, 123} 89	**14.** {81, 76, 121, 54} 83
5. {37, 38, 54} 43	**10.** {137, 76, 75} 96	**15.** {465, 392, 436} 431

Reflected answers, Set 21: 1. 5, 6. 25, 11. 282

Set 22 | *For use with page 141*

Find the quotients and remainders.

1. 2)15 7 R1	**11.** 4)234 58 R2	**21.** 9)198 22	**31.** 9)5264 584 R8	**41.** 9)42 345 4705
2. 4)30 7 R2	**12.** 7)537 76 R5	**22.** 4)297 74 R1	**32.** 5)3957 791 R2	**42.** 6)27 386 4564 R2
3. 3)19 6 R1	**13.** 6)498 83	**23.** 5)596 119 R1	**33.** 3)1832 610 R2	**43.** 5)24 730 4946
4. 5)34 6 R4	**14.** 8)637 79 R5	**24.** 8)166 20 R6	**34.** 7)3254 464 R6	**44.** 8)61 723 7715 R3
5. 7)52 7 R3	**15.** 3)224 74 R2	**25.** 3)856 285 R1	**35.** 4)2376 594	**45.** 5)32 176 6435 R1
6. 9)65 7 R2	**16.** 6)465 77 R3	**26.** 8)340 42 R4	**36.** 7)4552 650 R2	**46.** 4)20 593 5148 R1
7. 2)13 6 R1	**17.** 9)710 78 R8	**27.** 8)701 87 R5	**37.** 8)5839 729 R7	**47.** 6)43 741 7290 R1
8. 3)23 7 R2	**18.** 2)157 78 R1	**28.** 6)164 27 R2	**38.** 9)5555 617 R2	**48.** 9)60 592 6732 R4
9. 6)29 4 R5	**19.** 3)408 136	**29.** 3)575 191 R2	**39.** 8)7356 919 R4	**49.** 7)78 700 11 242 R6
10. 5)38 7 R3	**20.** 7)163 23 R2	**30.** 4)437 109 R1	**40.** 4)4859 1214 R3	**50.** 5)44 394 8878 R4

Reflected answers, Set 22: 1. 7R1, 2. 7R2, 11. 58R2, 12. 76R5, 21. 22, 22. 74R1, 31. 584R8, 32. 791R2, 41. 4705, 42. 4564R2

Set 23 *For use with page 145*

Find the quotients and remainders.

1. 40)332 8 R12	**9.** 40)2560 64	**17.** 60)1964 32 R44	**25.** 50)2150 43
2. 70)701 10 R1	**10.** 30)2460 82	**18.** 20)1735 86 R15	**26.** 80)4530 56 R50
3. 20)129 6 R9	**11.** 80)2290 28 R50	**19.** 90)4983 55 R33	**27.** 70)5540 79 R10
4. 60)916 15 R16	**12.** 70)3842 54 R62	**20.** 60)4560 76	**28.** 90)7020 78
5. 50)487 9 R37	**13.** 20)6621 331 R1	**21.** 70)5833 83 R23	**29.** 60)8365 139 R25
6. 80)552 6 R72	**14.** 50)2398 47 R48	**22.** 80)2169 27 R9	**30.** 20)4224 211 R4
7. 30)264 8 R24	**15.** 60)1176 19 R36	**23.** 40)8825 220 R25	**31.** 30)8183 272 R23
8. 90)675 7 R45	**16.** 40)2871 71 R31	**24.** 50)9704 194 R4	**32.** 40)3005 75 R5

Reflected answers, Set 23: 1. 8R12, 2. 10R1, 9. 64, 10. 82, 17. 32R44, 18. 86R15, 25. 43, 26. 56R50

Set 24 *For use with page 157*

Find the quotients and remainders.

1. 29)946 32 R18	**8.** 51)1020 20	**15.** 75)4782 63 R57	**22.** 61)3729 61 R8
2. 39)948 24 R12	**9.** 65)5385 82 R55	**16.** 31)9362 302	**23.** 36)2976 82 R24
3. 58)875 15 R5	**10.** 41)2876 70 R6	**17.** 59)4200 71 R11	**24.** 93)5760 61 R87
4. 42)846 20 R6	**11.** 72)5167 71 R55	**18.** 78)6054 77 R48	**25.** 28)1378 49 R6
5. 38)856 22 R20	**12.** 83)6572 79 R15	**19.** 84)6720 80	**26.** 59)38 562 653 R35
6. 39)978 25 R3	**13.** 35)1050 30	**20.** 42)8735 207 R41	**27.** 72)53 676 745 R36
7. 52)679 13 R3	**14.** 62)4965 80 R5	**21.** 76)6450 84 R66	**28.** 79)48 320 611 R51

Reflected answers, Set 24: 1. 32R18, 2. 24R12, 8. 20, 9. 82R55, 15. 63R57, 16. 302, 22. 61R8, 23. 82R24

Set 25 *For use with page 161*

Find the quotients and remainders. Use the shortcut.

1. 32)256 — 8	**9.** 61)3176 — 52 R4	**17.** 46)3875 — 84 R11	**25.** 63)5463 — 86 R45
2. 43)306 — 7 R5	**10.** 35)2329 — 66 R19	**18.** 29)1672 — 57 R19	**26.** 65)5476 — 84 R16
3. 35)216 — 6 R6	**11.** 64)5462 — 85 R22	**19.** 37)9786 — 264 R18	**27.** 58)4683 — 80 R43
4. 20)147 — 7 R7	**12.** 41)3457 — 84 R13	**20.** 19)1456 — 76 R12	**28.** 29)2465 — 85
5. 58)558 — 9 R36	**13.** 45)1574 — 34 R44	**21.** 72)3972 — 55 R12	**29.** 26)9541 — 366 R25
6. 20)185 — 9 R5	**14.** 35)2234 — 63 R29	**22.** 68)4763 — 70 R3	**30.** 68)5432 — 79 R60
7. 59)236 — 4	**15.** 39)2654 — 68 R2	**23.** 65)2976 — 45 R51	**31.** 54)7490 — 138 R38
8. 37)357 — 9 R24	**16.** 57)3528 — 61 R51	**24.** 52)2934 — 56 R22	**32.** 49)4538 — 92 R30

Reflected answers, Set 25: 1. 8, 2. 7R5, 9. 52R4, 10. 66R19, 17. 84R11, 18. 57R19, 25. 86R45, 26. 84R16

Set 26 *For use with page 163*

Find the quotients and remainders. Check your work.

1. 61)18 625 — 305 R20	**4.** 38)58 364 — 1 535 R34	**7.** 78)23 580 — 302 R24	**10.** 74)29 656 — 400 R56
2. 89)65 871 — 740 R11	**5.** 85)58 403 — 687 R8	**8.** 87)10 055 — 115 R50	**11.** 22)16 387 — 744 R19
3. 26)16 382 — 630 R2	**6.** 92)74 614 — 811 R2	**9.** 94)65 386 — 695 R56	**12.** 18)13 428 — 746

Solve each story problem.

13. If 1152 students want yearbooks, how many dozen yearbooks must be ordered? 96

14. 14,168 people wish to go to a football game. If each bus can hold 28 people, how many buses are needed? 506

Reflected answers, Set 26: 1. 305R20, 4. 1535R34, 7. 302R24, 10. 400R56

Set 27 — *For use with page 165*

Find the answers to these money problems.

1. $\$3.42 \times 5$ = $17.10
2. $\$0.86 \times 6$ = $5.16
3. $\$10.67 \times 8$ = $85.36
4. $\$3.50 \times 34$ = $ 119.00
5. $\$7.35 \times 67$ = $492.45

6. $\$4.31 \times 54$ = $232.74
7. $\$5.17 \times 12$ = $62.04
8. $\$8.06 \times 34$ = $274.04
9. $\$56.30 \times 25$ = $1407.50
10. $\$47.85 \times 56$ = $2679.60

11. $\$20.57 \times 98$ = $2015.86
12. $\$32.70 \times 86$ = $ 2812.20
13. $\$20.42 \times 9$ = $183.78
14. $\$53.02 \times 27$ = $1431.54
15. $\$26.57 \times 74$ = $1966.18

16. 5)$0.55 — $.11
17. 7)$6.30 — $.90
18. 4)$2.88 — $.72
19. 3)$12.15 — $4.05
20. 4)$16.40 — $4.10

21. 29)$1.45 — $.05
22. 51)$4.08 — $.08
23. 75)$4.50 — $.06
24. 65)$4.55 — $.07
25. 31)$2.17 — $.07

26. 31)$18.60 — $. 60
27. 49)$98.98 — $ 2.02
28. 32)$20.48 — $. 64
29. 76)$72.20 — $. 95
30. 23)$10.81 — $.47

Solve the problems.

31. Dave earns $4.95 a week delivering papers. How much will he earn in 6 weeks? $29.70

32. A 12-gram tube of toothpaste costs $0.84. How much is this per gram? $0.07

33. Glenn spends $1.65 each week for lunches. How much does he spend in one month (4 weeks) for lunches? $6.60

34. Mr. Wall paid $2.48 for gas for his car. If he got 14 litres of gas, how much did he pay per litre? $ 0.177 or 17.7¢

35. Nick saves $20.50 each month. How much will he save in one year? $246.00

36. Bill wants to save enough for a $66.40 bicycle in 16 weeks. How much must he save each week? $4.15

Reflected answers, Set 27: 1. $17.10, 2. $5.16, 3. $85.36, 4. $119.00, 5. $492.45, 26. $.60, 27. $2.02, 28. $.64, 29. $.95, 30. $.47

Set 28 — *For use with page 173*

List all the factors of each number.

1. 18 {1, 2, 3, 6, 9, 18} **3.** 40 {1, 2, 4, 5, 8, 10, 20, 40} **5.** 56 {1, 2, 4, 7, 8, 14, 28, 56} **7.** 64 {1, 2, 4, 8, 16, 32, 64} **9.** 72 {1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72}

2. 24 {1, 2, 3, 4, 6, 8, 12, 24} **4.** 54 {1, 2, 3, 6, 9, 18, 27, 54} **6.** 63 {1, 3, 7, 9, 21, 63} **8.** 69 {1, 3, 23, 69} **10.** 75 {1, 3, 5, 15, 25, 75}

Give the missing numbers.

11. If 10 is a factor of a number, then 2 and 5 are factors of that number.

12. If 14 is a factor of a number, then 2 and 7 are factors of that number.

13. If 2 and 11 are factors of a number, then 22 is a factor of the number.

14. If 3 and 5 are factors of a number, then 15 is a factor of the number.

Reflected answers, Set 28: 1. {1, 2, 3, 6, 9, 18}, 3. {1, 2, 4, 5, 8, 10, 20, 40}, 5. {1, 2, 4, 7, 8, 14, 28, 56}, 7. {1, 2, 4, 8, 16, 32, 64}, 9. {1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72}

Set 29 — *For use with page 177*

For each exercise, give the union and the intersection of the two sets.

1. S = {4, 7, 10} S∪T = {2, 4, 7, 10}
T = {2, 4} S∩T = {4}

2. A = {0, 1, 2, 3, 4, 5} A∪B = {0, 1, 2, 3, 4, 5, 6, 8, 10}
B = {2, 4, 6, 8, 10} A∩B = {2, 4}

3. M = {8, 9, 10} M∪N = {8, 9, 10, 12, 13}
N = {9, 10, 12, 13} M∩N = {9, 10}

4. C = {0, 1, 2, 3, 4, 5, 6, 7, 8}
D = {8, 9, 10} C∪D = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
C∩D = {8}

5. Q = {0, 1, 2} Q∪R = {0, 1, 2, 4, 5, 6}
R = {4, 5, 6} Q∩R = { }

6. E = {4, 5, 6, 7, 8, 9, 10}
F = {1, 2, 3, 4} E∪F = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
E∩F = {4}

Reflected answers, Set 29: 1. S ∩ T = {4}, S ∪ T = {2, 4, 7, 10}, 2. A ∩ B = {2, 4}, A ∪ B = {0, 1, 2, 3, 4, 5, 6, 8, 10}

Set 30 | *For use with page 179*

1. List the factors of 10. {1,2,5,10}
2. List the factors of 15. {1,3,5,15}
3. List the common factors of 10 and 15. {1,5}
4. What is the greatest common factor of 10 and 15? 5

5. List the factors of 12. {1,2,3,4,6,12}
6. List the factors of 27. {1,3,9,27}
7. List the common factors of 12 and 27. {1,3}
8. What is the greatest common factor of 12 and 27? 3

Give the greatest common factor of each pair of numbers.

9. 8, 20 4	**12.** 28, 35 7	**15.** 36, 24 12	**18.** 10, 16 2
10. 9, 24 3	**13.** 26, 6 2	**16.** 12, 21 3	**19.** 15, 20 5
11. 4, 17 1	**14.** 14, 28 14	**17.** 2, 13 1	**20.** 18, 30 6

Reflected answers, Set 30: 1. {1, 2, 5, 10}, 2. {1, 3, 5, 15}, 3. {1, 5}, 4. 5

Set 31 | *For use with page 181*

1. List the multiples (to 36) of 6. {0,6,12,18,24,30,36}
2. List the multiples (to 40) of 4. {0,4,8,12,16,20,24,28,32,36,40}
3. List the common multiples of 4 and 6. {0,12,24,36}
4. Give the least common multiple of 4 and 6. 12

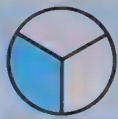
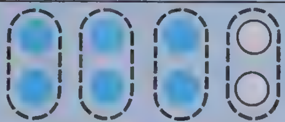
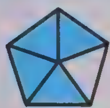
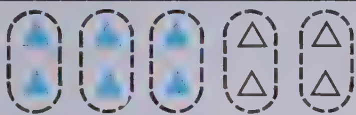
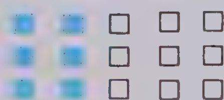
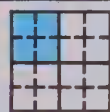
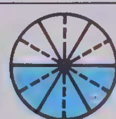
Give the least common multiple for each pair of numbers.

5. 2 and 7 14	**9.** 13 and 3 39	**13.** 10 and 3 30	**17.** 12 and 8 24
6. 8 and 3 24	**10.** 5 and 8 40	**14.** 4 and 12 12	**18.** 2 and 9 18
7. 9 and 5 45	**11.** 2 and 6 6	**15.** 9 and 6 18	**19.** 12 and 2 12
8. 7 and 4 28	**12.** 6 and 7 42	**16.** 10 and 2 10	**20.** 3 and 9 9

Reflected answers, Set 31: 1. {0, 6, 12, 18, 24, 30, 36}, 2. {0, 4, 8, 12, 16, 20, 24, 28, 32, 36, 40}, 3. {0, 12, 24, 36}, 4. 12, 5. 14, 9. 39, 13. 30, 17. 24

Set 32 | *For use with page 191*

Find the missing fractions.

1. $\frac{1}{3}$		____ is equivalent to $\frac{2}{6}$.	
2. $\frac{4}{10}$		$\frac{2}{5}$ is equivalent to ____.	
3. $\frac{3}{4}$, $\frac{6}{8}$		____ is equivalent to ____.	
4. $\frac{4}{5}$, $\frac{8}{10}$		____ is equivalent to ____.	
5. $\frac{5}{6}$, $\frac{10}{12}$		____ is equivalent to ____.	
6. $\frac{3}{5}$, $\frac{6}{10}$		____ is equivalent to ____.	
7. $\frac{3}{7}$, $\frac{6}{14}$		____ is equivalent to ____.	
8. $\frac{2}{5}$, $\frac{6}{15}$		____ is equivalent to ____.	
9. $\frac{1}{4}$, $\frac{4}{16}$		____ is equivalent to ____.	
10. $\frac{3}{6}$, $\frac{6}{12}$		____ is equivalent to ____.	

Reflected answers, Set 32: 1. $\frac{1}{3}$ 2. $\frac{4}{10}$ 3. $\frac{3}{4}$ $\frac{6}{8}$

Set 33 *For use with page 193*

Give the next three fractions in each set.

1. $\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{5}{15}, \ldots\}$ $\frac{6}{18}, \frac{7}{21}, \frac{8}{24}$
2. $\{\frac{2}{5}, \frac{4}{10}, \frac{6}{15}, \frac{8}{20}, \frac{10}{25}, \ldots\}$ $\frac{12}{30}, \frac{14}{35}, \frac{16}{40}$
3. $\{\frac{5}{6}, \frac{10}{12}, \frac{15}{18}, \frac{20}{24}, \frac{25}{30}, \ldots\}$ $\frac{30}{36}, \frac{35}{42}, \frac{40}{48}$
4. $\{\frac{1}{8}, \frac{2}{16}, \frac{3}{24}, \frac{4}{32}, \frac{5}{40}, \ldots\}$ $\frac{6}{48}, \frac{7}{56}, \frac{8}{64}$
5. $\{\frac{3}{7}, \frac{6}{14}, \frac{9}{21}, \frac{12}{28}, \frac{15}{35}, \ldots\}$ $\frac{18}{42}, \frac{21}{49}, \frac{24}{56}$
6. $\{\frac{5}{9}, \frac{10}{18}, \frac{15}{27}, \frac{20}{36}, \frac{25}{45}, \ldots\}$ $\frac{30}{54}, \frac{35}{63}, \frac{40}{72}$
7. $\{\frac{1}{10}, \frac{2}{20}, \frac{3}{30}, \frac{4}{40}, \frac{5}{50}, \ldots\}$ $\frac{6}{60}, \frac{7}{70}, \frac{8}{80}$
8. $\{\frac{7}{10}, \frac{14}{20}, \frac{21}{30}, \frac{28}{40}, \frac{35}{50}, \ldots\}$ $\frac{42}{60}, \frac{49}{70}, \frac{56}{80}$
9. $\{\frac{3}{5}, \frac{6}{10}, \frac{9}{15}, \frac{12}{20}, \frac{15}{25}, \ldots\}$ $\frac{18}{30}, \frac{21}{35}, \frac{24}{40}$
10. $\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}, \ldots\}$ $\frac{18}{24}, \frac{21}{28}, \frac{24}{32}$

Reflected answers, Set 33: 1. $\{\frac{6}{18}, \frac{7}{21}, \frac{8}{24}\}$ 6. $\{\frac{30}{54}, \frac{35}{63}, \frac{40}{72}\}$

Set 34 *For use with page 197*

Which pairs of fractions are equivalent?

1. $\frac{3}{4}, \frac{7}{9}$ Not equivalent
2. $\frac{1}{5}, \frac{20}{100}$ Equivalent
3. $\frac{5}{3}, \frac{25}{15}$ Equivalent
4. $\frac{2}{3}, \frac{12}{15}$ Not equivalent
5. $\frac{4}{7}, \frac{7}{10}$ Not equivalent
6. $\frac{4}{5}, \frac{17}{20}$ Not equivalent
7. $\frac{3}{10}, \frac{30}{100}$ Equivalent
8. $\frac{1}{2}, \frac{3}{5}$ Not equivalent
9. $\frac{8}{3}, \frac{40}{16}$ Not equivalent
10. $\frac{3}{2}, \frac{6}{4}$ Equivalent
11. $\frac{3}{10}, \frac{15}{50}$ Equivalent
12. $\frac{9}{5}, \frac{7}{4}$ Not equivalent
13. $\frac{1}{3}, \frac{3}{8}$ Not equivalent
14. $\frac{8}{5}, \frac{17}{10}$ Not equivalent
15. $\frac{2}{5}, \frac{5}{10}$ Not equivalent
16. $\frac{6}{15}, \frac{4}{10}$ Equivalent
17. $\frac{7}{10}, \frac{13}{20}$ Not equivalent
18. $\frac{8}{6}, \frac{4}{3}$ Equivalent
19. $\frac{9}{3}, \frac{12}{4}$ Equivalent
20. $\frac{15}{20}, \frac{3}{4}$ Equivalent
21. $\frac{24}{20}, \frac{18}{15}$ Equivalent
22. $\frac{12}{5}, \frac{14}{6}$ Not equivalent
23. $\frac{18}{4}, \frac{15}{3}$ Not equivalent
24. $\frac{75}{100}, \frac{15}{20}$ Equivalent
25. $\frac{24}{15}, \frac{16}{10}$ Equivalent
26. $\frac{8}{3}, \frac{14}{5}$ Not equivalent
27. $\frac{17}{9}, \frac{13}{6}$ Not equivalent
28. $\frac{5}{2}, \frac{25}{20}$ Not equivalent
29. $\frac{7}{10}, \frac{35}{50}$ Equivalent
30. $\frac{20}{6}, \frac{50}{15}$ Equivalent

Reflected answers, Set 34: 1. not equivalent, 2. equivalent, 11. equivalent, 12. not equivalent, 21. equivalent, 22. not equivalent

Set 35 | *For use with page 201*

Give the lowest-terms fraction for each of the following:

1. $\frac{5}{25}$ $\frac{1}{5}$	7. $\frac{8}{20}$ $\frac{2}{5}$	13. $\frac{9}{24}$ $\frac{3}{8}$	19. $\frac{12}{20}$ $\frac{3}{5}$	25. $\frac{25}{100}$ $\frac{1}{4}$	31. $\frac{30}{100}$ $\frac{3}{10}$	37. $\frac{5}{10}$ $\frac{1}{2}$
2. $\frac{15}{20}$ $\frac{3}{4}$	8. $\frac{4}{30}$ $\frac{2}{15}$	14. $\frac{9}{15}$ $\frac{3}{5}$	20. $\frac{8}{30}$ $\frac{4}{15}$	26. $\frac{8}{18}$ $\frac{4}{9}$	32. $\frac{12}{30}$ $\frac{2}{5}$	38. $\frac{40}{60}$ $\frac{2}{3}$
3. $\frac{6}{10}$ $\frac{3}{5}$	9. $\frac{8}{24}$ $\frac{1}{3}$	15. $\frac{12}{20}$ $\frac{3}{5}$	21. $\frac{4}{50}$ $\frac{2}{25}$	27. $\frac{4}{20}$ $\frac{1}{5}$	33. $\frac{35}{100}$ $\frac{7}{20}$	39. $\frac{60}{90}$ $\frac{2}{3}$
4. $\frac{3}{30}$ $\frac{1}{10}$	10. $\frac{6}{15}$ $\frac{2}{5}$	16. $\frac{6}{30}$ $\frac{1}{5}$	22. $\frac{8}{10}$ $\frac{4}{5}$	28. $\frac{10}{30}$ $\frac{1}{3}$	34. $\frac{10}{40}$ $\frac{1}{4}$	40. $\frac{15}{30}$ $\frac{1}{2}$
5. $\frac{2}{4}$ $\frac{1}{2}$	11. $\frac{7}{35}$ $\frac{1}{5}$	17. $\frac{25}{100}$ $\frac{1}{4}$	23. $\frac{2}{3}$ $\frac{2}{3}$	29. $\frac{25}{50}$ $\frac{1}{2}$	35. $\frac{5}{20}$ $\frac{1}{4}$	41. $\frac{25}{40}$ $\frac{5}{8}$
6. $\frac{60}{100}$ $\frac{3}{5}$	12. $\frac{5}{30}$ $\frac{1}{6}$	18. $\frac{1}{10}$ $\frac{1}{10}$	24. $\frac{9}{30}$ $\frac{3}{10}$	30. $\frac{70}{100}$ $\frac{7}{10}$	36. $\frac{14}{30}$ $\frac{7}{15}$	42. $\frac{14}{21}$ $\frac{2}{3}$

Reflected answers, Set 35:

Set 36 | *For use with page 211*

Give the correct point for the fractional number that is indicated by the set of equivalent fractions.

1.

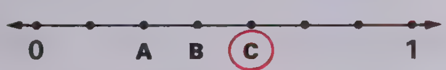

$\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots\}$

2.


$\{\frac{3}{5}, \frac{6}{10}, \frac{9}{15}, \frac{12}{20}, \ldots\}$

3.

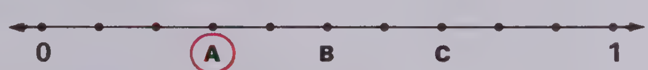

$\{\frac{1}{6}, \frac{2}{12}, \frac{3}{18}, \frac{4}{24}, \ldots\}$

4.


$\{\frac{4}{7}, \frac{8}{14}, \frac{12}{21}, \frac{16}{28}, \ldots\}$

5.


$\{\frac{7}{8}, \frac{14}{16}, \frac{21}{24}, \frac{28}{32}, \ldots\}$

6.


$\{\frac{3}{10}, \frac{6}{20}, \frac{9}{30}, \frac{12}{40}, \ldots\}$

Reflected answers, Set 36:

Set 37 — *For use with page 213*

Give the missing numerator or denominator.

1. $\frac{1}{2} = \frac{\square}{4}$ 2	**6.** $\frac{3}{8} = \frac{6}{\square}$ 16	**11.** $\frac{1}{4} = \frac{\square}{20}$ 5	**16.** $\frac{5}{7} = \frac{10}{\square}$ 14	**21.** $\frac{1}{10} = \frac{\square}{10}$ 1
2. $\frac{1}{10} = \frac{5}{\square}$ 50	**7.** $\frac{2}{3} = \frac{\square}{9}$ 6	**12.** $\frac{7}{10} = \frac{14}{\square}$ 20	**17.** $\frac{3}{10} = \frac{\square}{20}$ 6	**22.** $\frac{4}{5} = \frac{12}{\square}$ 15
3. $\frac{1}{7} = \frac{\square}{14}$ 2	**8.** $\frac{3}{5} = \frac{6}{\square}$ 10	**13.** $\frac{1}{6} = \frac{\square}{30}$ 5	**18.** $\frac{7}{10} = \frac{35}{\square}$ 50	**23.** $\frac{1}{4} = \frac{\square}{4}$ 1
4. $\frac{1}{3} = \frac{5}{\square}$ 15	**9.** $\frac{3}{4} = \frac{\square}{100}$ 75	**14.** $\frac{5}{10} = \frac{10}{\square}$ 20	**19.** $\frac{1}{8} = \frac{\square}{16}$ 2	**24.** $\frac{7}{10} = \frac{21}{\square}$ 30
5. $\frac{2}{5} = \frac{\square}{15}$ 6	**10.** $\frac{1}{5} = \frac{4}{\square}$ 20	**15.** $\frac{4}{5} = \frac{\square}{25}$ 20	**20.** $\frac{1}{9} = \frac{\square}{27}$ 3	**25.** $\frac{2}{5} = \frac{\square}{20}$ 8

Reflected answers, Set 37: 1. 2 6. 3 11. 5 16. 14 21. 1

Set 38 — *For use with page 215*

Give the correct sign ($>$, $=$, or $<$) for each $\square$.

1. $\frac{1}{4} \square \frac{1}{3}$ $<$	**11.** $\frac{3}{4} \square \frac{9}{10}$ $<$	**21.** $\frac{2}{3} \square \frac{3}{4}$ $<$	**31.** $\frac{28}{50} \square \frac{1}{2}$ $>$
2. $\frac{1}{2} \square \frac{2}{5}$ $>$	**12.** $\frac{3}{5} \square \frac{2}{3}$ $<$	**22.** $\frac{3}{5} \square \frac{7}{10}$ $<$	**32.** $\frac{5}{30} \square \frac{1}{6}$ $=$
3. $\frac{1}{10} \square \frac{1}{4}$ $<$	**13.** $\frac{1}{2} \square \frac{50}{100}$ $=$	**23.** $\frac{25}{100} \square \frac{1}{4}$ $=$	**33.** $\frac{1}{2} \square \frac{1}{3}$ $>$
4. $\frac{1}{4} \square \frac{2}{10}$ $>$	**14.** $\frac{1}{3} \square \frac{5}{20}$ $>$	**24.** $\frac{1}{2} \square \frac{5}{8}$ $<$	**34.** $\frac{1}{4} \square \frac{2}{8}$ $=$
5. $\frac{7}{8} \square \frac{3}{4}$ $>$	**15.** $\frac{7}{8} \square \frac{3}{4}$ $>$	**25.** $\frac{1}{3} \square \frac{2}{6}$ $=$	**35.** $\frac{10}{30} \square \frac{3}{9}$ $=$
6. $\frac{1}{3} \square \frac{4}{6}$ $<$	**16.** $\frac{10}{20} \square \frac{4}{8}$ $=$	**26.** $\frac{2}{3} \square \frac{5}{6}$ $<$	**36.** $\frac{3}{4} \square \frac{3}{8}$ $>$
7. $\frac{4}{9} \square \frac{5}{8}$ $<$	**17.** $\frac{5}{8} \square \frac{3}{8}$ $>$	**27.** $\frac{9}{10} \square \frac{4}{5}$ $>$	**37.** $\frac{2}{3} \square \frac{3}{5}$ $>$
8. $\frac{1}{3} \square \frac{2}{5}$ $<$	**18.** $\frac{3}{10} \square \frac{1}{3}$ $<$	**28.** $\frac{3}{4} \square \frac{15}{20}$ $=$	**38.** $\frac{7}{10} \square \frac{3}{4}$ $<$
9. $\frac{3}{4} \square \frac{15}{20}$ $=$	**19.** $\frac{5}{6} \square \frac{4}{5}$ $>$	**29.** $\frac{4}{5} \square \frac{11}{15}$ $>$	**39.** $\frac{1}{3} \square \frac{2}{5}$ $<$
10. $\frac{3}{5} \square \frac{9}{15}$ $=$	**20.** $\frac{4}{7} \square \frac{3}{4}$ $<$	**30.** $\frac{3}{8} \square \frac{2}{5}$ $<$	**40.** $\frac{2}{3} \square \frac{5}{7}$ $<$

Reflected answers, Set 38: 1. $<$ 2. $>$ 11. $<$ 12. $<$ 21. $<$ 22. $<$ 31. $>$ 32. $=$

Set 39 *For use with page 217*

Solve each short story problem.

1. Gail's hand: 9 centimetres long.
   Mark's hand: 1 decimetre.
   Who has the longer hand? Mark

2. Henry ate 4 eggs.
   Angie ate $\frac{1}{4}$ dozen eggs.
   Who ate more eggs? Henry

3. Bob ate dinner in $\frac{1}{4}$ hour.
   It took Alex 20 minutes to eat dinner. Who ate faster? Bob

4. Ann bought $\frac{1}{2}$ metre of material.
   Joan bought $\frac{3}{5}$ metre.
   Who bought more material? Joan

5. Frank worked $\frac{3}{4}$ hour.
   Pete worked 50 minutes.
   Who worked longer? Pete

6. Sam read $\frac{3}{8}$ of his book.
   Tony read $\frac{1}{2}$ of the same book.
   Who read more? Tony

Reflected answers, Set 39: 1. Mark 2. Henry

Set 40 *For use with page 219*

Solve each story problem.

1. The ratio of boys to girls is 2 to 3. There are 15 girls.
   How many boys are there? 10

2. The ratio of tires to cars is 4 to 1. There are 24 tires.
   How many cars are there? 6

3. The ratio of tables to chairs is 1 to 6. There are 5 tables.
   How many chairs are there? 30

4. The ratio of swings to students is 4 to 15. There are 30 students. How many swings are there? 8

5. The ratio of eggs to cartons is 12 to 1. There are 5 cartons.
   How many eggs are there? 60

6. The ratio of dogs to bones is 3 to 8. There are 9 dogs.
   How many bones are there? 24

Reflected answers, Set 40: 1. 10 2. 6

Set 41 — *For use with page 227*

Find the sums and differences.

1. $\frac{1}{5} + \frac{3}{5}$ $\frac{4}{5}$
2. $\frac{7}{8} + \frac{7}{8}$ $\frac{14}{8}$ or $1\frac{3}{4}$
3. $\frac{6}{4} - \frac{5}{4}$ $\frac{1}{4}$
4. $\frac{4}{25} + \frac{7}{25}$ $\frac{11}{25}$
5. $\frac{4}{7} + \frac{5}{7}$ $\frac{9}{7}$ or $1\frac{2}{7}$
6. $\frac{6}{10} - \frac{1}{10}$ $\frac{5}{10}$ or $\frac{1}{2}$
7. $\frac{7}{8} + \frac{4}{8}$ $\frac{11}{8}$ or $1\frac{3}{8}$
8. $\frac{8}{30} + \frac{2}{30}$ $\frac{10}{30}$ or $\frac{1}{3}$
9. $\frac{3}{8} + \frac{2}{8}$ $\frac{5}{8}$
10. $\frac{21}{100} + \frac{19}{100}$ $\frac{40}{100}$ or $\frac{2}{5}$
11. $\frac{3}{9} - \frac{2}{9}$ $\frac{1}{9}$
12. $\frac{7}{25} - \frac{4}{25}$ $\frac{3}{25}$
13. $\frac{6}{7} - \frac{1}{7}$ $\frac{5}{7}$
14. $\frac{7}{15} - \frac{5}{15}$ $\frac{2}{15}$
15. $\frac{7}{10} + \frac{9}{10}$ $\frac{16}{10}$ or $1\frac{3}{5}$
16. $\frac{2}{15} + \frac{12}{15}$ $\frac{14}{15}$
17. $\frac{7}{10} - \frac{6}{10}$ $\frac{1}{10}$
18. $\frac{2}{10} + \frac{7}{10}$ $\frac{9}{10}$
19. $\frac{3}{5} + \frac{3}{5}$ $\frac{6}{5}$ or $1\frac{1}{5}$
20. $\frac{4}{5} - \frac{1}{5}$ $\frac{3}{5}$
21. $\frac{6}{5} - \frac{2}{5}$ $\frac{4}{5}$
22. $\frac{9}{20} - \frac{4}{20}$ $\frac{5}{20}$ or $\frac{1}{4}$
23. $\frac{25}{100} - \frac{20}{100}$ $\frac{5}{100}$ or $\frac{1}{20}$
24. $\frac{3}{10} + \frac{3}{10}$ $\frac{6}{10}$ or $\frac{3}{5}$
25. $\frac{9}{20} + \frac{7}{20}$ $\frac{16}{20}$ or $\frac{4}{5}$

Solve each short story problem.

26. Drove $\frac{3}{10}$ kilometre.
then drove $\frac{6}{10}$ kilometre farther.
Drove how far? $\frac{9}{10}$ km

27. Read $\frac{3}{8}$ of a book,
then read $\frac{1}{8}$ more. Read
how much of the book? $\frac{4}{8}$ or $\frac{1}{2}$

Reflected answers, Set 41: 1. $\frac{4}{5}$, 6. $\frac{5}{10}$ or $\frac{1}{2}$, 11. $\frac{1}{9}$, 16. $\frac{14}{15}$, 21. $\frac{4}{5}$

Set 42 — *For use with page 231*

Make lists of equivalent fractions until you find 2 with the same denominator.
Then find the sum or difference.

1. $\frac{1}{3} + \frac{1}{6}$ $\frac{3}{6}$ or $\frac{1}{2}$
2. $\frac{1}{3} - \frac{1}{6}$ $\frac{1}{6}$
3. $\frac{1}{2} + \frac{1}{4}$ $\frac{3}{4}$
4. $\frac{1}{2} - \frac{1}{4}$ $\frac{1}{4}$
5. $\frac{3}{10} + \frac{1}{4}$ $\frac{11}{20}$
6. $\frac{3}{10} - \frac{1}{4}$ $\frac{1}{20}$
7. $\frac{3}{4} + \frac{1}{8}$ $\frac{7}{8}$
8. $\frac{3}{4} - \frac{1}{8}$ $\frac{5}{8}$
9. $\frac{1}{4} + \frac{3}{8}$ $\frac{5}{8}$
10. $\frac{3}{8} - \frac{1}{4}$ $\frac{1}{8}$
11. $\frac{2}{3} - \frac{1}{5}$ $\frac{7}{15}$
12. $\frac{2}{3} + \frac{1}{5}$ $\frac{13}{15}$
13. $\frac{2}{3} - \frac{2}{9}$ $\frac{4}{9}$
14. $\frac{1}{2} + \frac{1}{10}$ $\frac{6}{10}$ or $\frac{3}{5}$
15. $\frac{1}{2} - \frac{1}{10}$ $\frac{4}{10}$ or $\frac{2}{5}$
16. $\frac{4}{5} + \frac{1}{10}$ $\frac{9}{10}$
17. $\frac{4}{5} - \frac{1}{10}$ $\frac{7}{10}$
18. $\frac{2}{3} - \frac{1}{2}$ $\frac{1}{6}$
19. $\frac{2}{3} + \frac{1}{2}$ $\frac{7}{6}$ or $1\frac{1}{6}$
20. $\frac{3}{5} + \frac{1}{10}$ $\frac{7}{10}$
21. $\frac{3}{5} - \frac{1}{10}$ $\frac{1}{2}$

Reflected answers, Set 42: 1. $\frac{3}{6}$ or $\frac{1}{2}$, 4. $\frac{1}{4}$, 7. $\frac{7}{8}$, 10. $\frac{1}{8}$, 13. $\frac{4}{9}$

Set 43	*For use with page 235*

Give the missing numerators.

1. $\frac{2}{5} = \frac{\square}{10}$ 4
3. $\frac{2}{3} = \frac{\square}{15}$ 10
5. $\frac{3}{10} = \frac{\square}{50}$ 15
7. $\frac{1}{2} = \frac{\square}{6}$ 3
9. $\frac{1}{5} = \frac{\square}{100}$ 20
2. $\frac{3}{5} = \frac{\square}{15}$ 9
4. $\frac{1}{2} = \frac{\square}{20}$ 10
6. $\frac{3}{5} = \frac{\square}{15}$ 9
8. $\frac{4}{10} = \frac{\square}{30}$ 12
10. $\frac{2}{5} = \frac{\square}{50}$ 20

Find the sums and differences.

11. $\frac{1}{4} + \frac{1}{10}$ $\frac{7}{20}$
12. $\frac{1}{4} - \frac{1}{10}$ $\frac{3}{20}$
13. $\frac{1}{4} + \frac{1}{6}$ $\frac{5}{12}$
14. $\frac{1}{4} - \frac{1}{6}$ $\frac{1}{12}$
15. $\frac{3}{10} + \frac{7}{20}$ $\frac{13}{20}$
16. $\frac{7}{20} - \frac{3}{10}$ $\frac{1}{20}$
17. $\frac{7}{10} - \frac{1}{4}$ $\frac{9}{20}$
18. $\frac{1}{4} + \frac{7}{10}$ $\frac{19}{20}$
19. $\frac{2}{3} - \frac{1}{2}$ $\frac{1}{6}$
20. $\frac{2}{3} + \frac{1}{2}$ $\frac{7}{6}$ or $1\frac{1}{6}$
21. $\frac{3}{8} - \frac{1}{4}$ $\frac{1}{8}$
22. $\frac{1}{4} + \frac{3}{8}$ $\frac{5}{8}$
23. $\frac{1}{2} + \frac{1}{6}$ $\frac{4}{6}$ or $\frac{2}{3}$
24. $\frac{3}{4} - \frac{1}{8}$ $\frac{5}{8}$
25. $\frac{7}{10} + \frac{3}{5}$ $\frac{13}{10}$ or $1\frac{3}{10}$
26. $\frac{7}{10} - \frac{3}{5}$ $\frac{1}{10}$
27. $\frac{3}{10} + \frac{41}{100}$ $\frac{71}{100}$
28. $\frac{8}{10} - \frac{7}{100}$ $\frac{73}{100}$
29. $\frac{1}{3} - \frac{1}{6}$ $\frac{1}{6}$
31. $\frac{5}{6} + \frac{2}{6}$ $\frac{18}{\text{}}$ or $1\frac{1}{3}$
33. $\frac{1}{4} + \frac{7}{8}$ $\frac{9}{8}$ or $1\frac{1}{8}$
35. $\frac{1}{3} + \frac{2}{9}$ $\frac{5}{9}$
37. $\frac{50}{100} - \frac{2}{10}$ $\frac{30}{100}$ or $\frac{3}{10}$
30. $\frac{1}{4} + \frac{1}{8}$ $\frac{3}{8}$
32. $\frac{2}{3} - \frac{2}{5}$ $\frac{4}{15}$
34. $\frac{1}{4} - \frac{1}{5}$ $\frac{1}{20}$
36. $\frac{7}{9} + \frac{2}{3}$ $\frac{13}{9}$ or $1\frac{4}{9}$
38. $\frac{2}{10} - \frac{18}{100}$ $\frac{2}{100}$ or $\frac{1}{50}$

Solve each short story problem.

39. $\frac{1}{3}$ litre milk.
$\frac{1}{2}$ litre water.
How much liquid in all? $\frac{5}{6}$ litre

40. Had $\frac{3}{4}$ box of cookies.
Ate $\frac{3}{8}$ box of cookies. What part of the box is left? $\frac{3}{8}$ box

41. John ran $\frac{7}{10}$ kilometre.
Jim ran $\frac{1}{2}$ kilometre.
How much farther did John run? $\frac{1}{5}$ km

42. Andy played in $\frac{5}{8}$ of the football game. David played in $\frac{1}{4}$ of the game. How much more did Andy play? $\frac{3}{8}$ game

16. $\frac{1}{20}$, 39. $\frac{5}{6}$ LITRE, 40. $\frac{3}{8}$ BOX
11. $\frac{7}{20}$, 12. $\frac{3}{20}$, 13. $\frac{5}{12}$, 14. $\frac{1}{12}$, 15. $\frac{13}{20}$
Reflected answers, Set 43: 1. 4, 3. 10, 5. 15, 7. 3, 9. 20

Set 44 *For use with page 239*

Change each mixed numeral to an improper fraction.

1. $6\frac{2}{3}$ $\frac{20}{3}$ 3. $1\frac{6}{7}$ $\frac{13}{7}$ 5. $5\frac{1}{2}$ $\frac{11}{2}$ 7. $1\frac{3}{5}$ $\frac{8}{5}$ 9. $3\frac{1}{8}$ $\frac{25}{8}$ 11. $4\frac{3}{10}$ $\frac{43}{10}$

2. $3\frac{7}{10}$ $\frac{37}{10}$ 4. $4\frac{45}{100}$ $\frac{445}{100}$ 6. $7\frac{1}{5}$ $\frac{36}{5}$ 8. $9\frac{2}{5}$ $\frac{47}{5}$ 10. $1\frac{9}{10}$ $\frac{19}{10}$ 12. $2\frac{3}{4}$ $\frac{11}{4}$

Change each improper fraction to a mixed numeral.

13. $\frac{36}{5}$ $7\frac{1}{5}$ 15. $\frac{17}{4}$ $4\frac{1}{4}$ 17. $\frac{23}{10}$ $2\frac{3}{10}$ 19. $\frac{143}{100}$ $1\frac{43}{100}$ 21. $\frac{34}{7}$ $4\frac{6}{7}$ 23. $\frac{25}{3}$ $8\frac{1}{3}$

14. $\frac{20}{7}$ $2\frac{6}{7}$ 16. $\frac{27}{4}$ $6\frac{3}{4}$ 18. $\frac{15}{4}$ $3\frac{3}{4}$ 20. $\frac{59}{9}$ $6\frac{5}{9}$ 22. $\frac{52}{10}$ $5\frac{2}{10}$ or $5\frac{1}{5}$ 24. $\frac{26}{3}$ $8\frac{2}{3}$

Solve each short story problem.

25. Bought $\frac{3}{4}$ kg of apples and $\frac{3}{4}$ kg of oranges. Bought how many kilograms of fruit? $\frac{6}{4}$ or $1\frac{1}{2}$ kg

26. Exercised $\frac{5}{6}$ of an hour on Monday and $\frac{2}{3}$ of an hour on Tuesday. Exercised for how long on these two days? $\frac{9}{6}$ or $1\frac{1}{2}$ h

Reflected answers, Set 44: 1. $\frac{3}{50}$ ... 15. $4\frac{1}{4}$ 17. $2\frac{3}{10}$ ... 21. $4\frac{6}{7}$ 23. $8\frac{1}{3}$

Set 45 *For use with page 241*

Find the sums. Give them in lowest terms.

1. $4\frac{1}{4} + 7\frac{1}{2}$ $11\frac{3}{4}$
2. $8\frac{1}{6} + 2\frac{1}{3}$ $10\frac{1}{2}$
3. $5\frac{3}{4} + 4\frac{1}{8}$ $9\frac{7}{8}$
4. $3\frac{1}{2} + 2\frac{1}{3}$ $5\frac{5}{6}$
5. $6\frac{1}{3} + 5\frac{1}{6}$ $11\frac{1}{2}$
6. $4\frac{3}{10} + 3\frac{1}{5}$ $7\frac{1}{2}$
7. $6\frac{7}{10} + 1\frac{1}{2}$ $8\frac{1}{5}$
8. $7\frac{1}{2} + 2\frac{1}{4}$ $9\frac{3}{4}$
9. $6\frac{3}{10} + 4\frac{1}{5}$ $10\frac{1}{2}$
10. $6\frac{1}{6} + 1\frac{3}{4}$ $7\frac{11}{12}$
11. $8\frac{1}{5} + 7\frac{1}{4}$ $15\frac{9}{20}$
12. $4\frac{2}{3} + 2\frac{1}{5}$ $6\frac{13}{15}$

Reflected answers, Set 45: 1. $11\frac{3}{4}$ 2. $10\frac{1}{2}$ 3. $9\frac{7}{8}$ 4. $5\frac{5}{6}$ 5. $11\frac{1}{2}$ 6. $7\frac{1}{2}$

Set 46 — *For use with page 243*

Solve each short story problem.

1. $4\frac{1}{3}$ hours going.
   $5\frac{1}{6}$ hours returning.
   How long for entire trip? $9\frac{3}{6}$ or $9\frac{1}{2}$ h

2. $2\frac{112}{1000}$ kg of peanuts.
   $1\frac{138}{1000}$ kg of almonds.
   How many kilograms of nuts? $3\frac{250}{1000}$ or $3\frac{1}{4}$ kg

3. $\frac{7}{10}$ cm of rain on Saturday.
   $\frac{3}{5}$ cm of rain on Sunday.
   How much rain in all? $\frac{13}{10}$ or $1\frac{3}{10}$ cm

4. Ran $\frac{1}{2}$ kilometre.
   Walked $\frac{9}{10}$ km more.
   How far in all? $\frac{14}{10}$ km or $1\frac{4}{10}$ km

5. Lost $1\frac{1}{2}$ kilograms one week
   and $\frac{3}{4}$ kilograms the next week.
   Lost how much in all? $1\frac{5}{4}$ or $2\frac{1}{4}$ kg

6. Drank $2\frac{1}{2}$ cups of milk
   and $1\frac{3}{4}$ cups of water.
   Drank how much in all? $3\frac{5}{4}$ or $4\frac{1}{4}$ cups

Reflected answers, Set 46:

Set 47 — *For use with page 245*

Find the differences. Give them in lowest terms.

1. $6\frac{7}{10} - 1\frac{1}{2}$ = $5\frac{1}{5}$	2. $7\frac{3}{5} - 2\frac{1}{4}$ = $5\frac{7}{20}$	3. $6\frac{7}{20} - 4\frac{1}{10}$ = $2\frac{1}{4}$	4. $8\frac{3}{8} - 7\frac{1}{4}$ = $1\frac{1}{8}$	5. $9\frac{9}{10} - 6\frac{1}{5}$ = $3\frac{7}{10}$	6. $5\frac{1}{2} - 2\frac{1}{4}$ = $3\frac{1}{4}$
7. $6\frac{1}{3} - 3\frac{1}{6}$ = $3\frac{1}{6}$	8. $9\frac{4}{5} - 1\frac{3}{10}$ = $8\frac{1}{2}$	9. $5\frac{4}{5} - 4\frac{1}{10}$ = $1\frac{7}{10}$	10. $9\frac{1}{3} - 8\frac{1}{4}$ = $1\frac{1}{12}$	11. $7\frac{2}{5} - 5\frac{1}{4}$ = $2\frac{3}{20}$	12. $3\frac{3}{5} - 1\frac{1}{10}$ = $2\frac{1}{2}$
13. $8\frac{3}{4} - 3\frac{3}{5}$ = $5\frac{3}{20}$	14. $7\frac{7}{10} - 6\frac{3}{5}$ = $1\frac{1}{10}$	15. $9\frac{2}{3} - 4\frac{1}{4}$ = $5\frac{5}{12}$	16. $6\frac{3}{4} - 1\frac{1}{6}$ = $5\frac{7}{12}$	17. $7\frac{3}{4} - 7\frac{1}{5}$ = $\frac{11}{20}$	18. $7\frac{7}{10} - 1\frac{1}{4}$ = $6\frac{9}{20}$
19. $9\frac{2}{5} - 2\frac{4}{15}$ = $7\frac{2}{15}$	20. $6\frac{21}{50} - 1\frac{1}{100}$ = $5\frac{41}{100}$	21. $7\frac{1}{4} - 3\frac{1}{8}$ = $4\frac{1}{8}$	22. $8\frac{23}{50} - 4\frac{17}{100}$ = $4\frac{29}{100}$	23. $5\frac{63}{100} - 3\frac{241}{1000}$ = $2\frac{389}{1000}$	24. $9\frac{47}{50} - 8\frac{81}{100}$ = $1\frac{13}{100}$

Reflected answers, Set 47:

Set 48 | *For use with page 253*

Give the missing numerator or denominator.

1. $7.2 = 7 + \frac{\square}{10}$ 2
2. $8.1 = 8 + \frac{1}{\square}$ 10
3. $28.3 = 28 + \frac{3}{\square}$ 10
4. $7.4 = 7 + \frac{\square}{10}$ 4
5. $2.6 = 2 + \frac{6}{\square}$ 10
6. $43.3 = 43 + \frac{\square}{10}$ 3
7. $8.2 = 8 + \frac{2}{\square}$ 10
8. $3.5 = 3 + \frac{5}{\square}$ 10
9. $79.4 = 79 + \frac{\square}{10}$ 4
10. $9.6 = 9 + \frac{6}{\square}$ 10
11. $6.2 = 6 + \frac{\square}{10}$ 2
12. $34.7 = 34 + \frac{7}{\square}$ 10

Give the correct decimal for each sum.

13. $3 + \frac{4}{10}$ 3.4
14. $26 + \frac{7}{10}$ 26.7
15. $41 + \frac{3}{10}$ 41.3
16. $86 + \frac{2}{10}$ 86.2
17. $45 + \frac{1}{10}$ 45.1
18. $9 + \frac{7}{10}$ 9.7
19. $3 + \frac{8}{10}$ 3.8
20. $15 + \frac{2}{10}$ 15.2
21. $17 + \frac{6}{10}$ 17.6
22. $19 + \frac{4}{10}$ 19.4
23. $26 + \frac{5}{10}$ 26.5
24. $42 + \frac{3}{10}$ 42.3
25. $91 + \frac{5}{10}$ 91.5
26. $36 + \frac{6}{10}$ 36.6
27. $21 + \frac{4}{10}$ 21.4
28. $62 + \frac{8}{10}$ 62.8

Reflected answers, Set 48: 1. 2, 5. 10, 9. 4, 13. 3.4, 17. 45.1, 21. 17.6, 25. 91.5

Set 49 | *For use with page 255*

Write each fractional number as in the example.
(Example: $9.362 = 9 + \frac{3}{10} + \frac{6}{100} + \frac{2}{1000}$)

1. 2.61 $2+\frac{6}{10}+\frac{1}{100}$
2. 3.82 $3+\frac{8}{10}+\frac{2}{100}$
3. 6.76 $6+\frac{7}{10}+\frac{6}{100}$
4. 81.6 $81+\frac{6}{10}$
5. 6.76 $6+\frac{7}{10}+\frac{6}{100}$
6. 62.31 $62+\frac{3}{10}+\frac{1}{100}$
7. 47.63 $47+\frac{6}{10}+\frac{3}{100}$
8. 48.74 $48+\frac{7}{10}+\frac{4}{100}$
9. 3.892 $3+\frac{8}{10}+\frac{9}{100}+\frac{2}{1000}$
10. 4.956 $4+\frac{9}{10}+\frac{5}{100}+\frac{6}{1000}$
11. 19.943 $19+\frac{9}{10}+\frac{4}{100}+\frac{3}{1000}$
12. 4.03 $4+\frac{3}{100}$
13. 3.057
14. 49.12
15. 16.804

(13) $3+\frac{5}{100}+\frac{7}{1000}$
(14) $49+\frac{1}{10}+\frac{2}{100}$
(15) $16+\frac{8}{10}+\frac{4}{1000}$

Give the correct decimal for each sum.

16. $8 + \frac{7}{10}$ 8.7
17. $9 + \frac{4}{10}$ 9.4
18. $5 + \frac{3}{10} + \frac{2}{100}$ 5.32
19. $7 + \frac{4}{10} + \frac{8}{100}$ 7.48
20. $12 + \frac{1}{10} + \frac{3}{100}$ 12.13
21. $15 + \frac{3}{10} + \frac{6}{100}$ 15.36
22. $24 + \frac{4}{10} + \frac{3}{100}$ 24.43
23. $16 + \frac{0}{10} + \frac{2}{100}$ 16.02
24. $8 + \frac{3}{10} + \frac{0}{100} + \frac{4}{1000}$ 8.304
25. $72 + \frac{4}{10} + \frac{2}{100} + \frac{2}{1000}$ 72.422
26. $41 + \frac{7}{10} + \frac{3}{100} + \frac{5}{1000}$ 41.735
27. $4 + \frac{0}{10} + \frac{0}{100} + \frac{3}{1000}$ 4.003

Reflected answers, Set 49: 16. 8.7, 20. 12.13, 24. 8.304

Set 50 — *For use with page 257*

Give the correct sign (<, =, or >) for each ●.

1. 83.2 ● 83.6 <	6. 43.4 ● 44.3 <	11. 7.634 ● 7.635 <
2. 42.6 ● 46.2 <	7. 7.15 ● 7.14 >	12. 4.158 ● 4.258 <
3. 16.7 ● 16.7 =	8. 5.32 ● 5.22 >	13. 3.749 ● 3.739 >
4. 65.8 ● 64.4 >	9. 7.08 ● 7.06 >	14. 2.584 ● 2.684 <
5. 41.9 ● 40.9 >	10. 6.16 ● 6.26 <	15. 3.629 ● 3.628 >

Find the sums and differences.

16. $\frac{5}{10} + \frac{4}{10}$ $\frac{9}{10}$

17. $0.5 + 0.4$ 0.9

18. $\frac{9}{10} - \frac{6}{10}$ $\frac{3}{10}$

19. $0.9 - 0.6$ 0.3

20. $\frac{53}{100} - \frac{25}{100}$ $\frac{28}{100}$ or $\frac{7}{25}$

21. $0.53 - 0.25$ 0.28

Reflected answers, Set 50: 1. < 6. < 11. < 16. $\frac{9}{10}$ 17. 0.9 18. $\frac{3}{10}$ 19. 0.3 20. $\frac{28}{100}$ or $\frac{7}{25}$ 21. 0.28

Set 51 — *For use with page 259*

Find the sums and differences.

1. $0.5 + 0.7$ = 1.2	2. $0.8 + 0.6$ = 1.4	3. $1.9 - 0.7$ = 1.2	4. $2.4 - 0.6$ = 1.8	5. $47.6 + 32.9$ = 80.5
6. $0.34 + 0.25$ = 0.59	7. $13.5 - 7.3$ = 6.2	8. $2.5 + 3.8$ = 6.3	9. $29.6 - 19.8$ = 9.8	10. $3.43 + 5.57$ = 9.00
11. $24.39 + 8.7$ = 33.09	12. $6.34 - 4.57$ = 1.77	13. $8.63 + 7.29$ = 15.92	14. $15.28 - 7.63$ = 7.65	15. $70.91 - 12.86$ = 58.05
16. $0.683 + 0.785$ = 1.468	17. $0.902 - 0.610$ = 0.292	18. $8.771 - 5.847$ = 2.924	19. $21.446 - 0.008$ = 21.438	20. $4.844 - 0.369$ = 4.475

Reflected answers, Set 51: 1. 1.2 2. 1.4 3. 1.2 4. 1.8 5. 80.5

Set 52 *For use with page 261*

Find the total amounts.

1. $8.76 + 4.32 = $13.08
2. $10.39 + 4.61 = $15.00
3. $4.98 + 3.49 = $8.47
4. $36.50 + 9.98 = $46.48
5. $57.86 + 29.98 = $87.84
6. $6.36 + 2.83 = $9.19
7. $15.49 + 4.51 = $20.00
8. $7.49 + 8.89 = $16.38
9. $49.99 + 23.46 = $73.45
10. $58.67 + 46.98 = $105.65
11. $104.95 + 87.75 = $192.70
12. $69.99 + 69.99 = $139.98
13. $127.49 + 98.98 = $226.47
14. $89.98 + 89.98 = $179.96
15. $721.11 + 34.87 = $755.98

Find the differences in the amounts.

16. $5.65 − 3.27 = $2.38
17. $6.57 − 4.92 = $1.65
18. $56.30 − 27.88 = $28.42
19. $64.50 − 3.98 = $60.52
20. $36.50 − 17.98 = $18.52
21. $29.45 − 16.59 = $12.86
22. $104.25 − 67.48 = $36.77
23. $110.07 − 39.95 = $70.12
24. $365.98 − 299.99 = $65.99
25. $532.48 − 398.99 = $133.49

Reflected answers, Set 52: 1. $13.08, 2. $15.00, 3. $8.47, 4. $46.48, 5. $87.84, 16. $2.38, 17. $1.65, 18. $28.42, 19. $60.52, 20. $18.52

Set 53 *For use with page 293*

Give the products.

1. $7 \times \frac{1}{5}$ $\frac{7}{5}$ or $1\frac{2}{5}$
2. $8 \times \frac{1}{7}$ $\frac{8}{7}$ or $1\frac{1}{7}$
3. $3 \times \frac{1}{6}$ $\frac{3}{6}$ or $\frac{1}{2}$
4. $\frac{1}{3} \times 9$ $\frac{9}{3}$ or 3
5. $\frac{1}{5} \times 4$ $\frac{4}{5}$
6. $\frac{1}{8} \times 7$ $\frac{7}{8}$
7. $\frac{1}{3} \times \frac{1}{7}$ $\frac{1}{21}$
8. $\frac{1}{5} \times \frac{1}{8}$ $\frac{1}{40}$
9. $\frac{1}{6} \times \frac{1}{7}$ $\frac{1}{42}$
10. $\frac{1}{9} \times \frac{1}{10}$ $\frac{1}{90}$
11. $\frac{1}{3} \times \frac{4}{5}$ $\frac{4}{15}$
12. $\frac{2}{3} \times \frac{4}{7}$ $\frac{8}{21}$
13. $\frac{2}{2} \times \frac{2}{5}$ $\frac{4}{10}$ or $\frac{2}{5}$
14. $\frac{3}{9} \times \frac{6}{5}$ $\frac{18}{45}$ or $\frac{2}{5}$
15. $\frac{2}{3} \times \frac{1}{7}$ $\frac{2}{21}$
16. $\frac{3}{4} \times \frac{7}{5}$ $\frac{21}{20}$ or $1\frac{1}{20}$
17. $\frac{5}{6} \times \frac{4}{9}$ $\frac{20}{54}$ or $\frac{10}{27}$
18. $\frac{5}{8} \times \frac{3}{3}$ $\frac{15}{24}$ or $\frac{5}{8}$
19. $\frac{3}{7} \times \frac{5}{10}$ $\frac{15}{70}$ or $\frac{3}{14}$
20. $\frac{1}{8} \times \frac{3}{5}$ $\frac{3}{40}$

Reflected answers, Set 53: 1. $\frac{7}{5}$ or $1\frac{2}{5}$, 5. [illegible], 9. $\frac{1}{42}$, 13. $\frac{4}{10}$ or $\frac{2}{5}$, 17. $\frac{20}{54}$ or $\frac{10}{27}$

Set 54 — *For use with page 295*

Solve the equations.

1. $r \times \frac{1}{3} = \frac{1}{6}$ $r = \frac{1}{2}$
   $\frac{1}{6} \div \frac{1}{3} = r$
2. $b \times \frac{1}{5} = \frac{1}{10}$ $b = \frac{1}{2}$
   $\frac{1}{10} \div \frac{1}{5} = b$
3. $n \times \frac{1}{3} = \frac{1}{21}$ $n = \frac{1}{7}$
   $\frac{1}{21} \div \frac{1}{3} = n$
4. $x \times \frac{1}{3} = \frac{5}{6}$ $x = \frac{5}{2}$
   $\frac{5}{6} \div \frac{1}{3} = x$
5. $w \times \frac{1}{5} = \frac{3}{10}$ $w = \frac{3}{2}$
   $\frac{3}{10} \div \frac{1}{5} = w$
6. $t \times \frac{1}{3} = \frac{4}{21}$ $t = \frac{4}{7}$
   $\frac{4}{21} \div \frac{1}{3} = t$
7. $d \times \frac{2}{3} = \frac{1}{3}$ $d = \frac{1}{2}$
   $\frac{1}{3} \div \frac{2}{3} = d$
8. $\frac{1}{10} \times h = \frac{3}{40}$ $h = \frac{3}{4}$
   $\frac{3}{40} \div \frac{1}{10} = h$
9. $q \times \frac{1}{3} = \frac{7}{36}$ $q = \frac{7}{12}$
   $\frac{7}{36} \div \frac{1}{3} = q$
10. $\frac{3}{10} \times c = \frac{15}{50}$ $c = \frac{5}{5}$
    $\frac{15}{50} \div \frac{3}{10} = c$
11. $s \times \frac{2}{3} = \frac{6}{30}$ $s = \frac{3}{10}$
    $\frac{6}{30} \div \frac{2}{3} = s$
12. $i \times \frac{1}{2} = \frac{3}{10}$ $i = \frac{3}{5}$
    $\frac{3}{10} \div \frac{1}{2} = i$

Reflected answers, Set 54:

Set 55 — *For use with page 297*

1. Ate $\frac{1}{2}$ of $\frac{1}{3}$ of a pie. Ate how much pie? $\frac{1}{6}$
2. $\frac{2}{3}$ of the boys play football. $\frac{1}{3}$ of these boys are on the first team. What part is on the first team? $\frac{2}{9}$
3. $\frac{3}{4}$ of the students like beans. $\frac{1}{3}$ of these students also like corn. What part likes both vegetables? $\frac{3}{12}$ or $\frac{1}{4}$
4. Travelled $\frac{9}{10}$ of a kilometre. Walked $\frac{1}{2}$ of this distance. Walked how far? $\frac{9}{20}$ km
5. Home $\frac{1}{2}$ of the day. Spent $\frac{1}{6}$ of this time eating. Spent how much of the day eating? $\frac{1}{12}$
6. $\frac{7}{10}$ of the students ride the bus. $\frac{1}{2}$ of these students ride at least 3 kilometres. What part ride at least 3 kilometres? $\frac{7}{24}$

Reflected answers, Set 55:

Books to Explore

Adler, Irving. ***The Giant Golden Book of Mathematics.*** New York, Golden Press, 1960. (Available from Whitman Golden Ltd., Cambridge, Ontario)

Have you ever wondered why a tree grows or why a volcano is shaped as it is or what makes a card trick work? This colorful book answers these and many other questions, through exploring the world of mathematics. You will find all kinds of exciting ideas about numbers and what they mean in our daily lives. Here are just a few of the interesting topics:

- The symbol Egyptians used to show a fraction 43
- When the shortest path between two points is a curve 61
- Archimedes' development of giant catapults 85
- How Maxwell's equations led to electronics 90

Bendick, Jeanne, and Levin, Marcia O. ***Take Shapes, Lines and Letters.*** New York, McGraw-Hill Book Co., 1962. (Available from McGraw-Hill Ryerson, Scarborough)

This book sparks your interest about mathematics in art, music, and everyday life; about shapes and curves in nature; and about drawings, graphs, and secret codes. Ideas to explore include:

- What Kepler discovered about planets 25
- What figure is a symbol of the universe 33
- A number system using letters 56
- Chinese tangrams 68

Hogben, Lancelot. ***The Wonderful World of Mathematics.*** New York, Doubleday, 1968. (Available from Doubleday Publishers, Toronto, Ontario)

This book shows that the story of how man became civilized is also the story of how mathematics became a science. You will enjoy going back to the time of the cave man and finding out how man learned to measure and to count, to build and to navigate, to design and to calculate with computers. Some enjoyable things in this book are:

- The Mayan Indian base-twenty number system 11
- Cannonball warfare 55
- Drawing an ellipse with two pegs 59
- Graphs of solid steel 65

Jonas, Arthur. ***New Ways in Math.*** Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1962. (Available from Prentice-Hall of Canada Ltd., Scarborough, Ontario)

Cartoon-style mathematics explains sets, probability, and algebra in a way you'll really enjoy. The chapter "Men in Math" includes both modern giants, like Einstein and Von Neumann, and history's great mathematicians, like Pythagoras and Archimedes. Other chapters you will probably find interesting include:

- The magic of two 24
- When 1001 = 9 31
- What is your hunch? 45
- Doughnuts and pretzels 50

You may also enjoy reading *More New Ways in Math* (1964), by Mr. Jonas.

Other books to explore in your library are listed below:

Adler, Irving. ***Logic for Beginners Through Games, Jokes, and Puzzles.*** New York, John Day, 1964. (Available from Longman Canada Ltd., Don Mills, Ontario)
Good thinking is needed to solve the 285 brain teasers collected here.

Barr, Donald. ***Arithmetic for Billy Goats.*** New York, Harcourt Brace Jovanovich, Inc., 1966. (Available from Longman Canada Ltd., Don Mills, Ontario)
Counting with his two front hoofs, young goat William Gruff invents a two-digit (binary) counting system.

Bendick, Jeanne. ***Take a Number: New Ideas + Imagination = More Fun.*** New York, McGraw-Hill Book Co., 1961. (Available from McGraw-Hill Ryerson, Scarborough)
A cleverly written book about the history of numeration, the binary system and computers, perfect and prime numbers, and fun with numbers.

Bendick, Jeanne, and Levin, Marcia. ***Mathematics Illustrated Dictionary.*** New York, McGraw-Hill Book Co., 1965. (Available from McGraw-Hill Ryerson, Scarborough)
A handy tool for students. If you need to know about ancient or contemporary mathematicians, mathematical terms and definitions or any facts and figures, use this dictionary.

Gardner, Martin. ***Perplexing Puzzles and Tantalizing Teasers.*** New York, Simon and Schuster, 1969. (Available from Musson Book Co., Don Mills, Ontario)
Have fun with riddles, teasers, illusions, tricky questions, word and picture puzzles. All the answers are included. You may enjoy "Sneaky Arithmetic," page 30, "Folding Money Fun," page 54, and "Mother Hubbard's Cupboard," page 67.

Kettlekamp, Larry. ***Puzzle Patterns.*** New York, William Morrow, 1963. (Available from George J. McLeod Ltd., Toronto, Ontario)
Entertaining historical background of puzzles including Sir Isaac Newton's famous problem about trees and the 4000-year-old Chinese tangram.

Leeming, Joseph. ***Fun With Puzzles.*** Philadelphia, J. B. Lippincott Company, 1946. (Available from McClelland and Stewart, Toronto, Ontario)
Also available in paperback from Scholastic Book Service, 1966.
A collection of more than 200 match, coin, paper-and-pencil, cutout, and word puzzles. The answers are all in the back.

Luce, Marnie. ***"Math Concept Books."*** Minneapolis, Lerner Publications Co., 1969. (Available from J. M. Dent & Sons (Canada) Ltd., Don Mills, Ontario)
These books will help you understand the way numbers work. The titles include *Counting Systems, Lines and Planes, Points, Polyhedrons,* and *Primes Are Builders*.

Meadow, Charles. ***The Story of Computers.*** Irvington-on-Hudson, New York, Harvey House, Inc., 1971. (Available from Burns & MacEachern Ltd., Don Mills, Ontario)
An introduction to the fascinating new world of electronic computers—how they are used and how they are programmed to solve problems.

Murray, William, and Rigney, Francis. ***Paper Folding for Beginners.*** New York, Dover, 1960. (Available from General Publishing, Don Mills, Ontario)
See what you can make by folding, tearing, and cutting paper.

Ruchlis, Hy, and Milgrom, Harry. ***Math Projects: Mathematical Shapes.*** Brooklyn, New York, Book Lab, Inc., 1968.
Use straws and pipe cleaners to make cubes, tetrahedrons, octahedrons, prisms, and pyramids.

Sobol, Ken. ***The Clock Museum.*** New York, McGraw-Hill Book Co., 1967. (Available from McGraw-Hill Ryerson, Scarborough, Ontario)
A brief and interesting history of clocks and time-keeping devices from prehistoric times to the present.

Wentworth, D., Stecher, A., Couchman, K., and MacBean, J. ***Mapping Small Places.*** Toronto, Holt, Rinehart and Winston of Canada Ltd., 1972.
This book suggests activities related to maps and map-making. They include measuring distances directly, using map scales, measuring angles and heights, and using the compass.

White, Lawrence B., Jr. ***Investigating Science With Paper.*** Reading, Addison-Wesley Publishing Co., 1970.
A chapter on ''paper engineering'' includes bridge building, testing and the like. Math puzzles and a little magic add to the fun.

addend Any one of a set of numbers to be added. In the equation $4 + 5 = 9$, the numbers 4 and 5 are addends.

addition An operation that combines a first number and a second number to give exactly one number. The two numbers are called addends, and the one number which is the result of combining the two numbers is called the sum of the addends.

angle Two rays from a single point.

approximation One number is an approximation of another number if the first number is suitably "close" (according to context) to the other number.

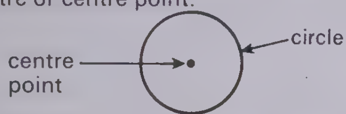
area The area of a closed figure or region is the measure of that region as compared to a given selected region called the unit, usually a square region in the case of area.

associative principle *See* grouping principle.

average (arithmetic mean) The average of a set of numbers is the quotient resulting when the sum of the numbers in the set is divided by the number of addends.

centimetre A unit of length. One centimetre is $\frac{1}{100}$ metre.

circle The set of all points in a plane which are a specified distance from a given point called the centre or centre point.



clock arithmetic A mathematical system using only the twelve numbers of a clock face. It is also called modular arithmetic or remainder arithmetic.

common factor When a number is a factor of two different numbers, it is said to be a common factor of the two numbers.

commutative principle *See* order principle.

compass A device for drawing models of a circle.

composite number Any whole number greater than 1 that is not prime.

cone Generally thought of as a right circular cone, which is illustrated below.

congruent figures Figures that have the same size and shape.

congruent triangles

co-ordinates Number pair used in graphing.

co-ordinate axes Two number lines intersecting at right angles at 0.

cube A rectangular prism (box) such that all faces are squares.

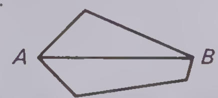
cylinder Generally thought of as a right circular cylinder, which is illustrated below.

decimal Any base ten numeral that uses place value to represent a fractional number.

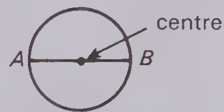
degree An angle unit that is $\frac{1}{90}$ of a right angle.

denominator The number indicated by the numeral below the line in a fraction symbol.

diagonal A segment joining two nonadjacent vertices of a polygon. In the figure, the diagonal is segment *AB*.



diameter A chord that passes through the centre point of the circle.



difference The number resulting from the subtraction operation.

digits The basic Hindu-Arabic symbols used to write numerals. In the base-ten system, these are the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

distributive principle *See* multiplication-addition principle.

dividend In the problem $33 \div 7$, 33 is called the dividend.

Example:
$$\begin{array}{r} 4 \\ 7\overline{)33} \leftarrow \text{dividend} \\ \underline{28} \\ 5 \end{array}$$

division An operation related to multiplication as illustrated:

$$3 \times 4 = 12 \begin{cases} \nearrow 12 \div 3 = 4 \\ \searrow 12 \div 4 = 3 \end{cases}$$

divisor In the problem $33 \div 7$, 7 is called the divisor.

edge An edge of a space figure is one of the segments making up any one of the faces of the space figure.

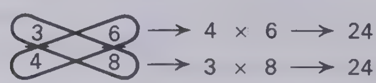
empty set A set that has no objects in it.

equality (equals; or =) A mathematical relation of being exactly the same.

equation A mathematical sentence involving the use of the equality symbol.

Examples: $5 + 4 = 9$; $7 + \square = 8$; $n + 3 = 7$

equivalent fractions Two fractions are equivalent when it can be shown that they each can be used to represent the same amount of a given object. Also, two fractions are equivalent if these two products are the same:



equivalent sets Two sets that may be placed in a one-to-one correspondence.

estimate To find an approximation for a given number. (Sometimes a sum, a product, etc.)

even numbers The whole-number multiples of 2 (0, 2, 4, 6, 8, 10, 12, . . .).

face The face of a given space figure is any one of the plane geometric figures (regions) making up the space figure. For example, in a cube each of the square regions is a face of the cube.

factor *See* multiplication. The equation $6 \times 7 = 42$ illustrates that both 6 and 7 are factors of 42.

fraction A symbol for a fractional number, usually written $\frac{2}{3}, \frac{3}{4}, \frac{1}{2}$, and so on.

fractional number The one number we think about for each set of equivalent fractions.

function The set of number pairs (input, output) generated by a function rule applied to a given set of numbers (input numbers).

graph (1) A set of points associated with a given set of numbers or set of number pairs. (2) A picture used to illustrate a given collection of data. The data might be pictured in the form of a bar graph, a circle graph, a line graph, or a pictograph. (3) To draw the graph of.

greater than (>) One of the two basic inequality relations.

Examples: $8 > 5$, $28 > 25$, $80 > 50$

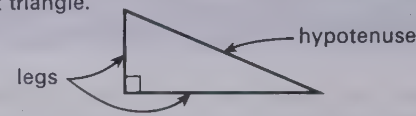
greatest common factor The largest, or greatest, number that is a factor of each of two numbers.

grouping principle (associative principle) When adding (or multiplying) three numbers, you can change the grouping and the sum (or product) is the same.

Examples: $2 + (8 + 6) = (2 + 8) + 6$
$3 \times (4 \times 2) = (3 \times 4) \times 2$

hexagon A six-sided polygon.

hypotenuse The side opposite the right angle in a right triangle.

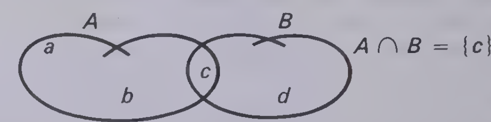


improper fraction A fraction in which the numerator is greater than or equal to the denominator.

Examples: $\frac{8}{5}, \frac{9}{6}, \frac{12}{3}, \frac{7}{7}$

inequality (<, ≠, >) In arithmetic, a relation indicating that the two numbers are not the same.

intersection of sets The intersection of two sets is the set of elements common to both of the sets. If A and B are sets, $A \cap B$ denotes the intersection of the sets.



least common denominator The least common multiple of two denominators. The least common denominator of $\frac{1}{4}$ and $\frac{5}{6}$ is 12.

least common multiple The smallest non-zero number that is a multiple of each of two given numbers. The least common multiple of 4 and 6 is 12.

length (1) A number indicating the measure of one line segment with respect to another line segment, called the unit. (2) Sometimes used to denote one dimension (usually the greater) of a rectangle.

less than (<) One of the two basic inequality relations. Examples: $5 < 8$, $25 < 28$, $50 < 80$.

line A line is a set of points that "goes on and on" in both directions. There is only one line through any two points.

line segment *See* segment.

lowest terms A fraction is in lowest terms if the numerator and denominator of the fraction have no common factor greater than 1.

measure (1) A number indicating the relation between a given object and a suitable unit. (2) The process of finding the number described in (1).

metre A unit of length in the Metric System. A metre is 100 centimetres.

midpoint A point that divides a line segment into two parts of the same size.

minus (−) Used to indicate the subtraction operation, as in $7 - 3 = 4$ (read, "7 minus 3 equals 4").

mixed numerals Symbols such as $2\frac{1}{2}$ and $3\frac{1}{4}$.

multiple A first number is a multiple of a second number if there is a whole number that multiplies by the second number to give the first number. Example: 24 is a multiple of 6 since $4 \times 6 = 24$.

multiplication An operation that combines a first number and a second number to give exactly one number. The two numbers are called factors, and the one number which is a result of combining the two numbers is called the product of the two numbers.

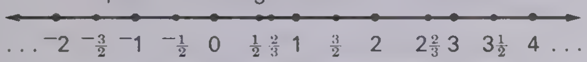
multiplication-addition principle (distributive principle) This principle is sometimes described in terms of "breaking apart" a number before multiplying.

Example: $6 \times (20 + 4) = (6 \times 20) + (6 \times 4)$

number line A line with a subset of its points matched with a subset of the real numbers. We say that the rational number line has "holes" in it because some points are not matched with rational numbers. The real number line is said to be "complete" because each point is matched with some real number.

The number line

A positive and negative whole number line



The rational number line

number pair Any pair of numbers. In this book, usually a pair of whole numbers.

numeral A symbol for a number.

numerator The number indicated by the numeral above the line in a fraction symbol.

odd number Any whole number that is not even.

one principle (for multiplication) Any number multiplied by 1 is that same number.

one-to-one correspondence A one-to-one correspondence exists between two sets when the elements of one can be matched with the elements of the other in such a way that each element of the first set is matched with exactly one element of the second set and each element of the second set is matched with exactly one element of the first set.

order principle (commutative principle) When adding (or multiplying) two numbers, the order of the addends (or factors) does not affect the sum (or product).

Examples: $4 + 5 = 5 + 4$
$2 \times 3 = 3 \times 2$

parallel lines Two lines which lie in the same plane and do not intersect.

parallelogram A quadrilateral with its opposite sides parallel.

parentheses A pair of curved symbols, ( ), used to indicate grouping or order of performing operations.

Examples: $(5 \times 4) - 2 = 18$
$5 \times (4 - 2) = 10$

pentagon A five-sided polygon.

perimeter The sum of the lengths of the sides of a given polygon.

period In arithmetic, each set of three digits indicated by spacing when writing a numeral is called a period. These periods are called (right to left) units' period, thousands' period, millions' period, and so on.

Example:	**3 4 2**	**6 7 4**	**2 0 8**
	millions' period	thousands' period	units' period

perpendicular lines Two lines that intersect in right angles are perpendicular to each other.

place value A system used for writing numerals for numbers, using only a definite number of symbols or digits. In the numeral 3257 the 5 stands for 50; in the numeral 36 289 the 6 stands for 6000.

plus (+) Used to indicate the addition operation, as in $4 + 3 = 7$ (read, "4 plus 3 equals 7").

polygon A closed geometric figure made up of line segments.

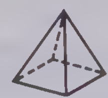
prime number A number greater than 1 whose only factors are itself and 1.

product The result of the multiplication operation. In $6 \times 7 = 42$, the product of 6 and 7 is 42.

prism A three-dimensional figure whose bases are congruent polygons in parallel planes and whose faces are parallelograms.

protractor An instrument used for measuring angles.

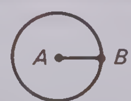
pyramid A three-dimensional figure with a polygonal base and triangular lateral faces.



quadrilateral A four-sided polygon.

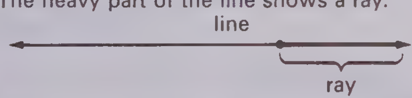
quotient The number (other than the remainder) that is the result of the division operation. It may be thought of as a factor in a multiplication equation.

radius (1) Any segment from the centre point to a point on the circle. (2) The distance from the centre point to any point on the circle.



ratio A pair of numbers used in making certain comparisons. The ratio of 3 to 4 is written $3{:}4$ or $\frac{3}{4}$.

ray The heavy part of the line shows a ray.



rectangle A quadrilateral that has four right angles.

regrouping A method of handling place value symbols in adding or subtracting numbers.

remainder: Example:

$$\begin{array}{r} 6 \\ 7\overline{)47} \\ \underline{42} \\ 5 \end{array} \longleftarrow \text{remainder}$$

repeated addition Finding the sum of a set of numbers, each of which is the same.

Example: $5 + 5 + 5 + 5$

repeated subtraction Starting with a number and repeatedly subtracting the same given number from each difference that is obtained.

rhombus A parallelogram with 4 sides of the same size.

right angle An angle that has the measure of 90 degrees.

right triangle A triangle that has one right angle.

Roman numerals Numerals used by the Romans. Used primarily to record numbers rather than for computing. Examples: IV, IX, XIV.

rotation A motion in which a given figure is turned about a fixed point.

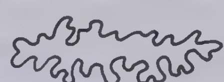
scale drawing A drawing constructed so the ratio of all the dimensions in the drawing to those of the actual object is the same.

segment Two points on a line and all the points on that line that are between the two points.

sequence A collection or set of numbers given in a specific order. Such numbers are commonly given according to some rule or pattern.

set A group or collection of objects.

simple closed curve Can be thought of as a loop of string on a flat surface that does not cross itself.



solution The number or numbers which result from solving an equation or a given problem.

solve To find the number or numbers which, when substituted for the variable or placeholder, make a given equation true.

square A quadrilateral that has four right angles and four sides that are the same length.

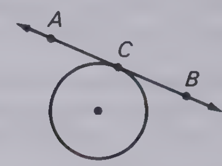
subtraction An operation related to addition as illustrated:

$$7 + 8 = 15 \begin{array}{l} \nearrow 15 - 8 = 7 \\ \searrow 15 - 7 = 8 \end{array}$$

sum The result obtained by adding any set of numbers.

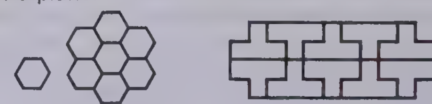
symmetric figure A figure that can be folded in half so the two halves match.

tangent A line is tangent to a circle if the two figures are in one plane and have exactly one point in common.



Line AB is tangent to the circle at point C.

tessellation A repeated pattern of regions that can cover a plane.



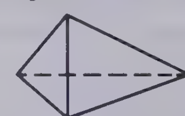
times ($\times$) Used to indicate the multiplication operation, as in $3 \times 4 = 12$ (read, "3 times 4 equals 12").

translation A motion in which each point of a figure is moved the same distance and the same direction.

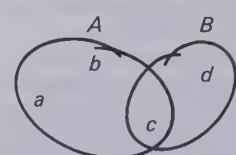
trapezoid A quadrilateral with at least one pair of parallel sides.

triangle A three-sided polygon.

triangular pyramid A 4-sided space figure that has triangular regions for all faces.



union of sets If A and B are sets, then $A \cup B$ (the union of A and B) is the set consisting of all elements that belong to at least one of the two sets.



$A \cup B = \{a, b, c, d\}$

unit An amount or quantity adopted as a standard of measurement.

vertex The point that the two rays of an angle have in common.

volume The measure, obtained by using an appropriate unit (usually a cube), of the interior region of a space figure.

whole number Any number in the set $\{\ldots, ^-3, ^-2, ^-1, 0, 1, 2, 3, \ldots\}$

zero principle (for addition) Any number added to zero is that same number.

Tables of Measures

LENGTH	
10 millimetres (mm) = 1 centimetre (cm)	1000 millimetres = 1 metre
10 centimetres = 1 decimetre (dm)	100 centimetres = 1 metre
10 decimetres = 1 metre (m)	10 decimetres = 1 metre
1000 metres = 1 kilometre (km)	1 / 1000 kilometres = 1 metre

TIME	
60 seconds (s) = 1 minute (min)	52 weeks = 1 year (yr)
60 minutes = 1 hour (h)	12 months (mo) = 1 year
24 hours = 1 day	365 days = 1 year
7 days = 1 week (wk)	366 days = 1 leap year

CAPACITY
10 millilitres (ml) = 1 centilitre (cl)
10 centilitres = 1 decilitre (dl)
10 decilitres = 1 litre (*l*)
1000 litres = 1 kilolitre (kl)

WEIGHT
1000 grams (g) = 1 kilogram (kg)
1000 kilograms = 1 tonne

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A Text for Teachers

Investigating Mathematics Learning

Phares G. O'Daffer

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INVESTIGATING MATHEMATICS LEARNING

I. Some Thoughts About Learning

Almost everyone has some observations on teaching and learning. A recent quote that has been making the rounds is: "If we tried to teach children to speak, they would never learn." However, in *The Process of Education* (Harvard University Press, 1960), Jerome Bruner observes, "Any subject can be taught effectively in some intellectually honest form to any child at any stage of development." But Linus (of *Peanuts* cartoon fame), considerably less optimistic, laments: "How can I learn 'New Math' with an 'Old Math' mind?"

In a more critical vein, John Holt in *How Children Fail* (Pitman Publishing Co., 1964) asserts: "In our classes, we begin with words, carry on with words, and often fail to get beyond words." He also says, "All too often the mathematics classroom becomes a temple of worship for the right answers, and the way to get ahead is to lay plenty of them on the altar." We know, of course, that many teachers for many years have been doing an excellent job helping elementary school children learn mathematics. Yet, it is worthwhile for us to reevaluate our approaches and, if possible, find even better ways to create situations where children learn more effectively.

The implications of the research of Piaget and others in how children learn mathematics and the observations of countless classroom teachers concerning the directions we should take are well summarized by a familiar Chinese proverb:

I hear and I forget.
I see and I remember.
I do and I understand.

The message of this proverb is that hearing and seeing are not enough: to learn with understanding, the child should experience *active involvement* with mathematical ideas. In order for the child to become actively involved, it has been found that the use of *physical materials* which contain the seeds of the mathematical ideas are valuable and often necessary. Coupled with the idea of active involvement with physical materials is the idea that teachers should encourage *student responsibility* and create conditions in which the student is not always encouraged to rely solely on the teacher but rather to take initiative for figuring out some things for himself.

Z. P. Dienes summarized a multitude of suggestions from researchers and teachers when he said: "It is suggested that we shift the emphasis from teaching to learning, from our experience to the children's, in fact, from our world to their world."

Teachers vary considerably in their views of how best to help children become actively involved with mathematics. While one teacher desires to convert his classroom immediately into a mathematics laboratory, another teacher may prefer a very modest beginning with a limited amount of active student involvement with physical materials inserted into his usual classroom approach. In this text we suggest a number of approaches for modest beginnings and indicate ways in which these approaches might be expanded to provide for a total laboratory approach and a more extensive individualized program.

To introduce one possible approach, let us simulate a teaching strategy by outlining one way to organize a specific lesson. Thus, suppose a teacher wanted to devise a lesson which would help children understand the idea of congruent segments in geometry. First the teacher provides each child with a geoboard and a sheet containing several 5-by-5 arrays of dots. Then he reviews, very briefly, the idea of a segment and the endpoints of a segment. Next, after helping the children see that they can use a rubber-band around two nails to represent a segment on the geoboard, he passes out the investigation suggested in Figure 1.

INVESTIGATION

How many different segments with endpoints just as far apart as these can you find on your geoboard? Record your results on dot paper.

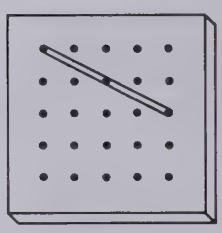


Figure 1

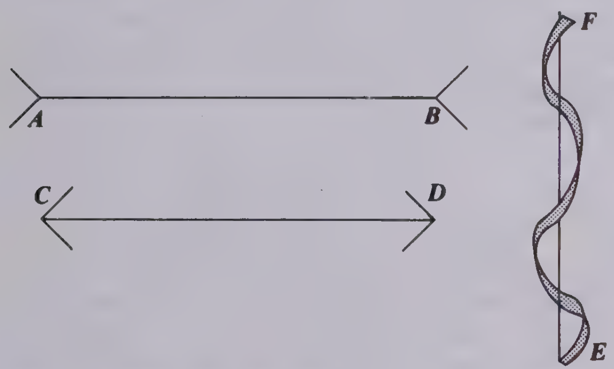
The teacher may choose to have chairs or desks rearranged so that children can communicate with each other as they become involved in the investigation. The teacher will check to be sure that everyone understands the investigation question; then he should encourage the children to find their own way to answer the question and record their findings. (To gain a fuller appreciation of an investigation situation, play the role of the child and complete the investigation yourself.)

Brief discussions among children or between teacher and children may occur during investigations, but the main discussion is most effective after the investigation has been completed. At this time, the teacher might ask such questions as: "How many different

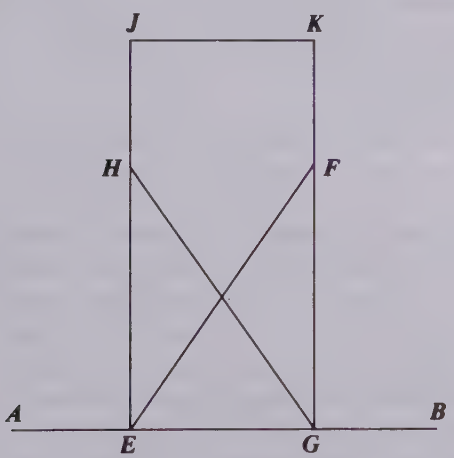
segments did you find?" "How can you be sure that you have found all such segments?" "How could you convince someone that each of your segments has endpoints just as far apart as all the others?" Such questions could then be followed up with a definition of congruent segments: When the endpoints of one segment are just as far apart as the endpoints of another segment, we say the segments are *congruent*. Then ask, "Can you think of some other ways to tell when two segments are congruent?" This question might lead into a discussion of how tracing paper, compasses, or marks on the edge of a piece of paper can be used to determine whether or not two segments are congruent.

After the children have discussed the ideas, the teacher may provide them with some problems which *utilize* these ideas. The child would probably be encouraged to use the ideas for testing congruence of segments that were developed in the discussion. The following are examples of possible exercises.

1. Find 2 segments below that are congruent to each other.



2. Name each pair of congruent segments in this picture.



One way to individualize a lesson is through an *extension* of the exercises. Extending the exercises can provide for remediation, reinforcement, or enrichment. As an extension to individualize this lesson, the teacher might give certain students the follow-up investigation below. (For a fuller appreciation of this lesson, complete the exercises and the investigation yourself.)

INVESTIGATION

Segment AB is not congruent to segment CD.

How many different segments (no two congruent) can you find on your geoboard? Record your results on dot paper.

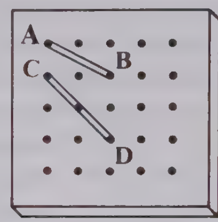


Figure 2

This abbreviated "lesson" provides a preview of one possible technique for encouraging children to become actively involved with physical materials in situations where they take more responsibility in the learning of mathematical ideas.

In the next section of this text, the parts of lessons such as the one described above will be analyzed and discussed. An outline for planning such lessons will be given, and various suggestions for carrying out each part of such a lesson will be proposed.

Since the investigation phase of the lesson provides the encouragement for active involvement by the child and since the kind of investigation used depends upon the type of learning involved, Section III in this text will focus on specific types of learning in elementary school mathematics. For example, the "lesson" described above helped children learn the *concept* of congruent segments; other lessons might be concerned with developing a *skill*, forming a *generalization*, learning a *fact*, or developing an *attitude*. Each type of learning will be analyzed and related to activity-oriented lessons that provide modest beginnings toward an active approach to mathematics learning.

Edith Biggs and James MacLean, in their book *Freedom to Learn* (Addison-Wesley, 1969), state: "A few schools scattered throughout the world are responding with some speed to a message which has been repeated with increasing urgency for some three hundred years. It is a simple message: Schools should be organized, not for teachers to teach, but for children to learn." In the same book, there appears an extensive list of "homemade" materials and devices that can be easily acquired for use in the mathematics classroom. Many materials, from newsprint and drinking straws, to string, popsicle sticks, beans, and homemade geoboards, can be made available to children at minimal cost. Rather than dismiss the possibility of actively involving children with materials in the classroom because no funds are available, a teacher should study this list carefully; he may be amazed by how much can be done with minimum expense.

Teachers sometimes feel that to involve children with physical materials and allow them to communicate with other children in the classroom is to invite

chaos. On the contrary, it has been found that, when children really become involved in using materials to investigate a situation, there may be a bit more low-keyed noise about the room but the usual discipline problems are almost nonexistent. It is helpful if there are tables available in the classroom so that children can work in small groups. If tables are not available, desks could be moved to assist in small-group work. On occasion, an investigation might call for children to leave their desks and to engage in other activity in the room. A simple set of "ground rules" should suffice to make the situation quite manageable.

It is interesting to consider the number of elementary school teachers who prefer to say that they are "helping children learn mathematics" rather than that they are "teaching mathematics." What one says, of course, does not always describe accurately what one does. It does seem important, however, in the light of recent studies and observations about how children learn mathematics, to focus on the child and try to create an environment in which the child has a greater opportunity to make decisions and to become really interested in his study. It is hoped that the following sections of this teachers' text will provide some ideas which may help you improve your ability to "help children learn mathematics."

EXERCISE SET 1

1. What was your reaction to the investigations in this section? **A** Did you become involved in the activity? **B** Were you interested? **C** Did you watch the clock? **D** Did you talk to anyone else while completing the investigation? (If so, was it helpful?) **E** Did the investigation situation help you better understand the idea involved? **F** What other feelings did you have?
2. Which quotation in this section seemed most significant to you? Why?
3. **A** Do you think most teachers teach the same way they were taught as elementary school children? **B** What do they do differently? **C** What are some ways you think our teaching of elementary school mathematics might be improved?
4. Look through the *Investigating School Mathematics* text at your grade level. How do the comments in this first section of the text relate to the approach taken in the child's text?

II. A Plan for a Learning Experience

First consider the practical matter of how the teacher proceeds in the daily task of helping children learn mathematics. A structured outline (inherently flexible) around which daily learning experiences may be planned can be a valuable organizational aid for the teacher and can give him a fresh insight into the role of new approaches to instruction.

Here is the outline that was used in planning the "lesson" in Section I. It has proven to be quite useful, especially for those teachers who have desired to make a beginning toward providing children more opportunities for active involvement with mathematical materials and ideas.

Preparation and Investigation
Discussion
Utilization and Extension

Since this outline offers a variety of possibilities for a teacher to reevaluate his approach to classroom instruction, the following sections provide an examination of its individual elements.

Preparation and Investigation

The investigation phase (often called simply "the investigation") is central to the learning experience. In this phase, the children are encouraged to become actively involved, individually or in groups, in the investigation of a situation that contains the seed for the central idea of the lesson. The investigation should be the "main event" in terms of pupil activity and involvement. The teacher should think of the investigation as a child-centred activity. Completion of the investigation in Figure 3 will help clarify the ideas of investigation.

INVESTIGATION

Can you find an investigation in a text from the *Investigating School Mathematics* series that

(a) uses centimetre strips?
(b) utilizes paper folding?
(c) has a question like "How many can you find?"
(d) involves the geoboard?
(e) encourages children to use graph paper?
(f) asks the children to record their findings?
(g) directs the children to use reference material?

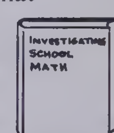


Figure 3

Homemade or commercially produced manipulative materials often provide the stimulus for the situation to be investigated. At other times, even more simple teacher-devised activities provide this stimulus. For example, the suggested investigation in Figure 4 might have been made by a teacher to initiate an investigation in a lesson designed to help children form the generalization, "You can rearrange three addends any way you please, and the sum will always be the same."

Sometimes by asking appropriate questions about a situation of interest to the children the teacher may involve them in an exploration of a central idea to be developed.

Regardless of how an investigation is initiated, a teacher should remember that the investigation situation is specifically designed to encourage children to

INVESTIGATION

Make three slips of paper like these.

2 3 4

Then turn them over and mix them up. Pick any two slips and add the numbers on them. Then add the number on the other slip.

If you do this five times, will you get the same number each time?

Figure 4

take responsibility for the thinking and exploring. Too much "teacher help" can hinder the achievement of these aims.

In an investigation, it is not uncommon to see children deeply involved and assuming full responsibility for completing the task at hand. The teacher, who plays a key role in initiating the investigation, may appear not to be needed as he moves about the room. Occasionally, a brief discussion between teacher and child occurs, but most of the larger-group discussion occurs after the investigation. The investigation itself should embody an attitude toward learning that could be easily stifled by too many words from the teacher. Perhaps, in an investigation, a new adage should replace the old: the teacher, rather than the children, should be "seen but not heard."

The investigation is predicated on the assumption that the best way to minimize the need for words is to substitute an appropriate question for a wordy explanation at a time when a child's interest in a mathematical situation is beginning to ripen.

For example, suppose a certain group of children understand the concept of a triangle and are ready to consider characteristics that distinguish one type of triangle from another. An appropriate question to initiate an investigation might be the one shown in Figure 5. (Try this investigation yourself.)

INVESTIGATION

How many triangles with different shapes can you make that have no nails inside?

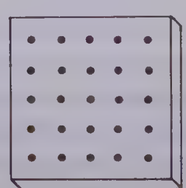


Figure 5

This question is both activity-stimulating and activity-sustaining. It helps involve the child in a search which he will continue with little further motivation. Notice also that the answer is not as important as the experiences the child will have as he responds to the question. Further, the question is sufficiently clear that the child immediately becomes involved with the challenge of the investigation rather than dissipating energy in efforts to understand the question. Another characteristic of this type of question is that it provides for individual differences: when the child is asked "How many can you find?" he can feel successful even if he finds only one. Of course, not all investigations can or should be introduced by this type of question, but it is important for the teacher to recognize that as the children respond to these questions, they will achieve in widely differing ways. In an investigation, the teacher should give recognition for all levels of achievement.

It should be noted that the amounts of time used for the investigations may vary considerably. One investigation may involve a very brief "happening" which sparks a simple idea within the child. Another investigation may utilize a large part of the period of time available for the mathematics lesson and might involve the child in a sustained exploration of a game or a set of manipulative materials.

To set the stage for an investigation of any duration, a preliminary *preparation* phase is sometimes needed. This phase provides for a brief review of key ideas needed for the investigation and for any motivational activity helpful in introducing it. This phase should be kept fairly short and care should be taken to see that this preliminary work does not preempt the central idea or activities involved in the investigation or the work that follows it.

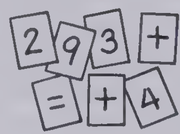
In summary, the investigation phase is the child-involvement phase. It often requires materials, and is usually motivated by a carefully selected question which focusses the student's attention on the central idea of the lesson. Proper consideration of this phase in your lesson planning can be highly rewarding.

EXERCISE SET 2

1. Find some investigations in the *Investigating School Mathematics* text that contain features not mentioned in Figure 4.
2. Choose a lesson from an *Investigating School Mathematics* text and write a description of the role you think the teacher would play in using the investigation phase of the lesson.
3. Choose an idea to be taught and prepare an investigation situation which has the potential of involving the child in working with this idea.
4. Two investigations follow. Give the central idea of a possible lesson based on the use of each one.

A

INVESTIGATION



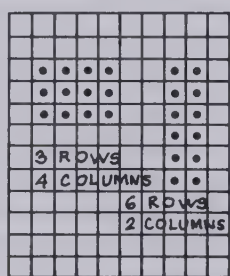
Cut out 7 slips of paper. Put one of these numerals or one of these signs on each one.

How many different equations with 3 addends can you write with your slips of paper? Record each equation you find.

B

INVESTIGATION

The graph paper shows two different ways to arrange 12 counters in a rectangular array.



How many different ways can you arrange 24 counters in a rectangular array? Record your findings by drawing pictures on graph paper.

5. Here is an interesting investigation you may like to try. Through it, you will be introduced to a basic idea of mathematics. Be sure to record your findings and be ready to discuss them further in the next section.

Copy and continue this array of numbers until you reach 52.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16
17	18	19	20
	. . .		

Then circle all the prime numbers in the array.

Notice that the numbers in the right-hand column can be written as $4 \times$ (a whole number). For example: $8 = 4 \times 2$, $12 = 4 \times 3$, and $20 = 4 \times 5$.

Can you make a statement about prime numbers that is suggested by this activity?

Discussion

Following the investigation, a *discussion* phase allows teacher and children to further share ideas in a discussion of what they found in the investigation. The teacher has an excellent opportunity in this phase to ask questions and to supply examples to help children further develop their understanding of the ideas germinated by the investigation.

Another valuable aspect of the discussion phase is that it provides additional opportunities for children to communicate with other children as a means of shaping their ideas. In a good discussion, it is not unusual for children, having reached an impasse in *their* thinking and communication about an idea, to ask the teacher if he can clarify the point. This is when the teacher as a resource person emerges. At other times, when ideas new to the teacher arise, the teacher participates in the discussion, not as a resource person, but as a fellow-learner. Both of these situations can contribute to a comfortable, meaningful discussion, but its potential benefits may never be realized if the teacher monopolizes the discussion to the extent that the children are denied the opportunity to draw their own inferences and make their own decisions. Since it is the child who is involved in the investigation, the child's ideas about the findings should be of primary importance, and the child should supply as many details leading to the understanding of the idea as possible.

By listening to the child and asking appropriate questions, the teacher can build on the child's initial ideas and help him develop a deeper understanding in preparation for further work. This understanding cannot be developed, however, by always asking questions which require simply that a child remember a fact correctly or perform a practical skill. Nor is it sufficient to ask questions to which a child can respond with a guess of "Yes" or "No." Rather, the questions that should be asked often are those that require a deeper thinking on the part of the child.

For examples of the more effective type of question, consider again the investigation described in Figure 7. This investigation, designed to set the stage for the development of the concepts of isosceles triangle, right triangle, scalene triangle, and equilateral triangle, might be followed by a discussion in which the teacher would ask questions such as the following:

1. Can you choose a pair of triangles you found and describe ways in which one is different from another?
2. In what ways are some triangles you found alike? (Note: Children may respond, "Some have a square corner," "Some have two sides the same," "Some have no sides the same," "Some are large," and so on.)
3. How would you describe a triangle that is different from any of the triangles you formed on the geoboard?

As the teacher asks thought-provoking questions and listens to the children's responses, he will be able to find ways to clarify the basic idea of the lesson and to prepare the children for the independent work which is to follow. It is in the latter stages of the discussion that the teacher may want to explain more carefully, show additional examples, and, in general, lead the child to a deeper mastery of the ideas involved.

EXERCISE SET 3

1. Can you find a question in the "Discussing the Ideas" section of an *Investigating School Mathematics* text which **A** asks the children to recall something previously learned? **B** asks the children to restate or explain an idea in their own words? **C** asks the children to interpret a diagram, picture, or explanation? **D** asks the children to analyze a given situation? **E** asks the children to evaluate a given situation?
2. What do you think about the effectiveness of the investigation described in Figure 5 as a means of meeting the goals indicated?
3. Write five questions you might ask while conducting a discussion in a mathematics lesson of your choice.
4. The following discussion exercises refer to the investigation presented in exercise 5 of Exercise Set 2. **A** What statement did you make about prime numbers? **B** Can you find a prime number that does not appear in the first or the third column? Can you find more than one? **C** $4 \times n$ is an algebraic expression. What algebraic expression can you devise to describe the prime numbers in the third column? in the first column? **D** Of the prime numbers less than 100, which type of prime occurs more often? **E** 113 is a prime number. Which type of prime is it?
5. Investigation questions may be open ("In how many ways can you measure a ball?") or closed ("Can you find the circumference and diameter of this ball?"). Discuss the merits of open and closed questions.

Utilization and Extension

The *utilization* phase allows each child to work on his own and to use the ideas developed in the investigation and discussion phases.

Often children need to practice recalling facts that have been developed or introduced in the lesson. Appropriate exercises requiring written answers are often valuable in providing this practice.

In another lesson, a child may have learned an algorithm or a skill. In order to refine this skill, he may need considerable practice using it. Appropriately designed written exercises which children complete independently can be quite helpful in polishing these skills.

In another lesson, a new idea may have been presented. In order to become more familiar with this idea and to understand how it relates to other ideas, the child may need thought-provoking problems which involve the idea. The *utilization* phase presents an opportunity for the child to solve problems which involve ideas that have been presented previously or to look at an idea that is different but closely related to one he has already encountered.

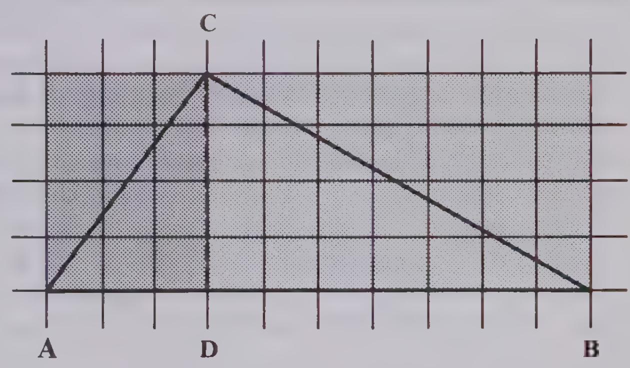
Creative activities for independent work can do much to extend the learnings developed in the investigation and discussion phases. The utilization exercises in examples A and B below are sequenced in such a way that the child has an opportunity to discover a new procedure or new ideas as a result of his work.

EXAMPLE A

Find the differences.

75	75	75	75	75	75	75
−32	−33	−34	−35	−36	−37	−38
43	42	41																								

EXAMPLE B

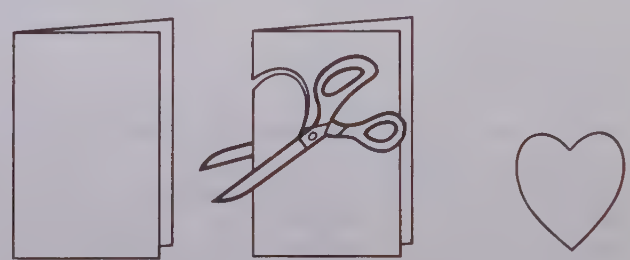


What is the area of the region shaded dark gray?
What is the area of the region shaded light gray?
What is the area of the two regions together?
What is the area of triangle ADC?
What is the area of triangle BDC?
What is the area of triangle ABC?
The area of triangle ABC is what part of the entire shaded region?

The teacher should appreciate the great potential value of discovery-sequenced exercises such as these, and should look for opportunities to make his own exercise sets using such sequences. Another set of utilization exercises might encourage the child to independently delve more deeply into the idea initiated in the investigation. Further activities with mathematical materials often provide opportunities for the child to use and extend the idea of the investigation. Example C provides an opportunity for the child to reinforce his concept of symmetry.

EXAMPLE C

Do this to make symmetrical figures.



Fold a piece of paper.

Make a cut that starts and ends on the fold.

Unfold the piece you cut out. It will be symmetrical.

Make cuts so that the unfolded shape will be:

A a rectangle	**D** a square	**G** a rocket
B a leaf	**E** a house	**H** a hexagon
C a triangle	**F** a pumpkin	**I** a butterfly

Regarding the utilization phase, it should be noted that on occasion it may be more valuable to have pairs or small groups of children work the exercises together.

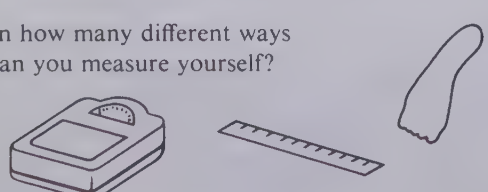
Finally, the *extension* phase provides for use of remedial, maintenance, or enrichment activities to further individualize the learning opportunities. This individualization offers numerous advantages. The slower children can avoid the frustration of having to proceed to new ideas before the previously presented ideas are understood, and the more capable children are spared the tedium of completing long lists of drill problems involving ideas they already understand.

The teacher might look for creative ways to meet individual differences in the ability to learn mathematics. For example, the slower child might profit from additional drill on certain facts and skills. Drill tapes or audio cassettes made by the teacher might provide a novel way to present the necessary practice. Duplicator masters and commercial workbooks are also available to provide extra work for those who need it. For other situations, an appropriate programmed instructional unit might serve the needs of the slower child. Single-concept film loops, which the child can play again and again, often are useful in helping him grasp an important concept. Appropriately conceived tutorial situations, in which classmates who understand the ideas work with children who do not, can be quite effective. Simple investigations utilizing physical objects which clarify more abstract ideas can also provide remedial work for certain children.

The teacher must also be concerned with those children who understand the basic ideas of the lesson and who can quickly work all the utilization exercises provided. These children can often become quite interested in activity cards which contain "open-ended" questions, such as the card shown below. (You are encouraged to try the suggested activity yourself.)

ACTIVITY CARD 10

In how many different ways can you measure yourself?



Make as many different measurements of **you** as you can and make a chart to show the information. Here are just a few suggestions:

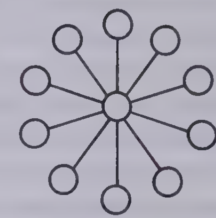
Pulse	Length of step
Height	Number of calories used
Weight	Area of bottom of foot
Arm span	Distance you can jump

Activities such as these give the child an opportunity to make his own decisions about which ideas he uses from the lesson and how he uses them.

Puzzles or riddles can also provide a useful extension of ideas for your children. Consider, for example, those shown in Figure 6.

Think

Draw a figure like this one on your paper. Place the numbers 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 in the circles so that the sum along any line is 21.



Think

I can be found halfway between Twenty-seven and seventeen.

27 ? 17

WHO AM I?

Figure 6

Conceptually fertile games can also provide valuable experiences to supplement the basic lesson. For example, the game "Sleuth" (3M Company) is fun for children and gives them valuable experience in classification and drawing logical inferences.

The methods suggested for extending the ideas for slower children are often suitable for use in certain situations with more capable children. Similarly, the more exciting modes of extension suggested for faster children can often be quite stimulating and valuable if used appropriately for the slower children.

It is to be hoped that the teacher will share a sense of excitement in providing extra stimulation to broaden the mathematical perspective of the children. Perhaps, he will also see that much of the extension activity can truly be fun for children while at the same time inspiring new interest and involvement in mathematical ideas. In using this suggested lesson outline, if the teacher chooses to maximize the investigation phase while deemphasizing the others, it might justly be said he is using the laboratory approach. On the other hand, should he maximize the discussion phase, he may find increased options for a guided discovery approach to mathematics learning. Also, it is possible that maximization of the utilization phase accompanied by appropriate student materials would allow the teacher to embark on a course of individually prescribed instruction.

EXERCISE SET 4

1. Find an example of an exercise set in which a learning sequence occurs in an *Investigating School Mathematics* text.

2. Choose a mathematics topic and write a set of exercises which might lead the student to discovery of a central idea.
3. Can you find a lesson in an *Investigating School Mathematics* text in which the "Using the Ideas" section provides for varying degrees of student ability.
4. Choose a learning experience appropriate for your children and list some possible specific activities for use in the extension phase of this learning experience.
5. Describe your views concerning the role of drill for slow, average, and bright children.
6. Select and play a game that could be used to extend a lesson with children.
7. In Exercise Set 2, you investigated an idea of mathematics. In Exercise Set 3, you had an opportunity to discuss this idea. The exercises below enable you to use the idea you learned, and suggest an extension of the idea.

Complete each exercise.

A List five prime numbers of the "$4n + 1$" type that are greater than 50.

B List five prime numbers of the "$4n - 1$" type that are greater than 50.

C 997 is the largest prime number less than 1000. Is it a "$4n + 1$" or a "$4n - 1$" prime?

D Suppose you used a continuation of the array of numbers shown below and circled all the prime numbers. What does this suggest about another way to classify the primes?

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
...					

III. A Focus on Specific Types of Learning

In considering the more specific aspects of mathematics learning it is helpful to categorize the general types of things children learn. A simplified categorization is given below.

Concepts
Skills
Generalizations
Facts
Attitudes

It is important to recognize that each of these types of learnings has unique characteristics. Because of this, the approaches and children's activities chosen to promote these learnings may often be quite different. In the sections that follow, we will consider each of these types of learning and suggest possible approaches and activities.

Concepts

Suppose that a child is having difficulty and comes to the teacher for assistance. When the teacher asks what the difficulty is, the child points to the multiplication 9×6 and says, "I can't do this because we haven't had it yet." This reflects a common attitude among children who have been in school for a few years. Somehow they learn to feel that they are incapable of figuring out anything new in mathematics. Literally, they can do nothing that they "haven't had yet."

If this child had confidence in his ability to "figure something out" and had a clear understanding of the *concept* of multiplication, he could have found the product by perhaps adding sixes, using sets, or making jumps on the number line. Another child who knew no division "facts" but who had a clear concept of division (as illustrated below) could use his knowledge of multiplication to find any of the basic quotients desired.

$$\overset{P}{72} \div \overset{F}{8} = \overset{F}{n} \leftarrow \text{You find this quotient,}$$

$$\text{when you find this factor.} \rightarrow \overset{F}{n} \times \overset{F}{8} = \overset{P}{72}$$

A *concept,* then, may be thought of as an idea which, when properly understood, will help the child to solve problems he "hasn't had yet," to figure something out for himself. As another example, consider the concept of prime number. Once a child understands that a prime number is a whole number with exactly two factors, he has the power, providing he understands how to find the factors of a number, to seek out and list those numbers that are prime. Of course, the task of deciding whether or not a given number is prime may be quite laborious, but understanding the concept does give the child the power to succeed.

To look more carefully at what concepts are and how they are taught, consider a model in which concept learning is relatively easy, namely, that of a set of attribute pieces. Suppose there are pieces of four different shapes (triangles, squares, circles, and rectangles), of three different colors (red, blue, and yellow), and of two different sizes (large and small), as pictured in Figure 7. (In the figures the colors red, blue, and yellow are denoted by the initials R, B, and Y.)

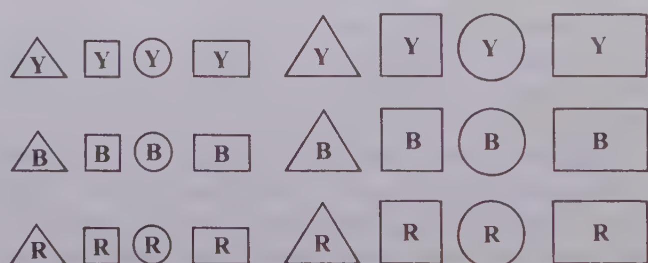


Figure 7

Now consider the Concept Card in Figure 8.

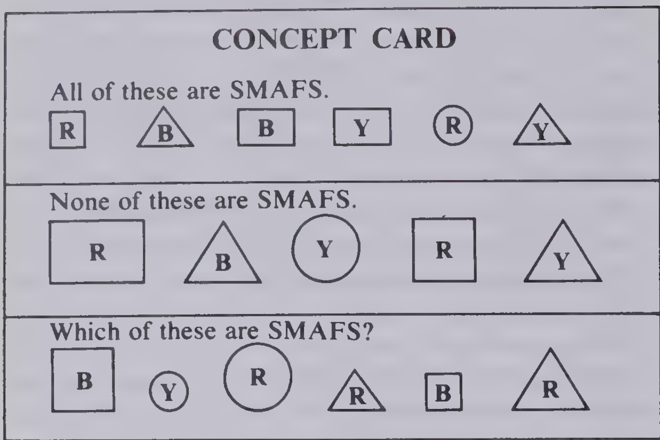


Figure 8

If you study the preceding Concept Card carefully, you will develop the simple concept of a SMAF. Notice that the key means used to teach this concept is by examples, along with *non-examples*. Both examples and non-examples play important roles in teaching many concepts in mathematics. The concept of a triangle may be taught to young children by using the Concept Card in Figure 9.

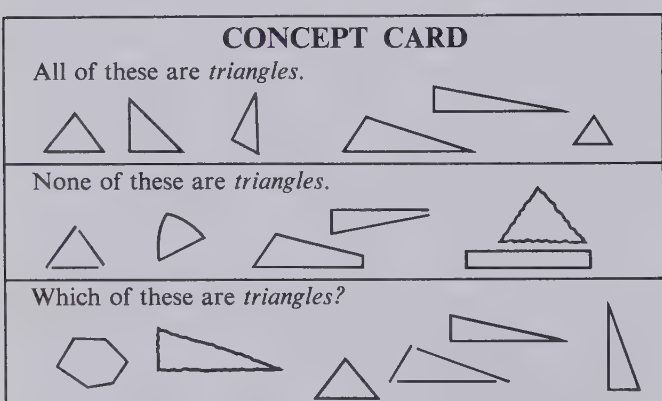


Figure 9

Clearly, the child would need further experiences in order to develop fully the concept of triangle, but the beginnings are embodied in the Concept Card shown in Figure 9.

One of the important ideas to remember when considering concepts is that concepts, unlike some other things that children learn, are developed over a period of time. Simple concepts may be developed very quickly, but other, more complicated concepts must be germinated when the child is very young and broadened through a spiralling return to the concept at various stages throughout the child's development. Many concepts are not fully developed until the child becomes an adult and encounters the idea in a variety of situations. For example, the concept of a fraction or fractional number may be introduced in grade 1 or grade 2, but a full understanding of this concept may not come until many years later. The child may acquire only an embryonic idea of a concept the first time it is presented, so it is important for the teacher to recognize the true nature of concepts and be willing to return often to the idea and carefully nurture its growth within the child. If he does not expect complete mastery after the initial presentation, he will spare himself considerable frustration when he recognizes later that the child needs further development of the basic idea.

Another key feature of concept learning suggested by the experiments of Piaget and supported and extended by the theories developed by Z. P. Dienes concerns the role of physical manipulative materials in young children's concept learning. In general, the implication of these authors' works is that it is through child involvement with physical environment that a firm basis for the development of more abstract concepts is laid. In fact, it is suggested that concept learning is facilitated by exposing children to as many different physical situations which embody the concept as possible.

It should be recognized that there are different levels of concept development and different types of concepts within these levels. For example, in the very earliest stages of mathematical learning, most concept learning involves the *concept of physical objects* such as balls, blocks, and circular or triangular objects. Very soon, the concept of certain *relations between objects* is developed: above, below, taller, shorter, larger, wider, longer, behind, and so on. A subsequent stage involves the concept of a *set of objects* such as a set of golf clubs, a set of dishes, a box of crayons, a set of blocks, a collection of stamps, or the children in a classroom. A slightly higher level of concept learning involves *relations between sets of objects:* equivalent, equal, has more than, has less than, and so on. It is at this stage that the important concept of *number* arises. For, in a sense, the concept of number involves a consideration of a set of equivalent sets. At a higher level of abstraction, the concept of certain *relations between numbers* (is less than, is greater than, is equal to, and so on) is developed. Ascending the ladder of abstraction, another level of development might involve the concept of *sets of numbers,* such as odds, evens, primes, composites, and perfects.

Clearly, the realm of concepts is vast, and the elementary teacher need not concern himself directly with many of the types of higher-level concepts. He must recognize, however, that the beginning stages in the development of many important concepts occur in the elementary school and that, through utilization of a variety of manipulative materials and appropriate strategies, he can do much to help the children learn concepts appropriate for their level.

EXERCISE SET 5

1. Use the attribute pieces shown in Figure 13. Invent a concept, name it, and make an appropriate concept card for it.

2. Choose at least two *Investigating School Mathematics* concepts from the list given below and develop concept cards which illustrate the use of examples and non-examples to teach the concepts you have chosen.
   A quadrilateral
   B simple closed curve
   C odd number
   D greater than (the relation)
   E right triangle
   F is congruent to (the relation)
   G lowest-terms fraction
   H parallelogram
   I diagonal of a polygon
   J parallel lines
   K one half
   L isosceles triangle
   M equivalent sets
   N symmetrical figure
3. Answer the questions on the sets of Creature cards from the set of attribute materials published by the Webster Division of McGraw-Hill Book Company (if available).
4. Choose an unusual concept of your own invention and make a concept card from which a person might discover your concept.
5. The investigation in Figure 1 was used to teach the concept of congruent segments. Make a card to teach this concept using examples and non-examples.
6. Complete "Learning a Concept" on pages I-18 and I-19; then answer the following questions.
   A What are some examples of the concept you learned?
   B Give some characteristics of the concept you learned.
   C What were your feelings about the lesson? How could the lesson be improved to make the learning of the concept easier?

Skills

Broadly speaking, there are several types of skills that children develop in the elementary school. Hopefully, many children will develop a skill in estimating distance, weight, capacity, and time. Some teachers may wish to help children develop skill in drawing geometric figures. Some teachers set goals for upper-grade children which include developing skills in reasoning and even in "proof" of simple ideas. In elementary mathematics the most fundamental skill, by far, is that of computation with whole and rational numbers. It is these specific computational skills involving addition, subtraction, multiplication, and division and the processes related to these operations with which we are particularly concerned in the discussion that follows.

Two types of skills, power skills and speed skills, are available for completing each arithmetic process. A *power skill* is any effective way to find an answer. A *speed skill* is the most efficient way to find an answer. A power skill is a process through which a given problem is attacked by means of some technique which, though possibly quite inefficient, can produce a correct solution. This power skill may involve a long, tedious process, one which may be totally unrelated to the most efficient method for arriving at the solution. On the other hand, when a speed skill is employed, the problem is attacked with the most efficient technique available, and the problem is solved relatively quickly, usually in a mechanical fashion.

For example, suppose a child wants to find the sum of 27 and 48. If he simply starts at 48 and counts on 27 more, he is using a power skill. If, however, he finds the answer by using the usual algorithm for addition, then a speed skill is being employed.

Two additional points are worth noting about the previous example. First, in order to utilize the power

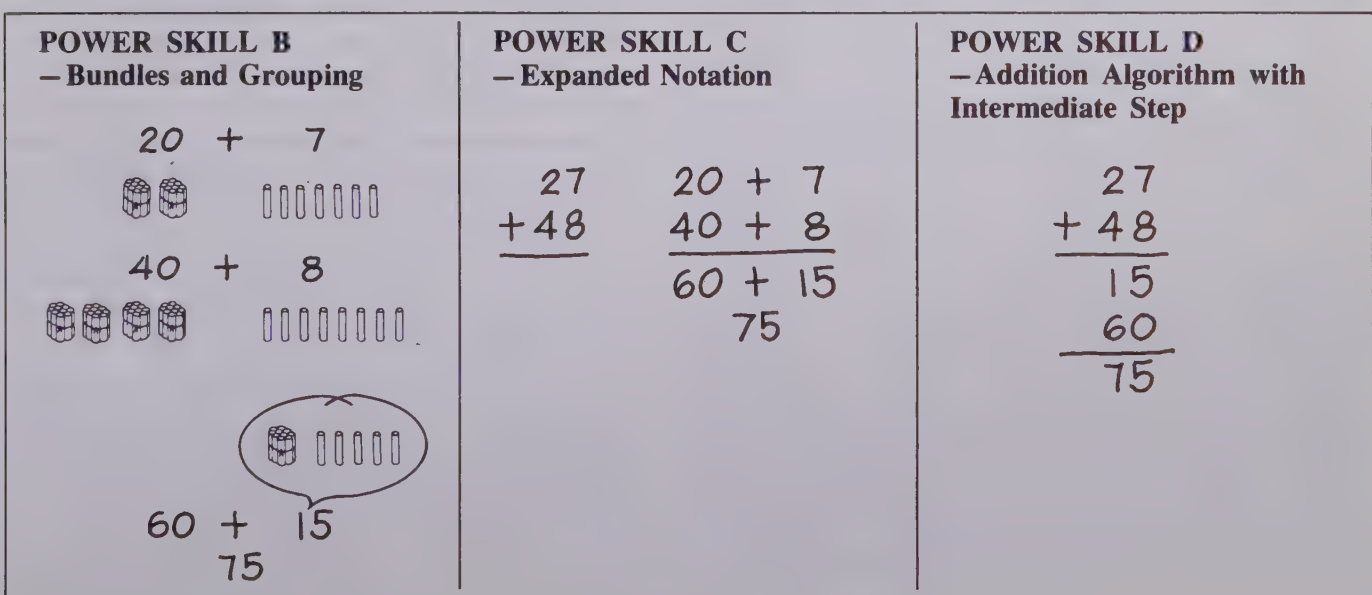


Figure 10

skill, the child needed a clear concept of addition as it relates to counting. Thus, a power skill relies on a previously learned concept. As the child uses the concept in a power-skill situation, he gains new confidence in his ability to do something he "hasn't had yet." Secondly, the teacher should observe the evolution from power to speed. In finding the sum of 27 and 48, the initial power skill involved a basic concept of addition and the counting process. In practice, the child may continue the evolutionary trek from power to speed by next utilizing power skills B, C, and D as shown in figure 10.

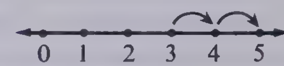
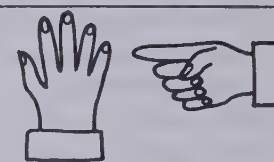
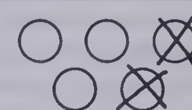
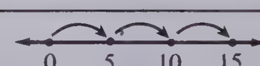
Note that each of these power skills represents a small step toward the ultimate, more efficient speed skill. When considering this process of evolution, it should also be noted that the earlier stages in a power-skill sequence often involve manipulative materials with subsequent power skills exhibiting a transition from the concrete to the more abstract. This physical beginning, which utilizes bundles and grouping, is illustrated as Power Skill B in Figure 10.

The use of power skill is available to all children. The slower child may well attempt the problem by the only means he knows, one which may often be quite laborious. For example, in finding the quotient $5863 \div 72$, the slower child might subtract 1 seventy-two at a time until he has reduced the dividend to some number less than seventy-two. The more able and creative child might tire of this method and attempt to subtract some multiple of seventy-two, such as 10 seventy-twos. Since each child is working on his own for a period of time, the development of power skill is extremely helpful in working with individual differences.

One decision that the teacher must make in relation to each child is the extent to which he should be encouraged to develop an efficient speed skill for a given algorithm. Obviously, skills are important and should be taught in elementary mathematics, yet it is the good judgment of the teacher that plays the crucial role in guiding a given child from power to speed. For certain processes, children should probably never be forced to attain a speed skill, but should be allowed to operate at the power-skill level. Other children should be directed toward the speed skill as quickly as possible in order that they may proceed to more interesting aspects of mathematics. In rare instances, a child might profit from an initial consideration of a speed skill with no previous power-skill development of a given process. The emphasis on the role of conceptual power in the performance of a skill is a key feature of the so-called "new" mathematics. It is quite probable that we cannot predict the future mathematical needs of children in our classes today, but we can help them develop the confidence, even in the area of learning skills, to utilize concepts previously learned to discover some of the basic processes for themselves.

EXERCISE SET 6

1. Write *power* or *speed* depending on the type of skill you think is being employed.

	Specific Skill	Example
A	Using sets to find sums	 $3 + 2 = \square$
B	Using number line to find sums	 $3 + 2 = \square$
C	Counting fingers to find sums	 $3 + 2 = \square$
D	Memorizing that $3 + 2 = 5$	Think: 3, 2 $\longrightarrow$ 5 $3 + 2 = 5$
E	Thinking about "take away" to find differences	 $5 - 2 = \square$
F	Using the inverse relation (missing addend) to find differences	Think: ? + 2 = 5 $5 - 2 = \square$
G	Memorizing that $5 - 2 = 3$	Think: 5, 2 $\longrightarrow$ 3 $5 - 2 = 3$
H	Using sets to find products	 $3 \times 6 = \square$
I	Using the number line to find products	 $3 \times 5 = \square$
J	Using logic (basic principles) to find products	Since $5 \times 5 = 25$, $6 \times 5 = \square$ or Since $3 \times 5 = 15$, $6 \times 5 = \square$

2. Four different power skills are shown for finding $91 \div 7$. These skills would lead up to finding this quotient by "ordinary short division."

$$7\overline{)91}^{\;13}$$

In what order should these be presented?

A

```
7)91
  70   (10)
  21   (3)
  21
   0
```

B Subtract 1 seven at a time.

```
 91
- 7
 84
- 7
  :
```

C

```
   13
7)91
  70
  21
  21
```

D Group 91 objects into sets of 7.

3. Complete the "Learning a Skill" lesson on pages I-19 and I-20; then do these exercises.
   - A Discuss the skill you learned and the way you learned in terms of power skills and speed skills.
   - B What part of the lesson helped you evolve a speed skill?
   - C What were your feelings about the lesson? How could it be improved?

Generalizations

Imagine that one of your students is engaged in an investigation in which he was asked to cut out a large quadrilateral and draw colored lines connecting the midpoints of each side of the quadrilateral. The question stimulating the investigation was, "Can you make an odd-shaped quadrilateral so that when you connect the midpoints you do not form a parallelogram?" As a result of this investigation and the subsequent discussion of his findings, the child was led to form a generalization: "The segments connecting the midpoints of any quadrilateral form a parallelogram."

In another lesson, a child might be responding to an investigation question which asked: "If you cut off the corners of a triangle and place the tips at the centre of a circle, what part of the circle can you cover? Can you find a triangle for which this is not true?"

As the child completes the investigation and engages in the discussion which follows, he forms this tentative, unproved *generalization:* "If a compass is used to draw arcs on the corners of any triangle and these corners are cut off along the arcs, then these corners will cover exactly one half of a circle drawn with the same compass opening." This tentative generalization, of course, is the forerunner of the familiar generalization that the sum of the degree measures of the three angles of any triangle is 180.

A generalization provides the economy of moving from consideration of isolated, specific cases to a general statement which holds true for a complete set of numbers or geometric figures. For example, the generalizations stated above deal with the set of all quadrilaterals and the set of all triangles. The regular occurrence of the word "any" in the generalization statements implies that the observation is true for *every* such geometric figure.

The key to teaching a generalization effectively is to provide children with appropriately chosen examples (or instances) which lead them to the generalization. An approach often used by teachers to help children learn generalizations is that of *guided discovery*. In this approach the teacher uses carefully sequenced questions and carefully chosen examples to focus the child's thought on the generalization to be discovered.

It is instructive for children in the upper elementary grades to have experiences in forming generalizations which seem obvious from a set of examples, but which, in fact, do not hold true. For example, consider the equations below.

$$\boxed{1} \times \boxed{1} - \boxed{1} + 11 = 11$$

$$\boxed{2} \times \boxed{2} - \boxed{2} + 11 = 13$$

$$\square \times \square - \square + 11 = ?$$

Figure 11

If 1 is written in the box and the operations are performed, the result is 11, which is a prime number. If 2 is written in the box, the result is 13, also a prime number. Upper-grade children are likely to conjecture that the sum is always a prime number. When they try 3, the sum is 17, also a prime. Similarly, the child finds that the numbers 4, 5, 6, 7, 8, 9, and 10, when written in the box produce a prime number. A child accustomed to forming generalizations from even fewer examples than this will likely conclude that this formula will always produce a prime number. It is instructive to note that when the next number, 11, is written in the box, the result is 121, which, being divisible by 11, is not a prime. This example illustrates the important idea that, even though the generalizations the child might make seem quite plausible and are most often true, it is only by means of a mathematical proof of a generalization that one can be completely sure that it is correct. These proofs, of course, are often not accessible to elementary school children. Thus, a healthy attitude might be characterized by references to generalizations which include phrases such as, "appears to be true," "is probably true," or "could most likely be proven."

Often a search for a generalization is initiated by a question such as, "Do you see any patterns?" For example, several simple generalizations might be formulated about the multiplication table in Figure 12. One child might observe that every number on the main diagonal of the table is a square number. Another student might observe that for every number on one side of this main diagonal, such as 10, there is a matching number symmetrically placed on the other side of the main diagonal.

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

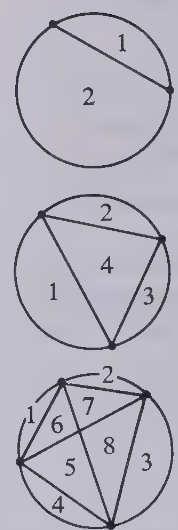
Figure 12

This last generalization is the table counterpart of the commutative principle for multiplication; that is, in the case of 12, $6 \times 2 = 2 \times 6$ or $4 \times 3 = 3 \times 4$. Another generalization that might be reached by careful consideration of the table is that the only primes in the table occur in the one-row or one-column of that table. Still another interesting generalization suggested by the table is that the sum of any number in the two-row and a number below it in the five-row will equal the number below these numbers in the seven-row. Of course, there are many other generalizations ranging from the very simple to the more complex that could be made about this multiplication table.

Perhaps the illustrations above will suggest that the mathematics available to the elementary school child is replete with possibilities for discovery of generalizations. The teacher's task is to create a learning environment in the classroom, not only in terms of physical materials and situations, but in terms of attitude toward learning and toward children, which provides opportunities for discoveries of generalizations and an atmosphere in which it is rewarding to make these discoveries. The teacher should be ever aware of the possibility that the habit of seeking generalizations may well be one of the most valuable things the child learns from his experiences in mathematics.

EXERCISE SET 7

1. Choose a text from the *Investigating School Mathematics* series and list some generalizations which the students who study this text might discover.
2. Investigate the Madison Project shoe boxes and complete the activities for at least two boxes.
3. The illustrations and the table which follow show that if you connect two points on a circle, you divide the interior of the circle into two regions; if you connect three points on a circle, you divide its interior into four regions; if you connect four points on a circle, you divide its interior into eight regions. Note that the points chosen should not be evenly spaced on the circle.



Number of points on a circle	Number of regions formed inside circle
2	2
3	4
4	8
5	
6	

   A Fill in the table to show how many regions are formed if five points on a circle are connected.
   B Form a generalization about the right-hand column of the table.
   C Test your generalization by finding out how many regions are formed inside when six points on a circle are connected.
4. Devise an investigation which might enable a student to discover this generalization: "The sum of the degree measures of the angles of a quadrilateral is 360."
5. Write some questions you would ask and show some examples you would use in guiding a child to discover one of the following generalizations.
   A The commutative principle for multiplication
   B The volume of a "box" is found by multiplying length times width times height.
   C In measuring length, the shorter the unit, the greater the measure.
   D Any angle inscribed in a semicircle is a right angle.
   E Every even number ends in 0, 2, 4, 6, or 8.
6. Complete the "Learning a Generalization" lesson on page I-20; then answer the following questions.
   A What generalization did you learn from the lesson?
   B How many specific examples did you consider before you understood the generalization?
   C In what way did you use the generalization after you discovered it?

Facts

In elementary mathematics, there are certain bits of information that are used so frequently that it is

beneficial for the child to be able to recall them quickly when they are needed. These items are ordinarily called *facts*. There are three main types of facts that are of major concern. The first type of fact is one which evolves from a concept. It might be an example of a specific concept ("Two is a prime number," "25 is a square number," "A parallelogram is a quadrilateral"), or it might be a characteristic of a specific concept, possibly even a part of the definition for the concept ("An isosceles triangle has two congruent sides," "An even number is a number divisible by two," "A pentagon has five sides"). Examples of, or characteristics of, concepts are not always considered as facts; only if such an example or characteristic is deemed important enough to be remembered for immediate recall, is it considered to be a fact and committed to memory.

A second type of fact is a fact derived from a generalization; that is, if a generalization is simple, or deemed important enough to remember for immediate recall, it might often be considered a fact. For example: "The sum of the squares of the lengths of the legs of a right triangle is equal to the square of the length of the hypotenuse of the right triangle"; or "The length of the segment joining the midpoints of two sides of a triangle is one half the length of the third side." Each of these statements might be considered facts since they are sometimes useful for immediate recall. A third type of fact—one that is given a great deal of attention in the elementary school mathematics program—is the type of fact derived from a power skill. For example, the child may have utilized a sequence of power skills for finding sums such as $4 + 3$. He may have used sets of counters, centimetre strips, jumps on the number line, or reasoning from facts such as $3 + 3 = 6$. These power skills, based on certain important concepts, provided the evolutionary progression toward the final speed skill used in finding sums. In this particular case, however, the speed skill used is simply that of memorizing the sum. Whenever the speed-skill stage involves memorization, the particular learning which was classified as a skill or a process during the power-skill stage is reclassified as a fact. The basic addition and multiplication facts fall into this category, and they are given major attention in the elementary school. It is these facts to which primary attention will be given in this section.

A first important point to be made in discussing the teaching of facts is that extensive power-skill work preceding the memorization stage can pay valuable dividends. The broad base of understanding provided by the power-skill work removes the aura of magic from this aspect of mathematics and not only makes the task of memorization of the facts easier, but helps the child view it as a "reasonable thing to do." Figure 13, for example, shows some of the power skills that might be utilized in the initial development of procedures for finding products. Careful development using some or all of these power skills can give the child a basic feeling for a procedure by which products may be found.

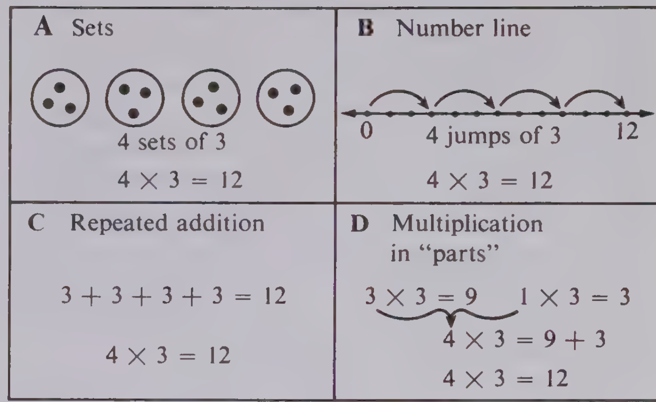


Figure 13

The teacher must use good judgment in deciding when a given child should move from this power-skill stage to memorization of the facts. The appropriate time could vary extensively depending upon the ability and experiences of the child. If the power-skill work is started early in the elementary grades, the child will have ample time to reap the benefits of this basic experience with materials and concepts before the transition to speed skill is made.

When the time has come to memorize the facts, it is important for the child to have a clear idea of the nature of this goal and the reasons it is appropriate. The teacher should even take time to help the child see the very clear difference between "figuring out the fact" and "memorizing the fact." Hopefully, he could help the child develop a feeling for situations in which the facts will be used and in which immediate recall would be quite valuable and time-saving for the child.

After the addition or multiplication facts to be memorized have been placed in perspective, the teacher should seek interesting situations and creative ways in which to practice recalling the facts. For example, the children might make their own flash cards and use a timer to see how long it takes them to give these facts. If desired, two children could work together and see which of them could go through the flash cards most quickly. Another game utilizes a pair of homemade colored dice and an empty multiplication table (Figure 14) for each child. As the game

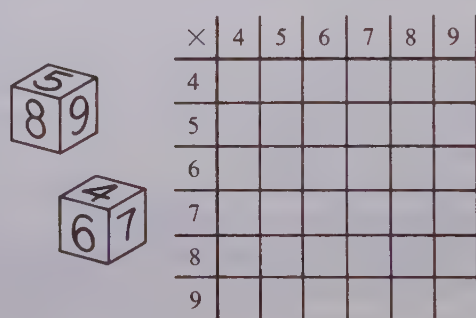


Figure 14

proceeds, a child rolls his dice and writes the product of the numbers on the dice in the appropriate space on his multiplication table. His partner then does the same thing when it is his turn. If a child arrives at an incorrect product or writes the product in the wrong space in the table, he is penalized by missing a turn. The object of the game is to see who can complete the table first. Various modifications of this game are possible, including one in which each child works independently and keeps a tally of the number of times he rolls the dice and also keeps track of the time it takes. The basic objective, of course, is to provide an interesting situation in which the child is motivated to recall multiplication facts rapidly.

Some children may need to spend considerable time in the power-skill stage before they begin to memorize. If there are children who have attempted to memorize the facts and find the job more difficult than anticipated, the teacher may want to consider allowing them to prepare a fact card on which they write the facts that they still do not know. Perhaps it would be realistic and beneficial to let some children use this fact card during the year whenever they desire, thus relieving the tension that could result from difficulties they encounter in memorizing the facts at one specific time. As the school year progresses, the teacher may want to suggest from time to time that a particular child concentrate on one of the troublesome facts and attempt to memorize it so that he can remove it from his fact card. The accomplishment of this goal, of course, would merit recognition and reward. After one fact is removed, the child might start working on removing another fact. The ultimate goal would be to remove all of the facts by the end of the year. Teachers who are interested in helping children learn mathematics in a comfortable way may find that a more realistic, less pressured approach to learning facts may enable the child to find greater enjoyment and success in his mathematical experience.

EXERCISE SET 8

1. Invent a game that could be used to help children practice recalling addition or multiplication facts.
2. Find a commercially produced game that is designed to help children practice recalling facts.
3. Complete the "Learning Some Facts" lesson on pages I-20 and I-21 of this text; then answer the following questions.
   - **A** How many of the facts did you know?
   - **B** What techniques did you use to help you memorize the remaining facts? Did you find this lesson difficult?
   - **C** Can you imagine some of the difficulties your children might have in learning facts?
   - **D** Did you find any mnemonic devices which were helpful in remembering the facts?

Attitudes

In his poem "Arithmetic," Carl Sandburg wrote:

Arithmetic is numbers you squeeze from your head
  to your hand to your pencil to your paper
  till you get the answer.
Arithmetic is where the answer is right
  and everything is nice
  and you can look out of the window
  and see the blue sky—
  or the answer is wrong
  and you have to start all over and try again
  and see how it comes out this time.

. . . . .

Arithmetic is where you have to multiply—
  and you carry the multiplication table in your head
  and hope you won't lose it.*

The attitude toward mathematics, school, one's ability, and learning in general that one senses on reading this part of Sandburg's poem is surely typical of many children in classrooms today. Perhaps, it was a feeling similar to this that caused Huckleberry Finn to say:

> I had been to school 'most all the time and could spell and read and write just a little and could say the multiplication table up to six times seven is thirty-five, and I don't reckon I could get any further if I was to live forever. I don't take no stock in math, anyway.

There are many different kinds of attitudes exhibited by children who have been exposed to classroom mathematical experiences in different parts of the world. There are, of course, the more general attitudes that a child has toward his teacher, toward his school, toward his fellow students, and toward the process of education. All too often the child's attitude toward education in general is that suggested by Charles Schulz in this *Peanuts* cartoon.

*From *Complete Poems*, copyright, 1950, by Carl Sandburg. Reprinted by permission of Harcourt Brace Jovanovich, Inc.

Two of the attitudes to be considered here, however, are the child's attitude toward mathematics and the child's attitude toward himself as he relates to mathematics. It has been said that the mathematical experiences of a child before the age of 11, and the responses he has been encouraged to make to those experiences, largely determine his potential mathematical development. If this is so, then a child's attitude toward mathematics and his feelings about how he relates to mathematics are extremely important considerations for the classroom teacher.

A moment's reflection on the number of people who are willing to say that they hate mathematics and on the multitude of others who seem to harbor a fear regarding their inability to cope with ideas of mathematics leads the teacher to realize that he does indeed teach attitudes, whether he tries to or not. Clearly, the teacher who conducts a classroom in which children's achievements are evaluated almost exclusively on the basis of how many right answers they can come up with must surely engender attitudes in children which differ greatly from those engendered by the sensitive teacher who recognizes the child's need to think his own thoughts and to become involved in an exciting exploration of ideas that interest him. Or, consider the difference between the teacher who teaches only speed skills and facts and the teacher who recognizes the central importance of concepts and generalizations, as well as the facts and skills. The child exposed to the first teacher must surely have a feeling toward mathematics, and his ability to interact with it, that is far different from that of the child who learned with the second teacher.

If what happens in the classroom is of such importance in developing attitudes within the child, then the teacher may want to reevaluate his approaches to instruction by reconsidering certain fundamental questions. What subject matter and methods most effectively instill within the child the feeling that mathematics is interesting, fun, and a source of adventure? Will these means provide an opportunity for the child to exercise his freedom of choice and to make decisions about what he does with mathematics? Aldous Huxley said: "A child is a genius until the age of ten." Could it be that our classroom approaches squelch this genius? Can we select mathematical experiences and materials that enable the children to experience success and thus maintain that sense of worthiness and prestige with peers that is of such importance? Can we structure these experiences in such a way that the child maintains within this atmosphere of freedom a sense of security and safety, thus avoiding the fear that can erode his ability to approach mathematical situations with confidence? Can we help children see the usefulness and importance of mathematics without boring them?

Clearly, the questions just raised are difficult to answer and specific techniques for developing healthy attitudes are hard to come by. But even though prescriptions for developing attitudes are scarce, many of the ideas about teaching suggested in earlier sections of this text can provide assistance for the teacher. The investigation, for example, provides the child with an opportunity to make independent decisions and to interact with mathematics and materials and encourages him to take responsibility for his own learning. As difficult as it may seem at times, a child's acceptance of responsibility for his own learning inculcates an attitude that is ultimately invaluable. Also, the manipulative materials or activities that are made available to the child in the investigation situation provide an interaction with the physical world that is often extremely valuable in making mathematics real to a student. Unless a child is ready for more abstract thinking, he cannot be induced to sense the adventure in mathematics without a physical environment to explore. Opportunities for attitude development are implicit not only in the investigation phase of a lesson but in the discussion as well. If a teacher can convince the child that his ideas are important, then the child finds himself in a situation, albeit a mathematical one, in which *he* feels important. His prestige with his peers increases and he feels successful. Exercises in the utilization phase of a lesson that begin simply and gradually increase in difficulty can also help the child feel that he can do mathematics on his own; and, of course, carefully selected extension activities can provide the child with a variety of opportunities to experience the fun of mathematics.

Not only do the phases of the learning experience provide unique opportunities of attitude development, but the particular types of learnings involved within these phases also have their effect. The teaching of concepts and generalizations provides the child with a feeling of power regarding mathematics, for when he experiences the thrill of discovering a concept or a generalization, or when he uses these to solve a problem, he is also developing a useful and wholesome attitude toward mathematics learning. He is developing a habit of reacting to a mathematical situation which will be invaluable when he later encounters mathematical situations possibly undreamed of today. Also, careful teaching of skills and facts can provide the child with that basic sense of security that comes simply from being able to do something or to remember something.

Figure 16

Regarding the child's level of confidence in his ability to cope with mathematical problems, one of the child's paramount needs is to experience success, and as mentioned previously, having entertaining experiences with mathematics might decrease the fear that can erode his confidence. To provide these experiences, the teacher might create in the classroom a "Fun with Mathematics" centre (see Figure 16) that contains mazes, puzzles, design materials, and so on. This centre represents an extra effort to encourage the child to successfully play with mathematics. Some of the materials that might be in such a centre are as follows: the soma cube, the tangram pieces, 2-cm cubes, materials for curve stitching, a kaleidoscope, pattern blocks, Cuisenaire rods, multi-base arithmetic blocks, geoboards, a wide variety of counters, attribute blocks, scales and balances, timers, calendars, measuring tapes and rulers, yarn and string, an assortment of boxes and cans, magazines and catalogues, mirrors, dice, play money, graph paper, assorted plane and solid shapes, abacus, pegboard, compass, mathematical balance, etc.

Perhaps, as you consider the attitudes more carefully and reevaluate the effects of your approaches to instruction, you will find other ways to help children develop a healthy attitude toward mathematics and an enthusiasm for the enjoyment it can offer. Each day as the teacher enters the classroom with plans for a learning experience, he might well ask himself: "What effect will *this* have on the attitudes of the students in my classes?"

EXERCISE SET 9

1. Select a text from the *Investigating School Mathematics* series and find at least five activities which could contribute to the child's development of a positive attitude toward mathematics.
2. Explain how you think some of the other types of learning might also contribute to better child attitude toward learning in general and mathematics specifically.
3. Complete the "Learning an Attitude" lesson on page I-21 of this text; then answer the following questions.
   A Was the lesson fun?
   B How did you feel when you had finished the lesson?
   C Did the lesson change any of your ideas about mathematics?

IV. Some Learning Experiences for the Teacher

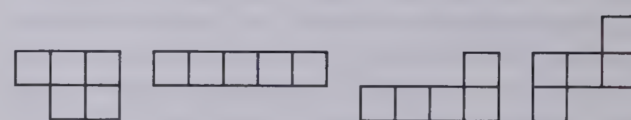
In Section II you were introduced to an outline for a learning experience which involved preparation, investigation, discussion, utilization, and extension. In Section III the types of things children learn—concepts, skills, generalizations, facts, and attitudes—were categorized. In this section, we combine these ideas and use them in presenting five learning experiences designed especially for the teacher. That is, in order to gain a first-hand view of lessons which develop these types

Lesson 1. Learning a Concept

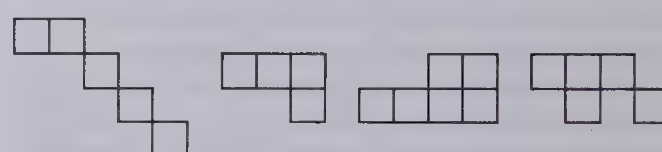
What is a pentominoe?

Investigating the Ideas

Each of these is a **pentominoe.**



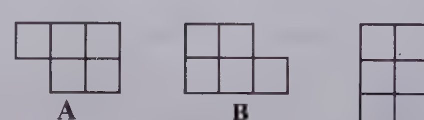
None of these is a **pentominoe.**



How many more pentominoes can you find and show on graph paper?

Discussing the Ideas

1. How many pentominoes did you find?
2. Can you give some characteristics of a pentominoe?
3. How would you "broadly classify" a pentominoe?
4. Can you define a pentominoe?
5. Are the pentominoes in Figures A, B, and C the same?



A B C

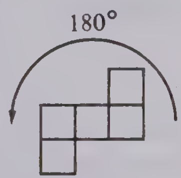
6. How could you convince someone that you have found all possible pentominoes?

of learning, the teacher will have experiences with each of these in the five lessons; and, in order to become more familiar with the suggested structure for a learning experience, each of these five lessons will involve an investigation, a discussion, a utilization, and an extension of the ideas.

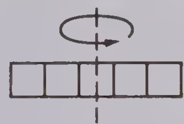
It might be valuable for the teacher, after he has become involved in each of these lessons and has completed the activities, to rethink and discuss his reactions to the various phases of the lesson structure and to the various types of learnings involved. In this way, he might gain a new insight into the way the children in his classes might react to these kinds of situations.

Using the Ideas

1. Which of the pentominoes can be folded to form a box with the "lid missing"?
2. Some pentominoes can be rotated about a point 180° and returned to their starting position. These pentominoes are said to have 180° rotational symmetry. Which pentominoes have 180° rotational symmetry?



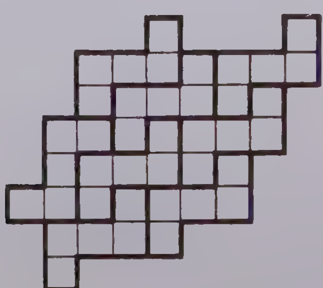
3. Some pentominoes can be flipped about a line and returned to their starting position. Such pentominoes are said to have reflectional symmetry. Which pentominoes have reflectional symmetry?



4. What do you think a hexominoe would be? How many hexominoes can you find?

Extension

Some pentominoes can be used to tessellate (fill without overlapping) the plane, as shown below. Can you find at least two more pentominoes that can be used to tessellate the plane? Show the tessellations on graph paper.



Lesson 2. Learning a Skill

Can you find the product of two 2-digit numbers "in your head"?

Investigating the Ideas

Follow these steps for writing the *answer only* for 74×36.

Step 1	Step 2	Step 3
Think $4 \times 6 = 24$	Think $4 \times 3 = 12$ $7 \times 6 = 42$ 54 Add 2 2 56	Think $7 \times 3 = 21$ Add 5 5 26
Write 4 Remember 2	Write 6 Remember 5	Write 26
$\begin{array}{r} 36 \\ \times 74 \\ \hline 4 \end{array}$	$\begin{array}{r} 36 \\ \times 74 \\ \hline 64 \end{array}$	$\begin{array}{r} 36 \\ \times 74 \\ \hline 2664 \end{array}$

Can you use this method to write answers only for the products below? Check your answer using the "long" method.

$\begin{array}{r} 53 \\ \times 48 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 37 \\ \times 62 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 45 \\ \times 23 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 67 \\ \times 32 \\ \hline \end{array}$

Discussing the Ideas

1. Explain this statement: In Step 1 you are finding the number of ones.
2. In Step 2 you are finding the number of __?__.
3. The 2 you remembered is really 2 __?__.
4. Explain what you are finding in Step 3.

Using the Ideas

Write answers only for each product.

1. $\begin{array}{r} 28 \\ \times 42 \\ \hline \end{array}$ 2. $\begin{array}{r} 46 \\ \times 33 \\ \hline \end{array}$ 3. $\begin{array}{r} 37 \\ \times 42 \\ \hline \end{array}$ 4. $\begin{array}{r} 82 \\ \times 56 \\ \hline \end{array}$ 5. $\begin{array}{r} 53 \\ \times 34 \\ \hline \end{array}$

6. $\begin{array}{r} 64 \\ \times 27 \\ \hline \end{array}$ 7. $\begin{array}{r} 29 \\ \times 63 \\ \hline \end{array}$ 8. $\begin{array}{r} 48 \\ \times 35 \\ \hline \end{array}$ 9. $\begin{array}{r} 53 \\ \times 53 \\ \hline \end{array}$ 10. $\begin{array}{r} 27 \\ \times 64 \\ \hline \end{array}$

Extension

1. Study the figures below for finding the product of two 3-digit numbers.

$\begin{array}{r} 352 \\ \times 436 \\ \hline 2 \end{array}$ $\quad$ $\begin{array}{r} 352 \\ \times 436 \\ \hline 72 \end{array}$ $\quad$ $\begin{array}{r} 352 \\ \times 436 \\ \hline 472 \end{array}$ $\quad$ $\begin{array}{r} 352 \\ \times 436 \\ \hline 3472 \end{array}$ $\quad$ $\begin{array}{r} 352 \\ \times 436 \\ \hline 153472 \end{array}$

2. Use the method shown in exercise 1 to find each product.

$$\begin{array}{cccc} 125 & 757 & 841 & 525 \\ \times 365 & \times 426 & \times 215 & \times 525 \\ \hline \end{array}$$

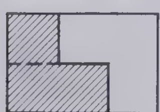
*3. Devise a rule for multiplying two 4-digit numbers.

Lesson 3. Learning a Generalization

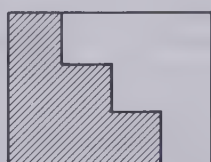
Can you find a pattern?

Investigating the Ideas

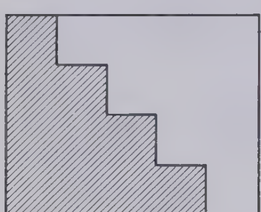
Use the small square in Figure A as the unit. Can you find the area of each shaded part in two different ways? For each part, write an equation to show that the two ways of calculating the area give the same result.



A



B



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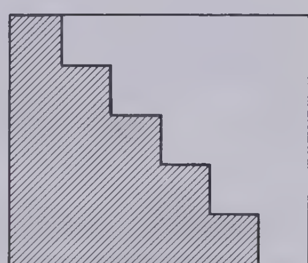
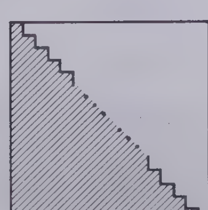


D

Discussing the Ideas

1. A Describe one way you found for finding area in the figures above.
   B Describe another way you found.
   C Did you find any other way?
2. Can you write an equation to show that these two methods give the same area?
3. A Suppose there are 50 vertical segments in the "stairsteps" of Figure E. What is the area of the shaded part?



   B Which of the two methods for finding the area would you use?
   C Can you write an equation about this?
4. Can you find the area of the shaded portion of Figure E if there are 100 vertical segments?
5. Can you use what you have learned so far to explain this generalization?

$$1 + 2 + 3 + 4 + 5 + \ldots + n = \frac{n \cdot (n+1)}{2}$$

Using the Ideas

1. Without adding each number, find the sum of the whole numbers through 25.
2. Find the sum of the first 75 whole numbers.
3. Find the sum of the first 200 whole numbers.
4. What is the sum of the first 1000 whole numbers?

Extension

1. What is this sum? $50 + 51 + 52 + 53 + \ldots + 99 + 100$
2. Can you find a short way to find the sum of
   A these even numbers? $0 + 2 + 4 + 6 + 8 + \ldots + 100$
   B these odd numbers? $1 + 3 + 5 + 7 + 9 + \ldots + 99$

*3. Can you state a rule for what you found in exercise 2 by using a variable?

Lesson 4. Learning Some Facts

Can you learn some "new" facts?

Investigating the Ideas

Many rapid "human Calculators" consider these products to be facts.

×	10	11	12	13	14	15
10						
11						
12						
13						
14						
15						

How many of these "facts" can you give without calculating?

(Record the facts you know and shade that portion of the table with a red pencil. Then fill in the remainder of the table by figuring out the remaining facts.)

Discussing the Ideas

1. Which facts in the table need not be memorized provided you know the others and also know the commutative principle? Shade these facts blue.
2. A How many facts altogether are in the table?
   B How many facts remain to be memorized?
3. A What is the "largest" fact?
   B Which facts are over 200?
   C Which facts are in the 190's?
   D Do you notice other patterns in the table that might help you remember certain facts?

USING THE IDEAS

1. Give these products as quickly as possible.

A 15 × 15	**E** 13 × 13	**I** 11 × 13
B 15 × 14	**F** 14 × 12	**J** 11 × 14
C 14 × 14	**G** 11 × 11	**K** 11 × 15
D 13 × 15	**H** 11 × 12	**L** 12 × 13

2. Make flash cards for the "facts" in exercise 1 that you do not know. Practice with a friend.
3. In exercise 1, start with part L and, following reverse order, give each of the products as quickly as possible.

*4. Make a large multiplication table with all numbers up to 20. Mark out the "facts" you know. How many of these "facts" are left to memorize?

*5. A person who knew the distributive principle and the facts in the table referred to in exercise 4 looked at the multiplication 143 × 15 and wrote 2145. How did he do it so quickly?

EXTENSION

Study the facts for these powers of 2.

$$2^2 = 2 \times 2 = 4$$
$$2^3 = 2 \times 2 \times 2 = 8$$
$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$
$$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$$

1. Give the next six powers of 2.

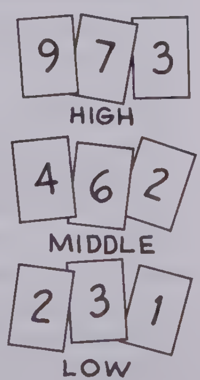
*2. Can you find some mnemonic aids to help you memorize the first ten powers of 2?

Lesson 5. Learning an Attitude

Let's try a place-value game.

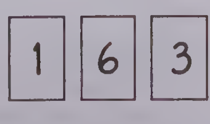
INVESTIGATING THE IDEAS

Use 3 sets of 9 cards, each with the digits 1 through 9. Shuffle the 27 cards and deal 3 to each player. Each player then forms a 3-digit numeral, places his cards face down in order, and declares (starting to dealer's left and rotating clockwise) whether his number is high, middle, or low. Play the game in groups of three players.

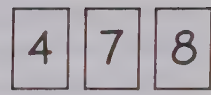


DISCUSSING THE IDEAS

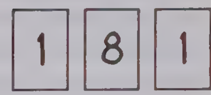
1. One player arranged his cards like this and declared that he would try for the low hand. What was wrong with his strategy?



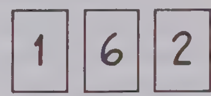
2. What is wrong with this arrangement for a middle hand?



3. If you were dealt these cards, would you try for a high or low? Why?



4. Suppose you are last to declare. Everyone else has declared either low or middle. What would you do with these cards?



USING THE IDEAS

1. Try playing this game with 2 or more other people.
2. Try the game with the rule that you can declare only high or low.
3. Make up rules for a game in which you turn up the cards one at a time starting with the ones' digit card.

EXTENSION

1. Invent a place-value game in which 4 or 5 cards are dealt to each player.

*2. Find or invent another game or activity that strengthens understanding of the concept of place value.

V. Some Thoughts About Evaluation

The strategy of preparation, investigation, discussion, utilization, and extension is a flexible organizational plan that allows each teacher an opportunity to make a modest beginning toward an activity-oriented mathematics program. The lesson categorization of concept, skill, generalization, fact, and attitude provides a framework that allows each teacher an opportunity to apply the teaching strategy to various types of learning situations. Since there are different types of learning, it is reasonable to assume that there should be different types of evaluation used to measure these learnings.

When considering the facts and skills, for example, emphasis should be placed on child accountability. The teacher should determine the learning outcomes, consider performance objectives for these outcomes, and help the child attain these objectives. The evaluation of this attainment is most easily completed by use of fact and skill tests which determine the child's level of achievement. Since the child needs considerable practice in remembering facts and performing skills, the procedure for helping them is reasonably straightforward.

When evaluating concepts, generalizations, and attitudes, however, the desired performance objectives are often quite difficult to verbalize. We have mentioned earlier that concept learning often takes place

over a relatively long time span, that concepts are extended and broadened, and that concepts mature with each subsequent set of related experiences. Clearly, it is difficult to write a performance objective which specifies the exact level of concept maturity appropriate for a given child at a given time. Whenever possible, objectives for simple concepts should be written, and an attempt should be made to write test items which will show whether children understand these concepts. These items should involve requests for children to give examples of concepts, characteristics of a concept, and even, in some cases, a definition of the concept. For more difficult concepts, the evaluation of children's progress might be made through observation and recorded by means of a check-list which specifies certain levels of development for the given concept. The teacher should be alert for situations in which the child actually uses the concept correctly and should recognize also that understandings which are only partially developed indicate positive achievement. The teacher should also search for instances where the child has shown an ability to form concepts, for this is one of the desired learnings.

When evaluating the child's understanding of generalizations, the teacher should specify the simple generalizations which should be learned by all children. Specific performance objectives and the subsequent test items should be written to evaluate these generalizations. Beyond this, the teacher should again evaluate in greater depth through personal observations or interviews with the children. In the area of generalizations, the teacher should be ever aware that a child who is in the habit of looking for patterns or generalizations has learned a great deal. The teacher should also recognize that a child who can form a generalization from a sequence of specific examples has developed an understanding of a process that is extremely important. We would be remiss if we evaluated only the factual part of the learning of generalizations. As noted earlier, however, although these are important goals of mathematics learning, it is very difficult to write performance objectives for these goals. Whenever possible, objectives should be written which go deeper than facts and skills, but in the absence of objectives, the teacher should feel free to use other means of evaluation, including interviews to evaluate student learning.

While attitudes are not easy to measure in a conventional way, it is suggested that teachers frequently observe children and talk to them about their feelings about mathematics. It is important to realize that one's philosophy toward testing can also have a marked influence on the child's attitude toward mathematics. Testing should be reasonable and realistic, and the child should understand its purpose. The spirit of evaluation should be one of helpful assessment, rather than of critical evaluation. If children participate with teachers in understanding (if not in developing) the goals of instruction, the testing procedure can be a positive influence on the child's attitude and ability to improve.

It is hoped that the teacher will constantly take a broad view toward evaluating mathematics learning among his children. In the long run, evaluation of a child's learning should depend upon the interaction of that child and his teacher. For this interaction to be successful, it may be necessary for the teacher to reexamine his own beliefs about how children learn mathematics. As each teacher makes modest beginnings toward an activity-oriented approach to mathematics learning, he might ask himself the following questions:

1. Do I respect each child as an individual with unique interests, abilities, sensitivities, and significant thoughts?
2. Does the learning environment of my classroom provide a natural, free atmosphere in which children can explore, make decisions, be independent, and encounter exciting new experiences?
3. Does the learning experience also include a supportive, non-judgmental atmosphere in which children have enough routine activities to provide a comfortable threshold of security?
4. Is the child's need for earned success recognized in my classroom?
5. Do I recognize and treat mathematics as a dynamic, ever-growing discipline which offers limitless new vistas to be explored and an inexhaustible variety of new problems to be investigated and solved?
6. Do I view mathematics as a subject of beauty and a source of pleasurable fulfillment of intellectual curiosity?
7. Do I appreciate the significance of my role as a fellow-learner rather than merely a source of information?
8. Is my overall attitude toward mathematics one that encourages a basic freedom to learn through use of manipulative materials in an investigative environment, and through free discussion and exchange of ideas?

As a teacher evaluates the children in his class, he should also reevaluate his approach to mathematical learning. The goal of this short text has been to help in that reevaluation by encouraging the teacher to read, study, observe, experience, experiment, and reconsider. If that goal has been achieved, perhaps his resulting basic beliefs about children, mathematics, and evaluation methods will help him create a new climate of interaction that will spark more effective learning experiences in his classroom.

EXERCISE SET 10

1. Give a set of performance objectives for each lesson completed in Section IV.
2. Create an evaluation tool for each set of behavioral objectives given in exercise 1.

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INTRODUCING THE METRIC SYSTEM

Canada is committed to the metric system of measurement. You may be aware of this but may not have a clear idea of exactly what the metric decision means to you as a *teacher*. It is hoped that this section will serve three purposes—

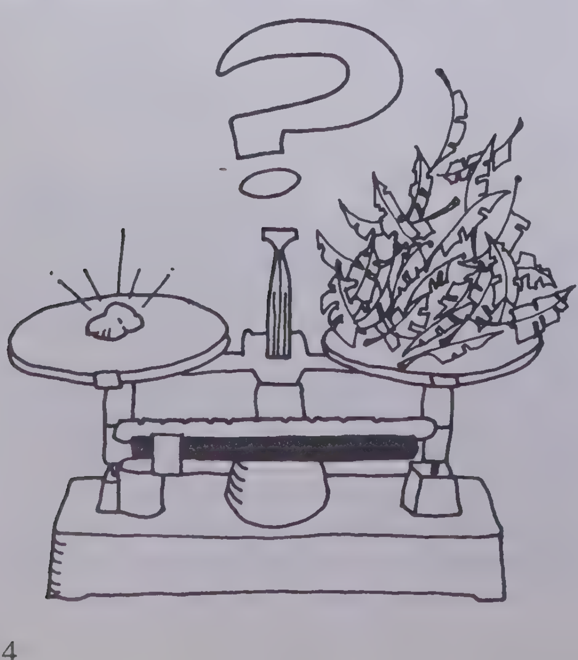
1. give you an idea of how the metric decision will affect you,
2. help you understand the metric system of measurement, and
3. give you some hints for teaching the metric system of measurement to your students.

History and Rationale

The English system of measurement developed from man's need to measure size and distances using units from the most readily available object—himself. He utilized his palm, span, finger, an ell, and a fathom for length; his foot, step, pace, an arrow's flight, and a day's journey for distance; and a handful, shellful, hornful, or gourdful for capacity.

There was little need for standardization until man began to travel and trade with other men. When "standard units" were developed, a new problem arose. Different countries used different definitions for the same unit. The foot was, at first, the length of any man's foot. In some countries, it was the length of the king's foot (since he was the "ruler") and this foot could change as the "rulers" changed. Later an effort was made to standardize some units; for example, England and Scotland decreed the foot to be 12 inches. Unfortunately, England and Scotland didn't use the same definition for the inch.

Today, in the age of technology, one still finds different units in those countries which are not yet metric. Canada and the United States are neighbouring countries, yet they use two different definitions for the gallon. A question at which people in metric countries must laugh is "Which is heavier, a pound of gold or a pound of feathers?" A pound of feathers is heavier since feathers are weighed by the avoirdupois pound (1 avoirdupois pound—7 000 grains) and gold is weighed by the troy pound (1 troy pound—5 760 grains). Which is heavier, an ounce of gold or an ounce of feathers? An ounce of gold is heavier. There are 12 ounces in the troy pound, so one ounce of gold weighs 480 grains; there are 16 ounces in the avoirdupois pound, so an ounce of feathers weighs 437.5 grains.



Out of such confusion there developed a need for a simple, standardized system of measurement. In 1670 Gabriel Mouton, a French abbé, developed a system of measurement organized according to the decimal system of numeration. It took over a hundred years for a system of measurement like the one Mouton put forth to get official sanction. In 1790 the French National Assembly appointed a committee to study the measurement situation and see if a rational system of measurement was possible. In 1795 France adopted a decimal system of measurement, defining the base unit of length to be the *metre* (from the Greek word *metron*, "a measure").

The metric system did not use parts of the human body as units. The metric system did not develop haphazardly adding more and more units as the need arose. The metre was defined as one ten-millionth of the distance from the North Pole to the equator, along the meridian passing near Dunkirk, Paris, and Barcelona. One can see that such a definition would be difficult to replicate in any one country. Also, the length of the metre changes as the position of the North Pole changes; at the time that the metre was defined, scientists were unaware that the position of the North Pole changed.

In 1870, because of the problem of replicating and comparing metric units from country to country, France called a meeting of the metric countries to develop a "unified metric system of measurement". In 1875, the *Treaty of the Metre* was signed to establish the General Conference on Weights and Measures which meets to determine the official definitions for the units used in the metric countries. In 1960 the Conference adopted the Système International des Unités (SI). It is this SI metric system that is most used throughout the world.

A Popular System

The popularity of the metric system stems from two characteristics—the high degree of standardization and its simplicity.

In the entire metric system there are only seven base units! They are **metre** (length), **kilogram** (mass), **second** (time), **ampere** (electric current), **degree kelvin** (thermodynamic temperature), **candela** (luminous intensity), and **mole** (amount of substance).

All units used in the metric system are related to these seven base units. The units you will be most concerned with (because they are the ones used in everyday living) appear in Table 1:

Table 1: Metric Units to be Studied

Quantity	Unit	Symbol
Length	metre	m
Mass	kilogram	kg
Capacity	litre	ℓ*
Temperature	degree Celsius	°C

*As a rule of thumb, the cursive letter (ℓ) is used as a symbol for the litre to avoid confusion with the numeral (1), however, in symbols such as ml (millilitre), kl (kilolitre) the cursive form is not used.

All other units to be discussed can be represented by the product of one of the units and a power of 10. For example, every possible unit of length can be developed by multiplying the number of metres by the appropriate power of 10.

Table 2: Metric Units of Length

Name (Symbol)	Metres
*kilometre (km)	10^3m or 1000 m
hectometre (hm)	10^2m or 100 m
decametre (dam)	10^1m or 10 m
*metre (m)	10^0m or 1 m
decimetre (dm)	10^{-1}m or $\frac{1}{10}$ m
*centimetre (cm)	10^{-2}m or $\frac{1}{100}$ m
*millimetre (mm)	10^{-3}m or $\frac{1}{1000}$ m

*preferred units

To make the system simpler the same prefixes are used with all units. For example, a millimetre (mm) is $\frac{1}{1000}$ of a metre, a millilitre (ml) is $\frac{1}{1000}$ of a litre, a milligram (mg) is $\frac{1}{1000}$ of a gram, etc.

According to the class, you may want to introduce the symbol "m" for metre, "cm" for centimetre, etc. The plurals, metres and centimetres, are also symbolized "m" and "cm", not "ms" or "cms." Remember, these are symbols and not abbreviations and no period is used after a symbol.

Countries which have been completely metric for several years find that some terms such as "decimetre" are not used in everyday living. People will talk of a book being 28 centimetres long rather than 2.8 decimetres long. You may wish to explain the term "decimetre," but it is not necessary.

Most people who feel that the metric system is complex are those who convert back and forth between the metric and English systems of measurement. When teaching the metric system, conversion to the English system is not necessary and should be avoided!

The metre is defined world-wide to be 1 650 763.73 wave lengths in a vacuum of the orange-red line of the spectrum of krypton 86. This is quite a definition! There are two reasons why such a complex definition was adopted—

1. the length never varies and

2. this measurement can be replicated in laboratories throughout the world.

From this brief history of the metric system it is hoped you will take three main thoughts—

1. The metric system resulted from concentrated effort to develop a rational system of measurement. It did not develop haphazardly.

2. The problem of standardization has been solved in the metric system.

3. The metric system is both popular and useful because of its simplicity.

Activities

Experience and activity are key words in the teaching of measurement. Measure things! The success of this material will depend upon the amount of experience each participant has with the activities. The limited number of activities that are presented should stimulate possibilities for many more. Although the content is approached through activities and measuring experiences, there is a need for exercises to further these experiences and to structure metric thinking. Two points should be emphasized—

1. It is *important* that *you* as well as your class do the activities in this section.

2. The activities will be more fun if done in a group situation.

Looking at Table 1 in the *History and Rationale* section, you will notice that you have to be concerned with only four base units. So, let's use the frontal attack, start right in on length, and begin inching our way down the metric road.

Length, Area, and Volume

In the groups where the metric system has been argued for years, there were two camps. One group wanted to use the centimetre, gram, and second for the core of the system and the other the metre, kilogram, and second. The latter group has prevailed.

It is strongly urged that first grade teachers **not** start with the metre. It is very difficult for first graders to handle a metre ruler. The same argument may be advanced for the kilogram and litre. Length will be approached as it should be covered with students, i.e., first measure with arbitrary units, then use the centimetre, next use the 10-centimetre (decimetre), and finally the metre. All measurement should be approached as a three step process—

1. Select a unit.

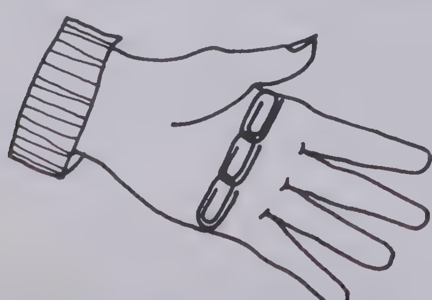
2. Partition the object to be measured into units.

3. Count the number of units used. That number is the measure of the object.

ACTIVITY 1

Measuring objects with an arbitrary unit. Students should do several activities of this type using arbitrary units such as their thumb, a paper clip, pencil, crayon, cutout of their shoe, width of their hand (a unit in the English system used for measuring the height of horses), cubit (another "English" unit, the length of the forearm from the elbow to the tip of the middle finger), or other selected units. For your experience measure the chalk eraser, the width of your hand, the width of this book, and the length of a pencil using a paper clip as the unit.

In the illustration, a "paper clip train" is being used to measure the width of a hand. Follow the three steps mentioned previously in the measurement process.



Record all answers. Then measure the object again using pieces of paper the length of a thumbnail. Repeat the process measuring other objects.

In class emphasize four points—

1. The first unit should be lined up with the "starting point" of the object.

2. The units should touch, but not overlap.

3. The "train" should be straight.

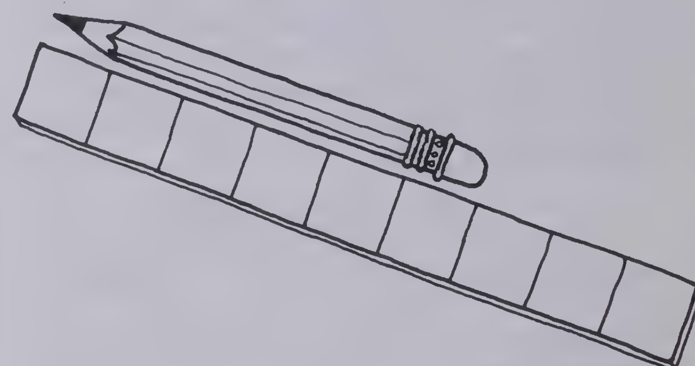
4. The units should be "rounded off" to the unit that has its right end nearest to the "finishing point" of the object.

In doing activities where arbitrary units are used, the need for standardized units becomes obvious. Ask several children to measure the same object, each with his own pencil. On the chalkboard, place their statements such as "The table (or whatever object you pick) is 5 pencils wide." "The table is 7 pencils wide." "The table is 8 pencils wide." Children will soon see that when pencils of differing lengths are used, different answers will result.

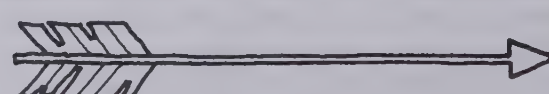
ACTIVITY 2

Developing the concept of a centimetre. Probably the first metric unit the children will make use of is the centimetre. You will need (and each student in the class will need) 9 centimetre strips—9 pieces of paper or cardboard 1 cm by 1 cm square.

The children, especially the younger ones, should have the experience of measuring many objects using centimetre strips. (If at the time you present this activity your students have studied two-digit numbers, have them measure objects longer than 9 cm.)

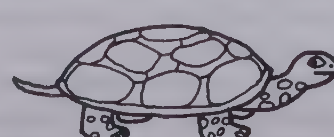


Using the centimetre strips, measure the length of a paper clip, a piece of chalk, the Cuisenaire 6-rod, the width of a hand, and the width of a thumb to the nearest centimetre. In this initial activity, actually use centimetre strips and not a ruler marked in centimetres. An exercise the children can do at their desks is to measure the pictures of objects drawn on a duplicator master. The pictures can be of predetermined length. Measure the pictures below.



The arrow is about ____ centimetres long.

The snail is about ____ centimetres long.



The turtle is about ____ centimetres long.

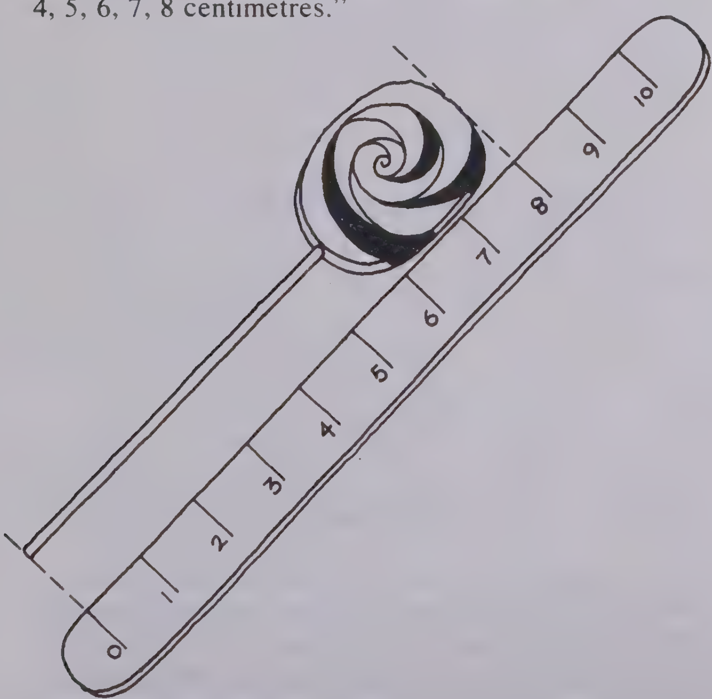
In exercises like these, the length can be controlled. Some answers should require "rounding up," and some "rounding down." The word "about" is important in the sentence since a measurement is an approximation. As the children progress you can have them write not only the number but also the name of the unit.

ACTIVITY 3

Measuring with centimetre rulers. When the children have learned to use the centimetre strips in the measurement process, a ruler marked off in centimetres (not millimetres) should be introduced. It is strongly urged that the child construct his own 10-cm ruler during his first introduction to metric measure. He can do this by constructing a 10-cm train on a 10-cm long piece of paper, pasting the train on the paper, then numbering the cars from 1 to 10. Another approach is to construct a 10-cm ruler in front of the class. Then hand out 10-cm long pieces of paper already marked off in centimetres and have the children number the centimetres from 1 to 10.

The next few activities should involve the measuring of an object with a centimetre train, a 10-cm ruler, and finally with only a 10-cm ruler. When measuring an object with a 10-cm ruler work toward getting your students to "read the ruler" rather than counting the centimetres as they did with the trains.

In the example illustrated the child should learn to round off to the nearest centimetre and then read the ruler, "8 centimetres," instead of counting "1, 2, 3, 4, 5, 6, 7, 8 centimetres."

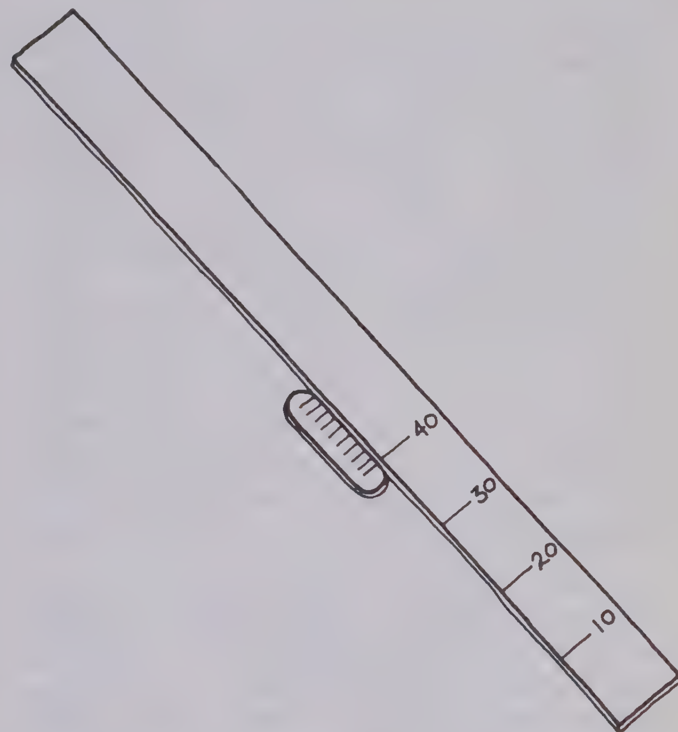


After the children have become skilled in using a 10-cm ruler, they should be given activities requiring them to measure objects which are longer than 10 cm. When working with 5-and-6-year olds, be careful that the measure of the object is not a number the children haven't studied. In the activities concerning measurement it is the process that should be emphasized; the numbers themselves should never be a source of difficulty.

Now, using your 10-cm ruler, measure the length and width of this book and length of your forearm, the length of your foot, and length of your span (what is your span?).

ACTIVITY 4

The metre and notation. Initially, you may want to have your students measure objects with metre-long strips of unmarked cardboard. Then ask them to number the centimetres on the metre strip in groups of 10 using their 10-cm strips. Before proceeding



further, have the class subdivide these cardboard metre rulers into centimetres. It is important that you do the activities with the same type of ruler your students will use. If you have a classroom set of wooden metre rulers, use one of them. Ideally, the rulers used should be marked off in centimetres, but if the ruler is marked off in centimetres (cm) and millimetres (mm) no harm is done. Measure the length, width, and height of your desk rounding off to the nearest metre.

The measurements for a desk, accurate to the nearest metre, might be 2 m long, 1 m wide, and 1 m high. Such measurements would not be helpful. The metre is used for much longer measurements, such as the length and width of the classroom, the playground, the school, the block, etc. To measure the dimensions of objects such as desks, tables, bookshelves, and people, a metre ruler may be used and the results recorded in centimetres. For example, a desk may be 152 cm long, 76 cm wide, and 74 cm high.

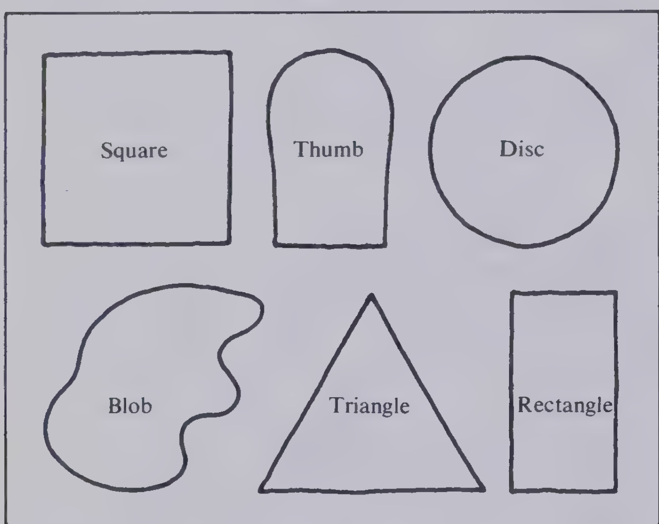
You might say: I am 178 cm tall; what is your height (in centimetres)?

Just as 153 cents is written as $1.53, 153 centimetres is written as 1.53 metres. This can be interpreted as 1 metre and 53 centimetres which is read as "one point five three" metres. Do not dwell on the mathematical use of the notation—it is not necessary!

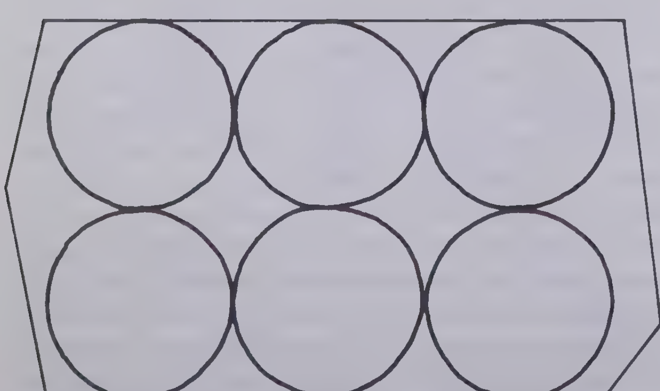
With your class, record the dimensions of your classroom, your desk, their desks, your height, and their heights in terms of centimetres, then in terms of metres using the decimal notation.

ACTIVITY 5

Area using arbitrary units. Here are some examples of area units:



Let the children give names to the units. Then follow the measurement process: select one of these units, match it against the area of some object, and count the number of units used. For example, the irregular figure below has an area of about 6 discs



(if disc is the name given to the unit used). Emphasize that you are trying to "cover" the object. The units should be "even with the edge" of the object, the units should touch, but not overlap, each other. Direct the children's attention to the parts of the object that are not "covered."

Make a cutout of some irregular area such as your thumb and make copies of it out of paper. Use your "thumb" to find the area of the top of a chalk eraser, of the irregular figure measured with the discs, of a cutout of your shoe, and of figure X.

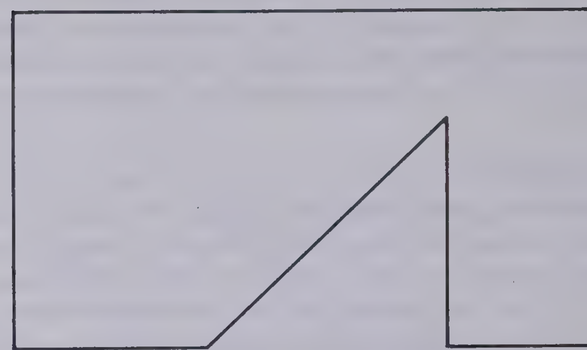


Figure X

Record the answers on the chalkboard in sentence form—

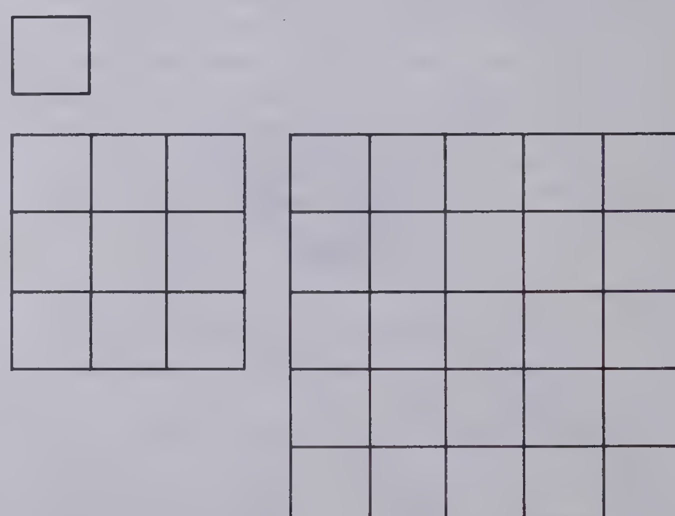
"The figure has an area of about ______ thumbs."

Have your class perform similar activities.

ACTIVITY 6

Area using the centimetre square (cm^2). Have the children make centimetre squares (or have them available for use). The children should have the experience of finding the area of many objects.

Make duplicator masters for some areas that the class can measure with their centimetre squares. The figures below are 1 cm^2, 9 cm^2, 25 cm^2, respectively.



You might point out that the square containing the 9 cm^2 has a side of 3 cm and the square containing the 25 cm^2 has a side of 5 cm.

Have the children use their centimetre squares to find the *area* of a stamp, a 10-cm ruler, the cutout of their thumb, the irregular figure which had an area of 6 discs, and figure X.

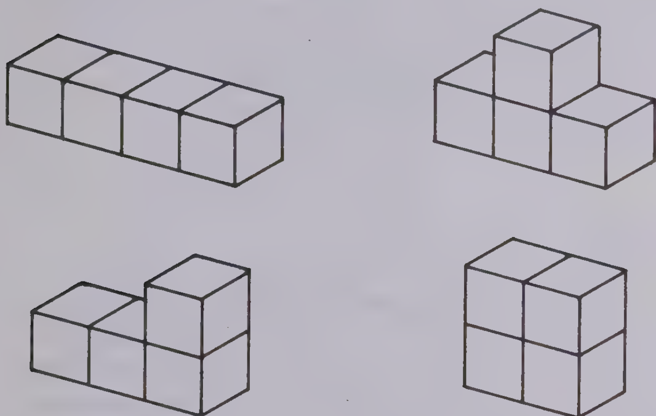
ACTIVITY 7

Volume, using the centimetre cube. In the initial development of the concept of volume, it is important

that children have the opportunity to construct several differently shaped objects each having the same number of volume units.

As with length and area, the study of volume should be introduced with activities making use of arbitrary units of volume, such as blocks, Cuisenaire rods, pencils, erasers, or even marbles.

Use 10 or 12 centimetre cubes in this activity. At first, let the children work on their own, constructing any objects they like. Encourage them to see that an object built of a specific number of cubes has a volume of the same number of cubes regardless of its shape. For example, the illustration shows 4 different constructions, each having a volume of 4 centimetre cubes (4 cm^3).



How many differently shaped objects can be constructed with a volume of 8 centimetre cubes? When those possibilities have been exhausted, try the activity with 10 cubes.

REVIEW: LENGTH, AREA, AND VOLUME

1. Have your class compare the length of their feet, spans, and cubits. Why are these units useless as standard units?
2. Complete these statements.
   - **a.** 128 cm = ____m **e.** 1.06 m = ____cm
   - **b.** 108 cm = ____m **f.** 10.01 m = ____cm
   - **c.** 15 cm = ____m **g.** 23.86 m = ____cm
   - **d.** 1010 cm = ____m **h.** 0.09 m = ____cm
3. What would be the length of the sides in a square containing:
   - **a.** 36 cm^2 $\rightarrow$ ____ cm?
   - **b.** 25 cm^2 $\rightarrow$ ____ cm?
   - **c.** 4 cm^2 $\rightarrow$ ____ cm?
   - **d.** 16 cm^2 $\rightarrow$ ____ cm?
4. How many different-shaped objects can you form with 6 centimetre cubes?

Capacity

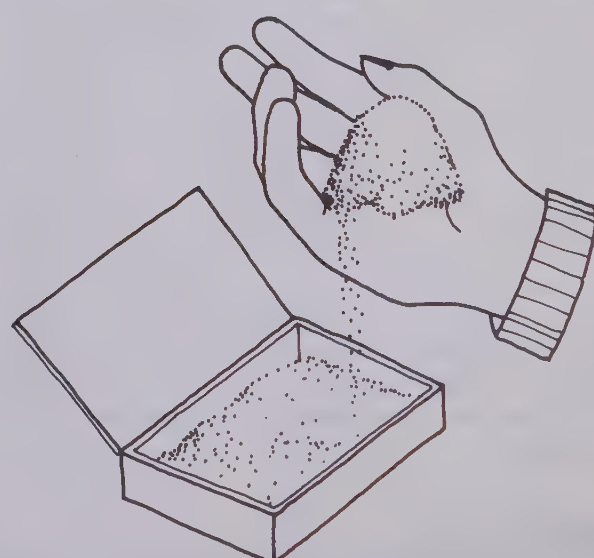
Capacity can be thought of as the amount of material a container will hold. Capacity is usually linked to liquid measure though you may have already had your classes measure capacity by using sand to avoid using liquids.

In the metric system of measurement, volume and capacity are directly related. A container with a volume of 1 cubic centimetre (1 cm^3) will hold 1 millilitre of water. One millilitre (1 ml) is one thousandth of a litre (0.001 ℓ).

The need for fractional names such as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{4}$ etc. will diminish. The parts of the whole which need emphasis are—0.1, 0.2, 0.3, . . . , 0.9. Of course, in measurement, fractions could disappear completely, since $\frac{3}{8}$ of a meter is 0.375 m or 375 mm. However, when working with the litre (the unit of capacity in the metric system) don't worry now about using $\frac{3}{4}\ \ell$, $\frac{2}{3}\ \ell$, etc. if it is the amount you want the children to see or work with. Since the metric system is based on 10 and since 1, 2, 5 and 10 are the only divisors of 10, we will probably talk about halves, fifths, and tenths of metric units. The decimal notation ($\frac{1}{2}$ is 0.5) will prevail eventually, even at the primary level.

ACTIVITY 8

Capacity and arbitrary units. The most obvious capacity units are handfuls. Give each child a container to fill with water or sand or other material you prefer to use. Have the children fill the container



(milk carton, ice cream carton, cigar box, etc.) with "handfuls" of material. Have them record their results on a piece of paper: "My carton holds ______ handfuls of ______." Compare the wide range of results. Re-emphasize the need for a standard unit to measure capacity. If further experience is necessary, you may want to repeat the project with cups brought from home (since there are so many different sized and shaped cups). Try the activity yourself or get several containers such as an ice cream carton, a milk carton, a wastebasket, a big cooking pan, and a litre container.

On a piece of paper write a pair of sentences for each container:

"The (name of container) holds about (guess) litres.

The (name of the container) actually holds (result) litres.

In the first blank "guestimate" the number of litres the container will hold. In the second, write in the results of measuring the object.

Don't forget the three step measuring process—

1. Select the unit—the litre.
2. Match the unit against the object—fill the object using the litre.
3. Count the number of units (litres) used.

When the container is full (it is best to have a "fill line" just below the top of the container) round off to the nearest whole litre according to whether more or less than half of the last litre was used.

ACTIVITY 9

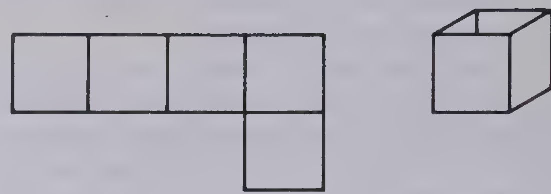
Working with the litre. Get a container that holds a litre of water(and, ideally, has submarkings for each 100 ml). When you are collecting containers for your classroom, try to get as many different shapes as you can. It is important, especially in early experiences, that the children see that litre containers can come in many different shapes. It is the quantity the container will hold, not its shape that determines a capacity of 1 litre.

Once you get a litre container you can make many more. Pour a litre of water into a container and mark a "fill line" for 1 litre on the outside with tape, or, if possible, cut the container so that it holds just 1 litre. Suggested existing containers which can be cut are quart, half-gallon, and gallon milk cartons, round quart, half-gallon, and gallon ice cream cartons. Containers that can be marked might be various shaped pans, cooking bowls, large tin cans, and bottles or jugs. Most activities for introducing the metric units should be accompanied by some estimation exercises. Have the students estimate and record how many litres a container will hold, then measure the container to see about how many litres it does hold. Compare records.

ACTIVITY 10

Introducing the millilitre. The litre is a unit for capacity that is used for milk, gasoline, paint, and other quantities of considerable size. The litre is not used to measure small quantities, such as toothpaste, soda pop, medicines, frozen orange juice, etc. The unit used for the smaller measures is the millilitre (ml). If your school is going to get a set of metric capacity containers, try to get them in these sizes—1 ℓ, 500 ml, 200 ml, 100 ml, 50 ml, 20 ml, and 10 ml. With such a set (whether bought, given, or constructed) one can do all the activities that are necessary.

Construct a container with a volume of 1 cubic centimetre (1 cm^3) to demonstrate the size of the millilitre (ml). Trace the figure below, then cut it out and tape it together along the edges. If you avoid spillage your cube will hold 1 ml of water.



The children need several activities measuring the capacity of objects and recording the results in millilitres. Have them first guess and then measure the capacity of a thimble, a match box, a tablespoon, and a teaspoon. Record the results in sentences like—

"I estimate that the thimble holds about ______ ml. It actually holds about ______ ml."

Mass

As the metric system becomes the predominant system of measurement you may hear talk about the difference between mass and weight. A lunar example may be the best way to show the difference. Now that we are in the space age, practically everyone knows that a man weighs less on the moon than he does on the earth. For example, a 300-kg man on earth would weigh about 50 kg on the moon, but he would have the same mass on the moon as he does on earth. Weight is dependent upon gravity, mass is not. Begin to stress the use of the correct metric term, mass.

The base unit of mass in the metric system is the kilogram (kg). For example, we say "I have a mass of 78 kg.

ACTIVITY 11

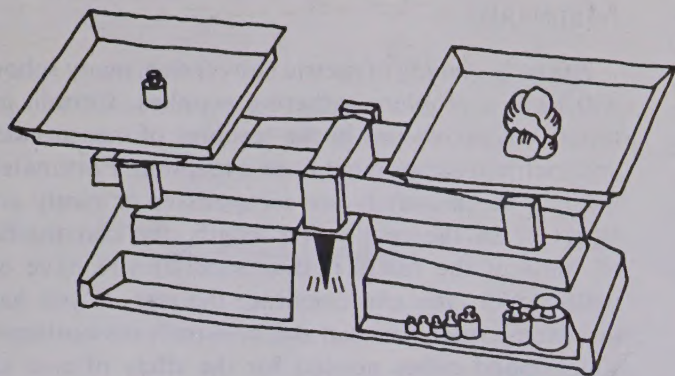
Arbitrary units of mass. To find the mass of an object you will need a balance and some arbitrary units such as paper clips, pencils, Cuisenaire rods, pennies, or other objects. Put a pencil on one side of the beam and then "balance the pencil" with pennies (or multiples of any other small unit). Record the results on paper in a sentence like:

"The pencil has a mass of about ______ pennies."

Repeat the activity with at least three other objects.

ACTIVITY 12

The unit used for small masses is the gram (g). This activity is very similar to the last. You will need gram masses. If you have a classroom set, that's great! If you don't, you can make one.



Put a gram mass on one side of the balance and balance it with a lump of clay or plasticine. Label your clay "1 g." In a similar manner make a set of clay or plasticine "masses" in multiples such as: 5 g, 10 g, 20 g, and 50 g. Use several small objects as test objects (a paper clip, a nickel, a penny, and a pencil). However, before you have the children put one of the test objects on the balance, ask them to estimate its mass in grams. Then find the mass of the object. Record both the guess and the result.

The quarter has a mass of about (guess) grams.
It actually has a mass of (result) grams.

Repeat the activity using other objects. Do you and the class get better at estimating mass?

ACTIVITY 13

Measuring mass using the kilogram. Hopefully, all schools will have metric scales available for finding the mass of children and other large objects using kilograms. For this activity, have each child find his own mass and then make and label a cutout of himself (perhaps using his projected shadow). Have him record his height and mass in metric units on the cutout.

Then you and your class might measure the mass of other objects, such as your own chairs, the textbooks used in the course of one day, litre of water (don't count the container—first find its mass when empty), a dictionary, and even the principal of the school (if he agrees). As mentioned earlier, there is a direct relationship between volume and capacity in the metric system of measurement. In fact, there is a direct relationship between volume, capacity, and mass. A container whose volume is 1 cubic cm (cm^3) holds 1 ml of water and the 1 ml of water has a mass of 1 g. A container whose volume is 1000 cubic cm (or 1 cubic decimetre) holds 1000 ml of water (or 1 litre), and the water has a mass of 1000 g (or 1 kilogram). What did you get for the mass of one litre of water?

N.B. It is a good idea to label some of the objects in the room as you introduce each metric unit. For example, the aquarium may be 70 cm long, 40 cm wide, 35 cm high; have a water surface area of 2 800 cm^2, volume of 98 000 cm^3; a capacity of 98 ℓ of water and a mass of 12 kg. If the children label the objects as they study particular units, they will begin to think metric.

REVIEW: CAPACITY AND MASS

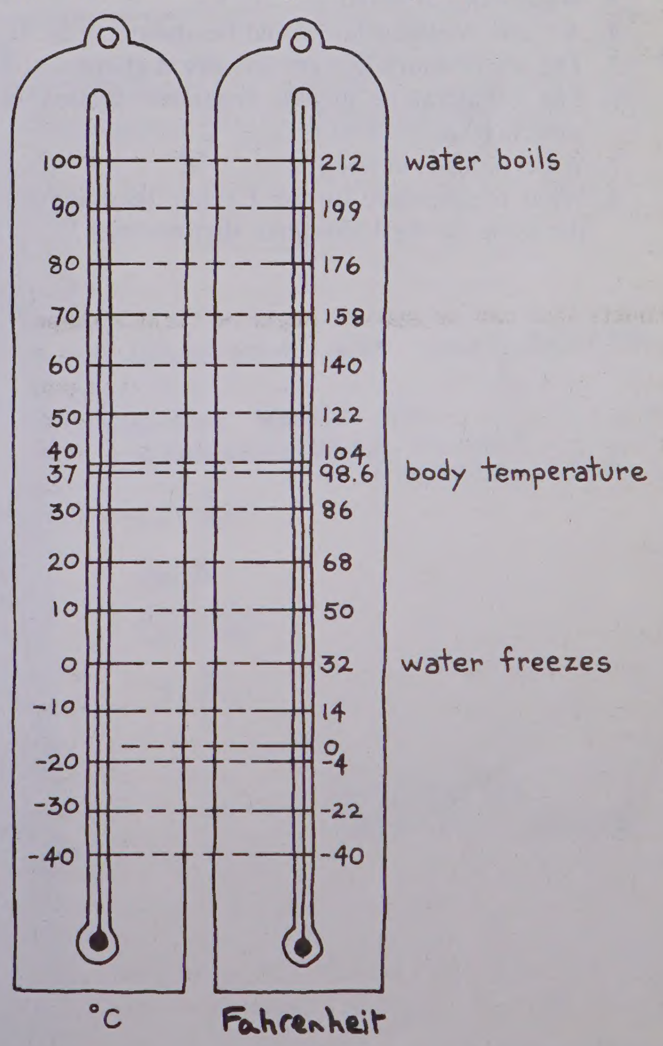
1. When finding the mass of something using a balance beam, how do you decide which unit to round off to?

2. Fill in the answers:
- **a.** 28 ml of water has a mass of about ______ grams.
- **b.** 170 ℓ is ______ ml.
- **c.** 3.12 kg is ______ g and 438 g or ______ kg.
- **d.** It would take ______ ml of water to balance 1 kg.

3. **a.** Will a car get a higher or a lower number of miles per litre than miles per gallon? (Is the litre larger or smaller than the gallon?)
- **b.** Will a car get a higher or a lower number of kilometres per gallon than miles per gallon? (Is the kilometre longer or shorter than the mile?)
- ★**c.** Gasoline consumption rates will be given in kilometres per litre. Will a car get a higher or a lower number of kilometres per litre than miles per gallon?

Temperature

This last section covers the introduction of a metric unit, the degree Celsius (°C), for which there is no physical model. On the Celsius scale for temperature, water boils at 100° C and freezes at 0° C. The unit is named after the Swedish scientist, Anders Celsius, who created the centigrade temperature scale. The



Celsius and centigrade scales are the same, but centigrade is no longer the proper term since the centigrade is a unit used to measure angles in the metric system.

The best way to get used to the Celsius temperature scale is to use it! It is almost a necessity that you have a Celsius thermometer. However, if you have a demonstration model of the Fahrenheit thermometer, you can rescale it using the nomograph shown here.

ACTIVITY 14

Graphing temperatures. Be sure to give the children lots of opportunities to read the temperature and record it in degrees Celsius (°C). Perhaps you could institute a morning weather report given by a different child each day to get the class to use Celsius thermometers and to give them a feeling for what the temperature is when expressed in degrees Celsius (°C). The previous day's high and low temperatures (taken from a newspaper account) could be recorded on a wall graph.

REVIEW: TEMPERATURE

1. My body temperature is about ______ °C.
2. Normal room temperature is about ______ °C.
3. Water boils at about ______ °C.
4. A warm summer day would be about ______ °C.
5. The temperature in a refrigerator is about ____ °C.
6. The temperature in the vegetable section of a supermarket is about ______ °C.
7. Water freezes at about ______ °C.
8. What temperature on the Celsius thermometer is the same on the Fahrenheit thermometer? ____ °C

Materials

At the beginning of metric conversion, many schools will have a problem gathering supplies. Certain materials are necessities in the teaching of measurement and metric measurement is no exception. Fortunately, most of the materials are inexpensive or easily constructed. In the section on length, the construction of some of the rulers is discussed. If you have one metric ruler, you can construct the rest. If you have one metric ruler, you can also construct the centimetre squares and cubes needed for the study of area and volume.

The construction of units of capacity and mass have also been discussed. When it comes to temperature you should have a thermometer available for classroom use. If it is a Fahrenheit thermometer, then you should rescale it to degree Celsius using the nomograph given earlier.

Following is a list of companies and government agencies that are currently producing materials or can give some assistance with this problem of teaching the metric system of measurement.

Addison-Wesley (Canada) Ltd. – Don Mills, Ontario
Buntin Gillies & Co. Ltd. – Ottawa, Ontario
Cameron Products – Bramalea, Ontario
Canadian Metric Association – (P.O. Box 35) – Fonthill, Ontario
Contrasts 20 – Calgary, Edmonton, Vancouver, Winnipeg, Regina (Nearest Barber-Ellis Office)
Kruger Pulp and Paper Ltd. – Moncton, Toronto, Hull, Montreal (Nearest Office)
Information Canada (Under Government of Canada) (Nearest Office)
Jack Hood School Supplies Co. Ltd. – Stratford, Ontario
Lufkin Rule Co. of Canada Ltd. – Don Mills, Ontario
Lily Cups Ltd. – Scarborough, Ontario
MacLean-Hunter Learning Materials Co. – Toronto 101, Ontario
Metric-Aids Ltd. – Toronto, Ontario
Moyer-Vico Ltd. – Moncton Weston, Winnipeg, Saskatoon, Edmonton, Vancouver and the Longueuil Co. in Chambly (Nearest Office)
The National Council of Teachers of Mathematics – 1906 Association Drive, Reston, Virginia 22091
Sargent-Welch Scientific Co. of Canada Ltd. – Weston, Ontario
Spectrum Education Ltd. – Toronto, Ontario
Spicars International Ltd. – Scarborough, Ontario
Toronto Dominion Bank (Nearest Office)

Three		ok Five	Book Six	
3 umerals: 30 nting sequences: 32, 36 iparing sets: 58 at's My Rule": 68 ction Machine: 70, 185, 215, 268, 28 iplication and sets: 122 counting sequences: 146–155 iplication patterns: 158 luct sets: 162 sion and sets: 170, 186–188, 276 n and intersection of sets: 184, 310 and even numbers: 214–217 mal logic: 16, 24, 61, 92, 134, 146–155, 6, 189, 296–297		8–11 quences: 58–61 section of sets: 176–179 nt fractions: 190–195 sequences: 212–213, 232 pairs: 302–319	Naming sets: 4–5 Classifying attribute pieces: 6–11 Logical reasoning: 10–11 Patterns: 12–15, 216–217 Union and intersection of sets: 130–135 Fractions and sets: 142–145 Sets of equivalent fractions: 144–145 Repeating decimal sequences: 248–249 Sets of ordered pairs: 282, 332	A
	Inequalities: 44–45, 81, 95, 225, 228–229, 231, 235–236, 240–243, 330–333 Base-ten machine: 52–53 Reading large numbers to 9 places: 48–51 Negative numbers: 312–313	bers: 20–21 cepts: 22–27 31, 214–215 Numeration systems other than base ten: 32–37 Fraction numerals: 186–189 Decimal numerals: 252–257 Integers: 316–319	Symbols for numbers: 18–19 Inequalities: 20–21, 150–153, 214–215, 228–229, 349 Billions: 22 Rounding numbers: 24–25, 234–235 Exponents and scientific notation: 26–29, 34, 96–97, 252–253 Number bases other than base ten: 30–33, 35 Decimals and scientific notation: 252–253 Integers: 340–352	B
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